Electron interactions in graphene in a strong magnetic field

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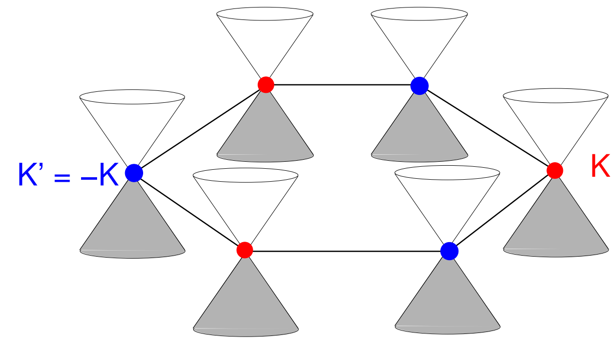


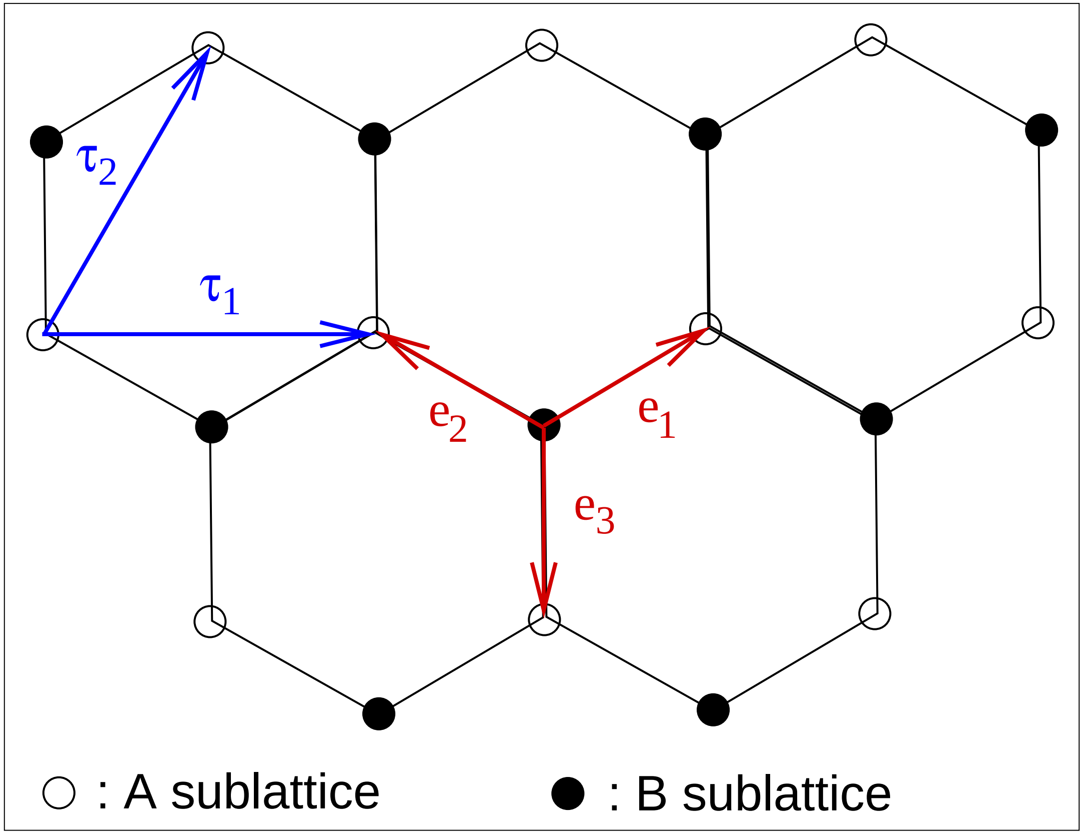
cond-mat/0604554

Overview

- Recent experiments: integer QHE in graphene
- Graphene background
  - band structure
  - length/energy scales
- Harper equation and continuum theory
- Interactions and SU(4) (spin $\times$ chirality) symmetry
  - what's special about $n = 0$
  - pseudopotentials: low and high Landau levels
  - easy-plane anisotropy due to 'backscattering'
  - effective stiffness
- Outlook

What is graphene?

- Graphene = $2D$ graphite
- Graphite = stack of weakly coupled graphene sheets
- Honeycomb lattice = triangular lattice with two-atom basis

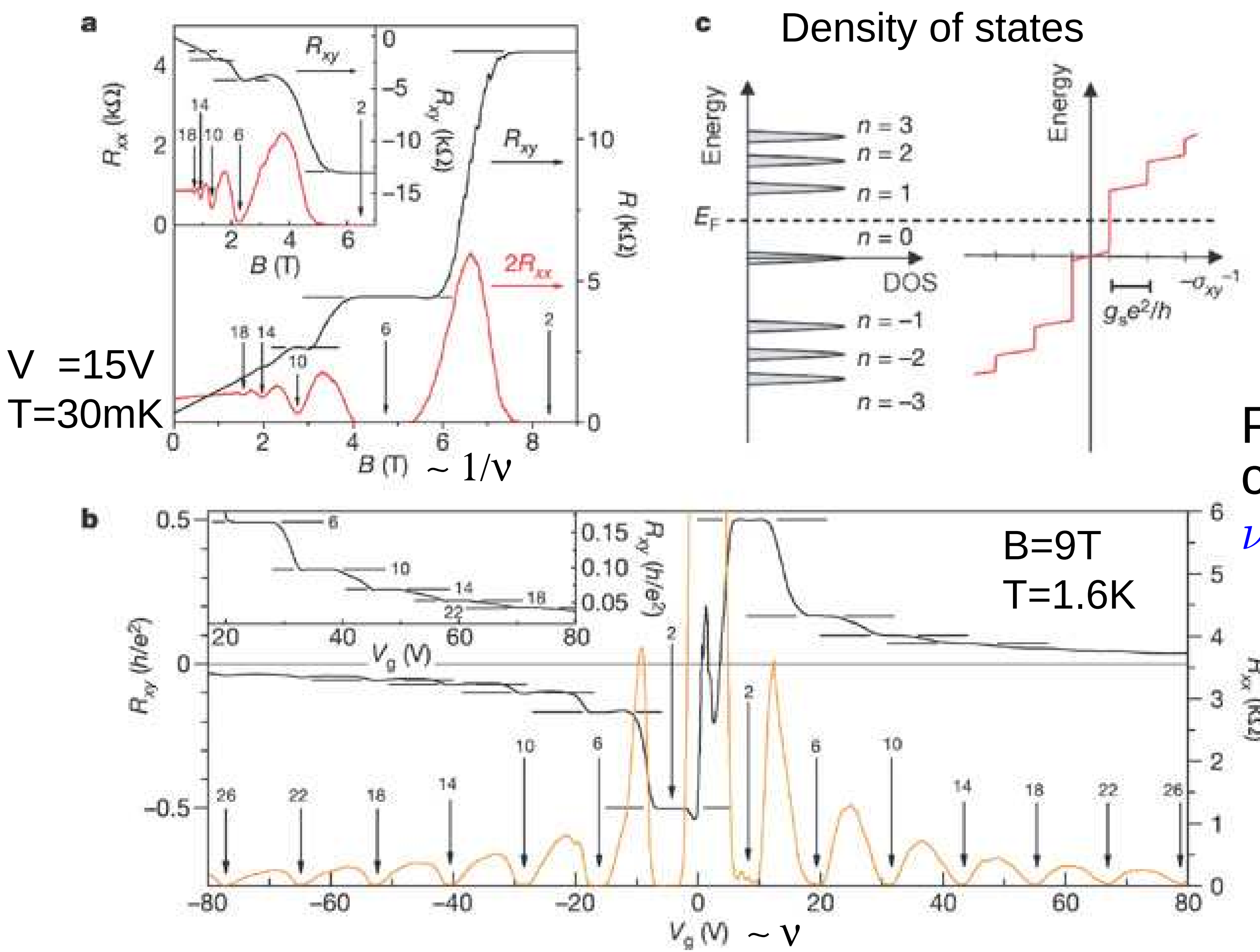


- Low-tech sample preparation: Scotchtape
- Technical difficulty: good contacts

IQHE in graphene

Novoselov et al., Nature 438, 197 (2005)

Zhang et al., Nature 438, 201 (2005)

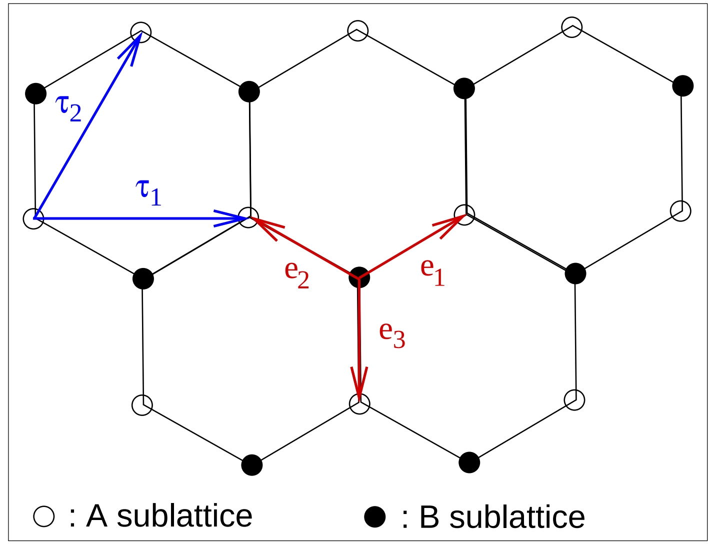


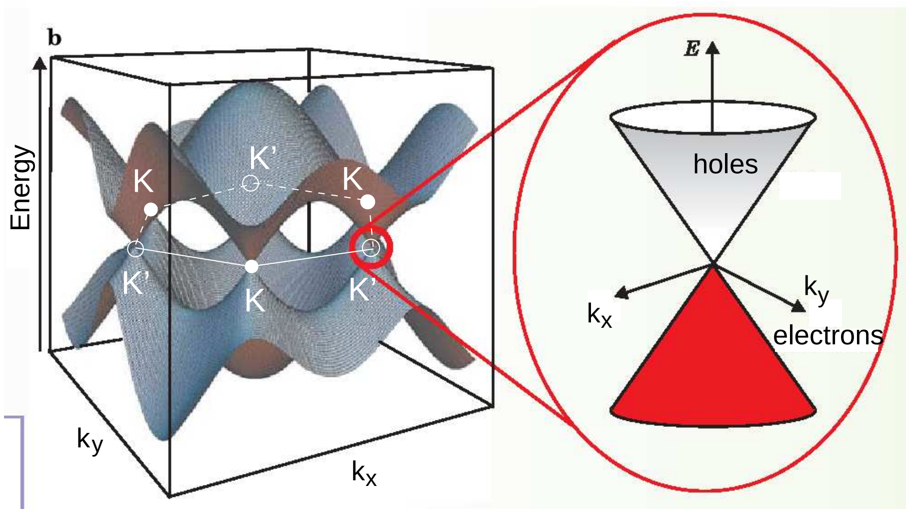
Plateaux corresp. to $\nu = 4n + 2$

Graphene band structure *papers from the 1950ies*

- Band-structure calculation in the tight-binding model

$$H_0 = -t \sum_{i \in A} \sum_{j=1}^{3} \left(b^\dagger_{\mathbf{R}_i + \mathbf{e}_j} a_{\mathbf{R}_i} + \mathrm{H.c.} \right)$$

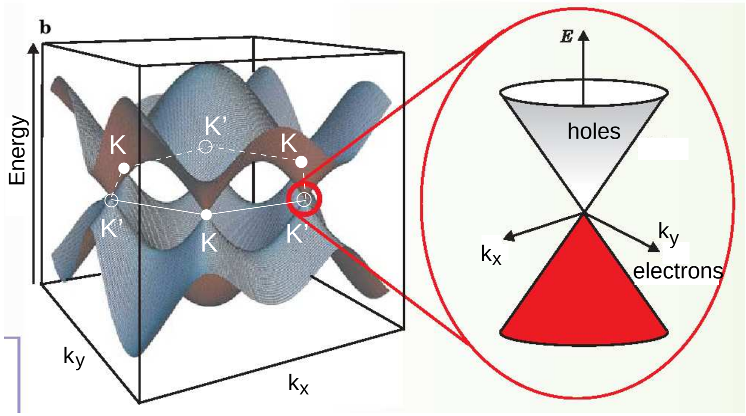




Energy dispersion:

$$\varepsilon_{\mathbf{k}} = \pm t \sqrt{\left[\sum_{j=1}^{3} \cos(\mathbf{k}\cdot\mathbf{e}_j)\right]^2 + \left[\sum_{j=1}^{3} \sin(\mathbf{k}\cdot\mathbf{e}_j)\right]^2}$$

Continuum theory with no magnetic field



- Zero-energy states:

$$\varepsilon^{\pm}_{\mathbf{K}} = 0 \Leftrightarrow \sum_{j=1}^{3} \cos(\mathbf{K}\cdot\mathbf{e}_j) = \sum_{j=1}^{3} \sin(\mathbf{K}\cdot\mathbf{e}_j) = 0$$

at K and K' points of the 1st BZ

- Continuum limit $\mathbf{k} = \mathbf{K}^{\pm} + \boldsymbol{\kappa}$ with $|\boldsymbol{\kappa}| \ll 1/a$:

$$\mathcal{H}^{\pm}(\boldsymbol{\kappa}) = \frac{3}{2}ta \begin{pmatrix} 0 & \kappa_1 \mp i\kappa_2 \\ \kappa_1 \pm i\kappa_2 & 0 \end{pmatrix} = \hbar v_F \left(\kappa_1\sigma^1 \pm \kappa_2\sigma^2\right)$$

- Energy dispersion (two-fold degenerate, chirality $\alpha = \pm$):

$$\varepsilon^{\alpha=\pm}_{\boldsymbol{\kappa}} = \pm\hbar v_F|\boldsymbol{\kappa}|$$

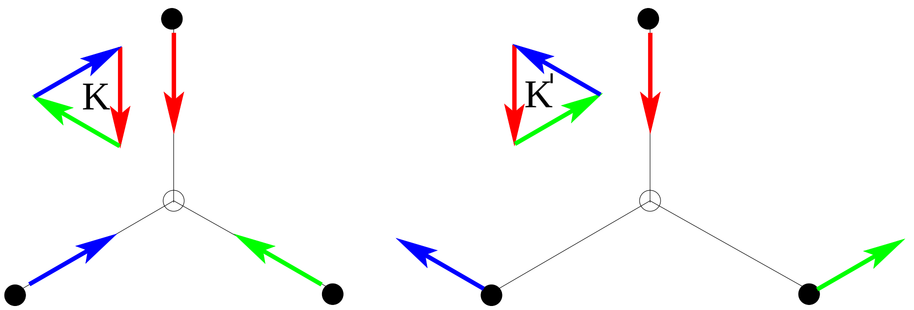
Zero-energy states at Dirac points

- Wavefunction $|\psi\rangle = \sum_i f_i |i\rangle$
- Zero-energy states: $H|\psi\rangle = 0 \Rightarrow \sum_{j:i} f_j = 0$
- Can choose wavefunction amplitudes $f_i = |f_i| \exp(i\phi_i)$ to be non-zero on $\mathcal{B}$ sublattice only
- For $|f_j|$ constant, require $\sum_{j:i} \exp(i\phi_j) = 0$

$\Rightarrow$ GS of triangular XY model

- These are distinguished by
  - irrel. global phase
  - chirality $= \pm \Leftrightarrow K,\ K'$

  $\Rightarrow$ Location of Dirac points



“Naïve” continuum theory with magnetic field (I)

- Usual route: Peierls substitution + minimal coupling:

$$\mathbf{k} \to \frac{\mathbf{p}}{\hbar} \to \frac{1}{\hbar}(\mathbf{p} + e\mathbf{A}) \equiv \frac{\mathbf{\Pi}}{\hbar}$$

- Non-commuting momenta (magnetic length: $l_B = \sqrt{\hbar/eB}$):

$$[x_\mu, p_\nu] = i\hbar\delta_{\mu,\nu} \Rightarrow [\Pi_x, \Pi_y] = -i\hbar^2/l_B^2$$

- Ladder operators $[a, a^\dagger] = 1$ as usual:

$$a = \frac{l_B}{\sqrt{2}\hbar}\left(\Pi_y + i\Pi_x\right), \qquad a^\dagger = \frac{l_B}{\sqrt{2}\hbar}\left(\Pi_y - i\Pi_x\right)$$

“Naïve” continuum theory with magnetic field (II)

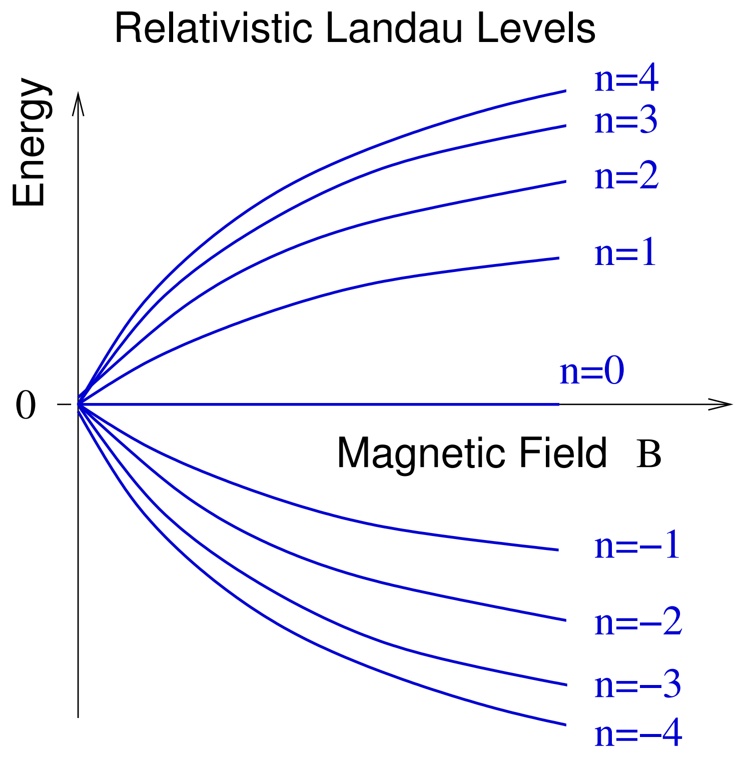
- 2×2 matrix Hamiltonians for each chirality (at K and K'):

$$H_K = \sqrt{2}\frac{\hbar v_F}{l_B}\begin{pmatrix} 0 & a^\dagger \\ a & 0 \end{pmatrix}, \qquad H_{K'} = \sqrt{2}\frac{\hbar v_F}{l_B}\begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

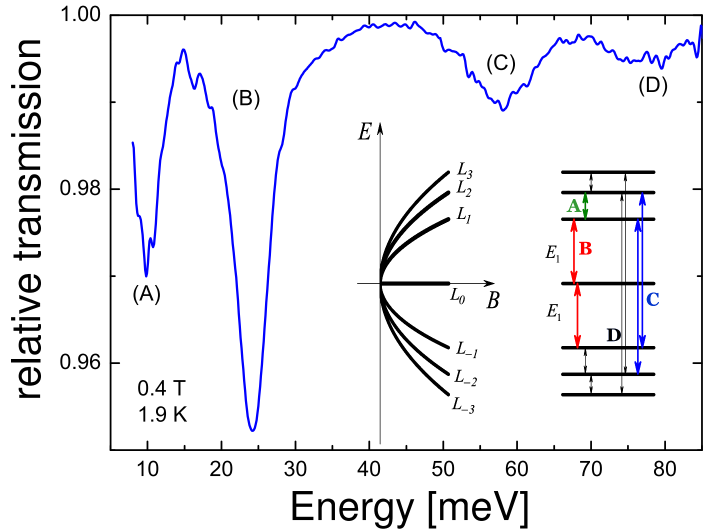
- Energy dispersion (degenerate in chirality quantum number α):

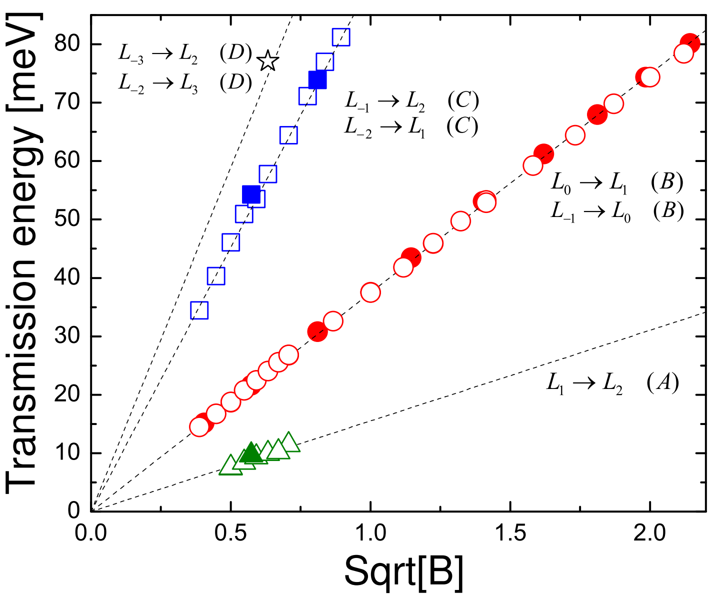
$$\epsilon_n = \pm\hbar\frac{v_F}{l_B}\sqrt{|n|} \propto \sqrt{B|n|}$$

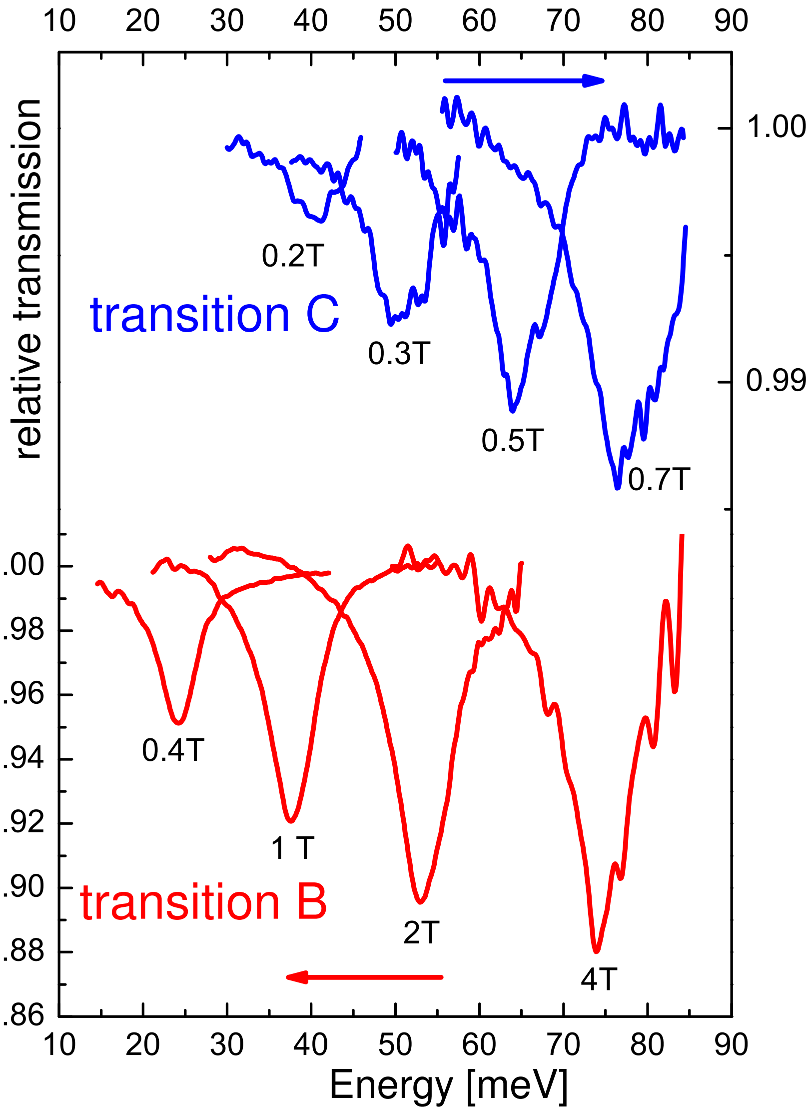
[Relativistic Landau levels (LLs)]



Infrared transmission spectroscopy







Sadowski et al., cond–mat/0605739

Degeneracy of relativistic Landau level

- Guiding center $\mathbf{R} = (X, Y)$ algebra as in non-relativistic case:
  $[H, \mathbf{R}] = 0, \qquad [X, Y] = il_B^2,$
- Degeneracy generated by $2^{\rm nd}$ set of ladder operators $b, b^\dagger$

$\Rightarrow$ Degeneracy: $N_\phi = A/2\pi l_B^2$ as per usual; $E(n) \propto \sqrt{nB}$

- Filling factor: $\nu = N_{el}/N_\phi$ measured from particle-hole symmetric point
- Chirality $\alpha = \pm$ and spin $\sigma = \uparrow, \downarrow$

  $\Rightarrow$ internal SU(2)$\times$SU(2) space

$\Rightarrow$ 4 copies of each Landau level

$\Rightarrow$ possibility of SU(4) symmetric Hamiltonian

Wavefunctions

- States $|n,m;\alpha\rangle$ are 2-spinors (entries: sublattice):

$$|n,m;+\rangle = \begin{pmatrix} ||n|,m\rangle \\ \mathrm{sgn}(n)||n|-1,m\rangle \end{pmatrix}$$

$$|n,m;-\rangle = \begin{pmatrix} \mathrm{sgn}(n)||n|-1,m\rangle \\ ||n|,m\rangle \end{pmatrix}$$

- Wavefunctions $|n,m\rangle$ are those of non-relativistic case!
  - both $|n,m\rangle,\ |n-1,m\rangle$ occur in the spinor
- Special case $n=0$:
  - electrons at K (K') live on $\mathcal{A}$ ($\mathcal{B}$) sublattice only
  - chirality=sublattice index

Continuum limit via Harper equation

- Define wavefunctions $g_{\{\mathcal{A}/\mathcal{B}\}}$ for sublattices $\mathcal{A}/\mathcal{B}$
  Near K ($l_B/a \gg 1,\ a \equiv 1$):

$$
\begin{aligned}
E g_{\mathcal{A}}(x) &= -2\cos\left\{2\pi/3 + (\sqrt{3}/2)\left[q_y + (x+1/4)B\right]\right\} g_{\mathcal{B}}(x+1/2) - g_{\mathcal{B}}(x-1) \\
E g_{\mathcal{B}}(x) &= -2\cos\left\{2\pi/3 + (\sqrt{3}/2)\left[q_y + (x-1/4)B\right]\right\} g_{\mathcal{A}}(x-1/2) - g_{\mathcal{A}}(x+1)
\end{aligned}
$$

- In Landau gauge $\mathbf{A} = Bx\mathbf{e}_y$; q_y is good quantum number
- Problem: Bx is unbounded, no matter how small B

$\Rightarrow$ For given $q_y \in 1^{st}\mathrm{B.Z.}$, define auxiliary 'q.n.' m such that $\sqrt{3}/2(q_y + Bx_m) = 2\pi m$ and write $x = x_m + \delta x$

- Strategy
  - solve for given $\{m, q_y\}$ assuming δx small
  - check consistency of solution with assumption

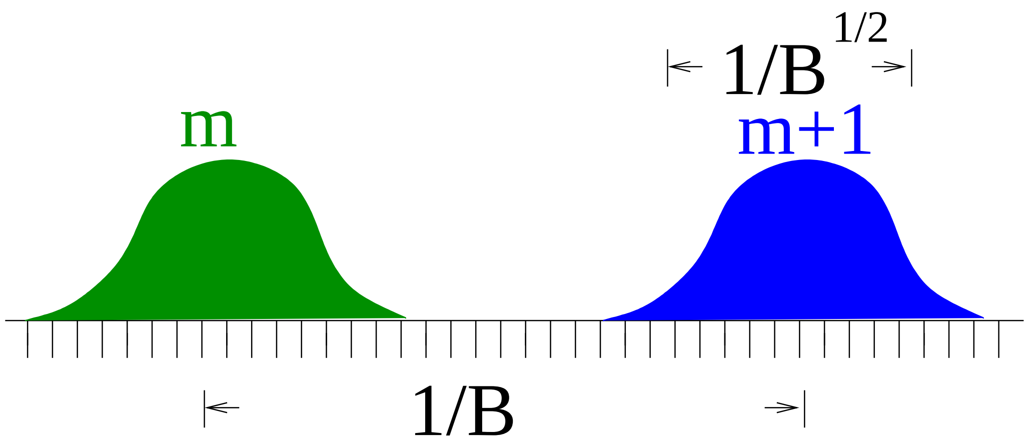
From Harper to Dirac

- Expansion near K, K' gives Dirac equation:

$$\begin{aligned} Eg_\alpha(x) &= -(3/2)(d/dx \pm B\delta x)g_\beta(x) \\ Eg_\beta(x) &= -(3/2)(-d/dx \pm B\delta x)g_\alpha(x) \end{aligned}$$

- Tunnelling between solutions $m \neq m'$ suppressed?

$\Rightarrow$ Require: $R_L = \sqrt{n} l_B \sim \sqrt{n/B} \ll 1/B \sim \Delta x_n$



- Equivalent to $E \ll t$ and $\rho \ll 1/a^2 \sim 10^{15} cm^{-2}$

Length and energy scales in graphene

Length scales

- Distance between neighbouring carbon atoms: $a = 0.14$ nm
- Magnetic length: $l_B = 26\text{nm}/\sqrt{B[\mathrm{T}]}$
- Larmor radius: $R_L = \sqrt{n} l_B$

Energy scales

- Band width: $t = 2.7$ eV
- Landau level 'spacing': $\hbar v_F / l_B = 3ta/2l_B \sim 20\sqrt{B[\mathrm{T}]}$ meV
- Landau level dispersion: $\exp(-a/R_L)$
- Zeeman splitting: $\Delta_z = g\mu_B B \sim 0.1 B[\mathrm{T}]$ meV
- Interaction energy: $e^2/\epsilon l_B \sim 2.4...12\sqrt{B[\mathrm{T}]}$ meV
- Lattice effects (anisotropies, etc.): $a/l_B \sim 0.005\sqrt{B[\mathrm{T}]}$

Interaction model – densities

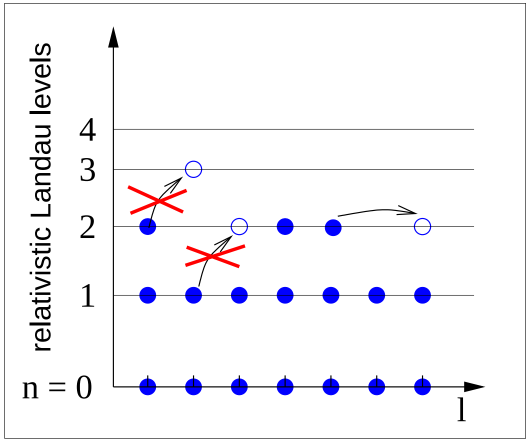
- Electrons in a single relativistic LL at $\nu \neq n$ (no spin):

$$H = \frac{1}{2}\sum_{\mathbf{q}} V(q)\rho^n(-\mathbf{q})\rho^n(\mathbf{q}), \qquad V(q) = \frac{2\pi e^2}{\epsilon q}$$

$$\rho^n(\mathbf{q}) = \rho^n_{\mathcal{A}}(\mathbf{q}) + \rho^n_{\mathcal{B}}(\mathbf{q})$$

$$\rho^n_\tau(\mathbf{r}) = \sum_{\alpha,\alpha'} \psi^\dagger_{n;\alpha;\tau}(\mathbf{r})\psi_{n;\alpha';\tau}(\mathbf{r})$$

with $\tau = \mathcal{A}/\mathcal{B}$, $\alpha = \pm$ (chirality)



- Projected densities $\rho^n(\mathbf{q}) = \sum_{\alpha,\alpha'} F_n^{\alpha\alpha'}(\mathbf{q})\bar{\rho}^{\alpha\alpha'}(\mathbf{q})$:

$$\bar{\rho}^{\alpha\alpha'}(\mathbf{q}) = \sum_{m,m'} \langle m|e^{-i[\mathbf{q}+(\alpha-\alpha')\mathbf{K}]\cdot\mathbf{R}}|m'\rangle c^\dagger_{n,m,\alpha}c_{n,m',\alpha'}$$

Interaction model (II)

- Graphene form factors $(l_B \equiv 1)$:

$$F_n^{++}(\mathbf{q}) = \frac{1}{2}\left[L_{|n|}\left(\frac{|\mathbf{q}|^2}{2}\right) + L_{|n|-1}\left(\frac{|\mathbf{q}|^2}{2}\right)\right] e^{-|\mathbf{q}|^2/4} = F_n^{--}(\mathbf{q}) \equiv \mathcal{F}_n(\mathbf{q})$$

$$F_n^{+-}(\mathbf{q}) = \left(\frac{-i(q+q^* - K - K^*)}{2\sqrt{2|n|}}\right) L_{|n|-1}^1\left(\frac{|\mathbf{q}-\mathbf{K}|^2}{2}\right) e^{-|\mathbf{q}-\mathbf{K}|^2/4}$$

$$F_n^{-+}(\mathbf{q}) = \left[F_n^{+-}(-\mathbf{q})\right]^*$$

- Model: $H = \frac{1}{2}\sum_{\alpha_1,\ldots,\alpha_4}\sum_{\mathbf{q}} v_n^{\alpha_1,\ldots,\alpha_4}(\mathbf{q})\bar{\rho}^{\alpha_1\alpha_3}(-\mathbf{q})\bar{\rho}^{\alpha_2\alpha_4}(\mathbf{q})$

  with interaction vertex:

$$v_n^{\alpha_1,\ldots,\alpha_4}(\mathbf{q}) = \frac{2\pi e^2}{\epsilon|\mathbf{q}|} F_n^{\alpha_1\alpha_3}(-\mathbf{q}) F_n^{\alpha_2\alpha_4}(\mathbf{q}),$$

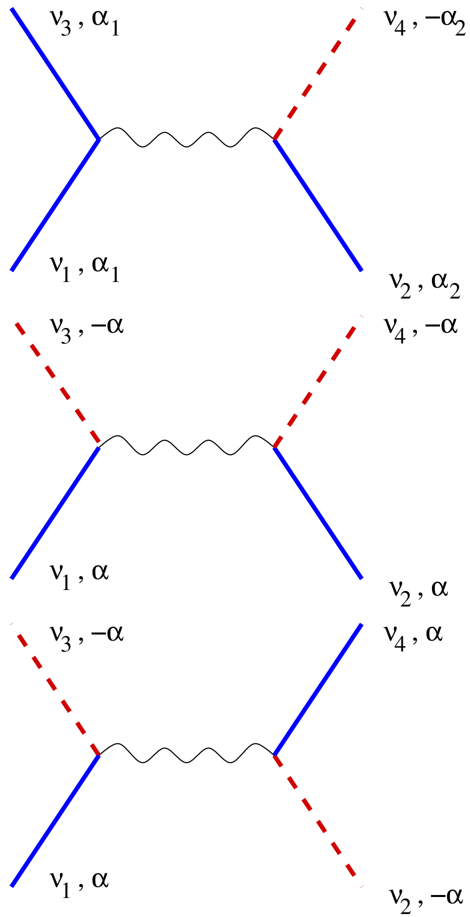
$\Rightarrow$ No SU(2) chirality symmetry so far !!

Interaction vertices

- Terms of the form $F_n^{\alpha,\alpha}(\mp\mathbf{q})F_n^{\alpha',-\alpha'}(\pm\mathbf{q})$:
  exp. suppressed $\sim \exp(-|\mathbf{K}|/8)$

- 'Umklapp' terms $F_n^{\alpha,-\alpha}(-\mathbf{q})F_n^{\alpha,-\alpha}(\mathbf{q})$:
  exp. suppressed $\sim \exp(-|\mathbf{K}|/2)$

- 'Backscattering' terms $F_n^{\alpha,-\alpha}(-\mathbf{q})F_n^{-\alpha,\alpha}(\mathbf{q})$:
  alg. small $\sim 1/|\mathbf{K}| \sim a/l_B$



$$\Rightarrow H_{SU(2)}^{n} = \frac{1}{2}\sum_{\alpha,\alpha'}\sum_{\mathbf{q}} \frac{2\pi e^2}{\epsilon|\mathbf{q}|}\left[\mathcal{F}_n(q)\right]^2 \bar{\rho}^{\alpha,\alpha}(-\mathbf{q})\bar{\rho}^{\alpha',\alpha'}(\mathbf{q}) + \mathcal{O}(a/l_B)$$

Leading [SU(2) invariant] interaction Hamiltonian

$$H^n_{SU(2)} = \frac{1}{2}\sum_{\mathbf{q}} v^G_n(q)\bar{\rho}(-\mathbf{q})\bar{\rho}(\mathbf{q});$$

with total projected density $\bar{\rho}(\mathbf{q}) = \bar{\rho}^{++}(\mathbf{q}) + \bar{\rho}^{--}(\mathbf{q})$ and

$$v^G_n(q) = \frac{2\pi e^2}{\epsilon q}\mathcal{F}^2_n(q)$$

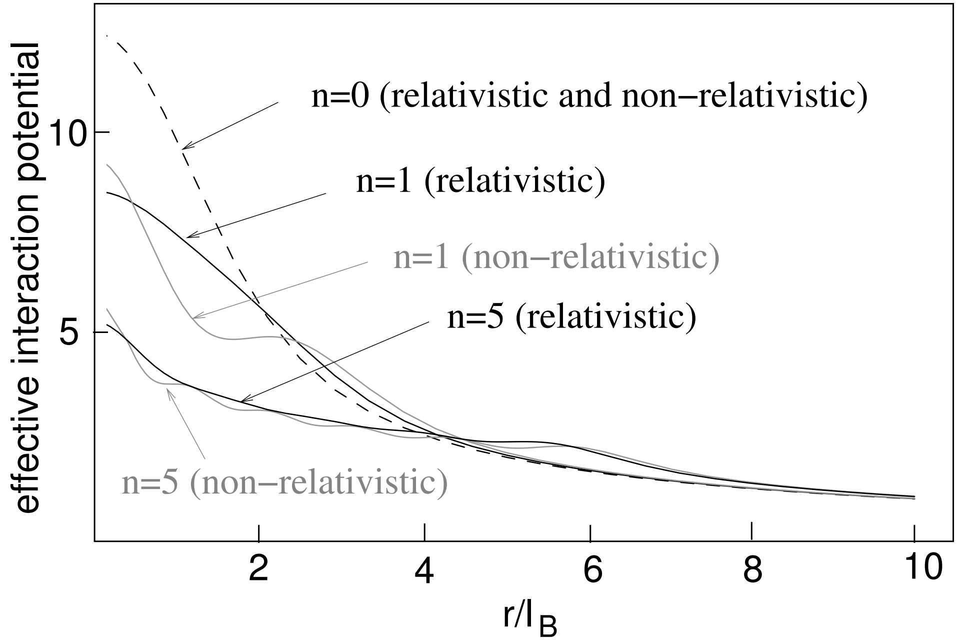
$$\mathcal{F}^2_0(q) = \exp(-q^2/2)$$

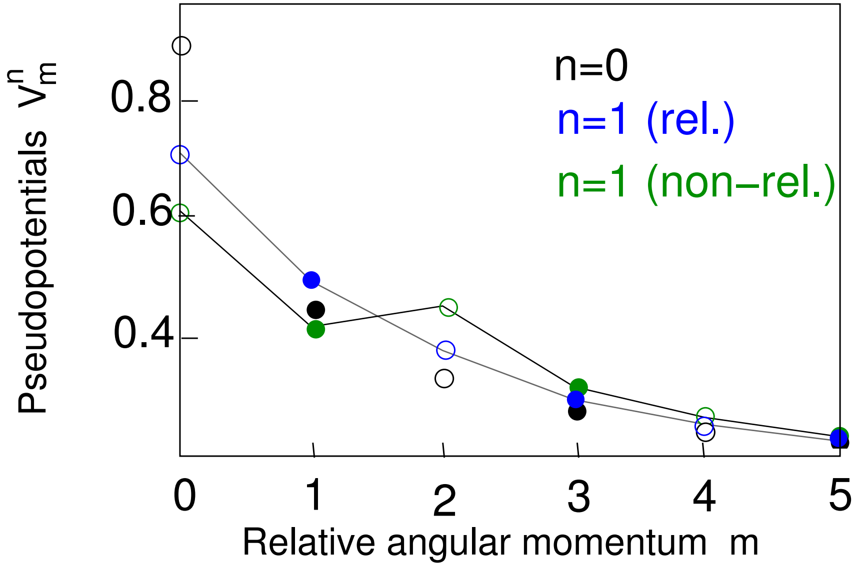
$$\mathcal{F}^2_n(q) = e^{-q^2/2}\left[L_{|n|}\left(\frac{q^2}{2}\right) + L_{|n|-1}\left(\frac{q^2}{2}\right)\right]^2$$

- Magnetic translation algebra for projected densities:

$$[\bar{\rho}(\mathbf{q}), \bar{\rho}(\mathbf{q}')] = 2i\sin\left(\mathbf{q}\wedge\mathbf{q}'/2\right)\bar{\rho}(\mathbf{q}+\mathbf{q}')$$

Effective SU(2) interaction potentials – FQHE





- Broadly similar to non-relativistic case
- Largest difference between rel. and non-rel. case in $n = 1$
- Similar behaviour of rel. interaction in $n = 0$ and $n = 1$:
  - absence of non-rel. $n = 1$ physics: Pfaffian at $\nu = 5/2$?
- unpolarised (chirality vs. spin) states

‘*Non-relativistic*’ *limit* $n \gg 1$

- For large n, can approximate envelope of $\mathcal{F}_n^2(q)$ as

$$\mathcal{F}_n^2(q) \simeq \frac{1}{4}\left[J_0(q\sqrt{2n-1}) + J_0(q\sqrt{2n+1})\right]^2$$

$$\simeq J_0^2(q\sqrt{2n}) \simeq \left[L_n\left(\frac{q^2}{2}\right)e^{-q^2/4}\right]^2$$

- Same as non-relativstic interactions
- Charge-density wave states in high Landau levels (stripes, bubbles)

What's special about the central Landau level

- Effective interaction looks the same as in non-relativistic LLL
- Only one sublattice occupied for each chirality
  - no backscattering term
  - unlike other LL, can easily arrange for CDW with $\lambda = a$

$\Rightarrow$ Hartree energy

  - details of energetics depend on structure of wavefunction on lattice scale
  - expect anisotropy of size a/l_B breaking down SU(2) chirality symmetry

SU(2) symmetry-breaking terms for $n \neq 0$

- Backscattering terms in $n \neq 0$:

$$H_{bs} = \frac{1}{2}\sum_{\alpha}\sum_{\mathbf{q}} v_n^{\alpha,-\alpha}(\mathbf{q})\bar{\rho}^{\alpha,-\alpha}(-\mathbf{q})\bar{\rho}^{-\alpha,\alpha}(\mathbf{q})$$

with interaction $v_n^{+-}(\mathbf{q}) = v_n^{-+}(-\mathbf{q})$

$$v_n^{+-}(\mathbf{q}) = \frac{\pi e^2 \mathrm{Re}(\mathbf{q}-\mathbf{K})^2}{\epsilon|\mathbf{q}|}\left[L^1_{|n|-1}\left(\frac{|\mathbf{q}-\mathbf{K}|^2}{2}\right)e^{-|\mathbf{q}-\mathbf{K}|^2/4}\right]^2$$

peaked near $\mathbf{q} = \pm\mathbf{K}$: $v_n^{+-}(q) \sim e^2/\epsilon|\mathbf{K}|l_B^2 \sim (e^2/\epsilon l_B)(a/l_B)$

Chirality quantum ferromagnetism

- Cf. exchange driven spin ferromagnetism at $\nu = 1$ in GaAs
- Review: Moon et al., PRB 51, 5138 (1995)

$$|\Psi\rangle = \prod_m \left(\sin\frac{\theta_m}{2} e^{-i\phi_m/2} c^\dagger_{m,+} + \cos\frac{\theta_m}{2} e^{i\phi_m/2} c^\dagger_{m,-} \right) |0\rangle \leftrightarrow \mathbf{n}_m = \begin{pmatrix} \sin\theta_m \cos\phi_m \\ \sin\theta_m \sin\phi_m \\ \cos\theta_m \end{pmatrix}$$

- Non-linear σ model (in coherent states: $m \sim \mathbf{r}$):

$$\mathcal{H} = \frac{\rho_s}{2} \int d^2r [\nabla \mathbf{n}(\mathbf{r})]^2 \qquad \rho_s = \frac{1}{16\sqrt{2\pi}} (e^2/\epsilon l_B)$$

- Easy-plane anisotropy due to backscattering terms in $n \neq 0$:

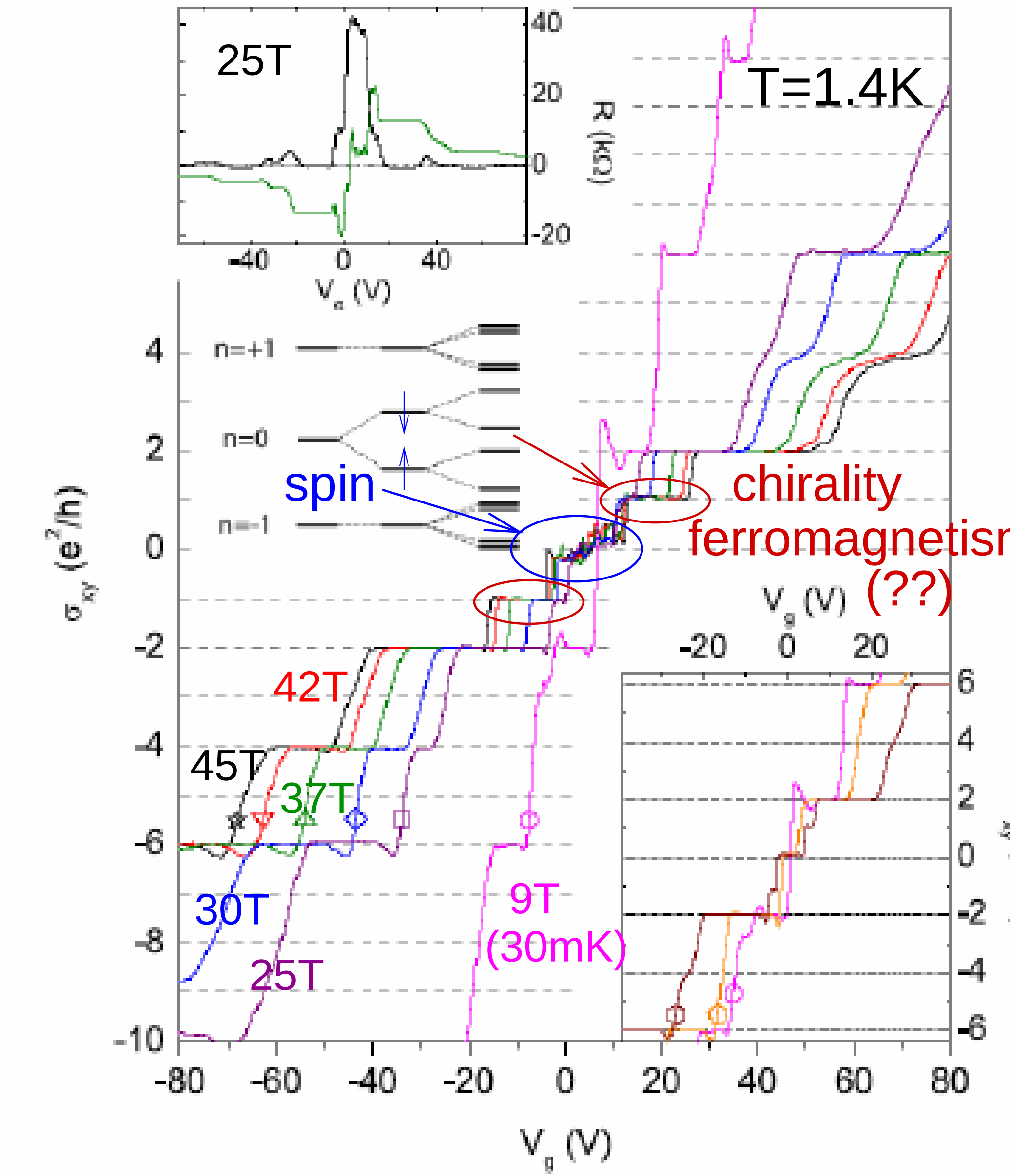
$$\mathcal{H}_Z = \Delta_z \int \frac{d^2r}{2\pi l_B} [n_z(\mathbf{r})]^2 \qquad \Delta_z = \frac{3\sqrt{3}}{64\pi^3} (e^2/\epsilon l_B)(a/l_B)$$

- Kosterlitz-Thouless physics

Graphene as a multi-component system

- Graphene is one of several multi-component systems
  - Spin; valley (Si); double layer; . . .
- Symmetry-breaking terms are quite weak (lattice effects – a/l_B)
  - highly isotropic
  - hard to probe?
  - need to investigate fate of unpolarised states
- Zeeman splitting much bigger than in GaAs $\Rightarrow$ not particularly close to SU(4)

Exprimental evidence for chirality polarisation



- Plateau transitions at $\nu = 0, \pm 1$ Zhang et al. PRL 06

$\Rightarrow$ Landau levels individually resolved

Related work

- Ferromagnetism in graphene: MacDonald *et al.*
- Effective description: Alicea+Fisher
- FQHE physics: Apalkov+Chakraborty

- Disorder and interaction effects (Guinea et al.)
- Edge states (Peres, Castro Neto, Guinea; Brey, Fertig; ...)
- Minimal conductance $\sim e^2/h$ (Beenaker's group, Katsnelson, ...)
- Disorder and weak (anti-)localisation (Altshuler et al, Guinea et al., Khveshchenko, ...)
- Dielectric breakdown (Baskaran *et al.*)
- $\ldots$

"We'll have to rewrite the theory of metals for this problem."

(Physics Today, January 2006, p. 21)

Summary

Graphene in a strong magnetic field

- continuum limit via Harper equation
- interaction Hamiltonian
  - chirality $SU(2)$ and its breaking by lattice effects
  - graphene form factor
  - pseudopotentials
  - the central Landau level
  - large-n limit
- chirality polarisation
  - exchange ferromagnetism
  - different routes to anisotropies

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Thank you for your attention!