

How to measure functional RG fixed-point functions for dynamics and at depinning

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Abstract – We show how the renormalized force correlator $\Delta(u)$, the function computed in the functional RG (FRG) field theory, can be measured directly in numerics and experiments on the *dynamics* of elastic manifolds in the presence of pinning disorder. For equilibrium dynamics we recover the relation obtained recently in the statics between $\Delta(u)$ and a physical observable. Its extension to depinning reveals interesting relations to stick-slip models of avalanches used in dry friction and earthquake dynamics. The particle limit ($d=0$) is solved for illustration: $\Delta(u)$ exhibits a cusp and differs from the statics. We propose that the FRG functions be measured in wetting and magnetic interfaces experiments.

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Models involving elastic objects driven through random media are important for magnets [1], superconductors [2], density waves [3], wetting [4], dry friction [5], dislocation and crack propagation [6], and earthquake dynamics [7]. There has been progress in qualitative understanding of, *e.g.* the depinning threshold for persistent motion at zero temperature $T=0$, the analogy with critical phenomena, collective pinning and roughness exponents, and ultra-slow thermally activated creep over diverging barriers. These phenomena are predicted by phenomenological arguments [2], mean-field models [8], functional renormalisation group [9–11], and were seen in numerical studies. Experimental evidence for creep was found in vortex lattices, in ferroelectrics, and in magnetic interfaces [1,12]. Some cases exhibit discrepancies with the simplest theories, *e.g.* the depinning of the contact line of a fluid [4,13].

Recent theoretical progress makes quantitative tests possible. For interfaces, powerful algorithms allow to find the exact depinning threshold and critical configuration on a cylinder [14] and to study creep dynamics [15]. The functional RG shows that differences between statics and depinning appear only at two loops [11]. The FRG is the candidate for a field theory of statics and depinning, beyond mean field. It introduces, rather than a single coupling as in standard critical phenomena, a function, $\Delta(u)$, of the displacement field u , which flows to a fixed point (FP) $\Delta^*(u)$. This FP is non-analytic, as is the effective action of the theory. Qualitatively, $\Delta(u)$ can be interpreted as the coarse-grained correlator of the random

pinning force. Its cusp singularity at the FP, $\Delta'(0^+) = -\Delta'(0^-)$, is related to shock singularities in the coarse grained force landscape, responsible for pinning. Until now however, comparison between experiments, numerics and FRG was mostly about critical exponents.

The aim of this letter is to make precise statements concerning *dynamical* FRG and propose experimental and numerical tests. Recently a relation was found [16] between the FRG coupling function $\Delta(u) = -R''(u)$ and *observables*, suggesting a method to measure these functions *in the statics*. The idea is to add to the disorder a parabolic potential (*i.e.*, a mass m) with a variable minimum location w . The resulting sample-dependent free energy $\hat{V}(w)$ defines a renormalized random potential whose second cumulant is proved to be *the same* $R(w)$ function as defined in the replica field theory —deviations arising only in higher cumulants [16]. This holds for any internal dimension d of the elastic manifold, any number of components N of its displacement field $u(x)$, and any T . At $T=0$, the (minimum energy) configuration $u(x;w)$ is unique and smoothly varying with w , except for a discrete set of shock positions where $u(x;w)$ jumps between degenerate minima. The limit of a single particle in a random potential ($d=0$) maps to decaying Burgers turbulence, and the statistics of the shocks can in some cases be obtained, yielding exact result [16] for $\Delta(u)$.

This method was used recently [17] to compute numerically the $T=0$ FRG fixed-point function $\Delta(u)$ in the statics, for interfaces ($N=1$), using powerful exact

minimization algorithms. Random bond, random field and periodic disorder were studied in dimensions $d=0,1,2,3$. $\Delta(u)$ was found close to the 1-loop predictions, with deviations consistent with 2-loop FRG and a linear cusp in any d . The cross-correlation for two copies of disorder was compared to a FRG study of chaos [18]. The main assumptions and central results of the FRG for the *statics* were thus confirmed. It is important to extend these methods to the dynamics of pinned objects and to the depinning transition.

In this letter we extend the method of ref. [16] to the dynamics. Using a slow, time-dependent, harmonic potential we show how the terms in the effective dynamical action identify with the FRG functions. The $T > 0$ equilibrium dynamics reduces to the definition used in the statics. To describe depinning at $T = 0$, the manifold is pulled by a quasi-static harmonic force (*i.e.* a spring of strength m^2), and the statistics of the resulting jumps directly yields the critical force and the FRG functions which converge to fixed forms as $m \rightarrow 0$. The model is similar to stick-slip models used, *e.g.*, in dry friction [5,19] and earthquake dynamics [7]. The present method provides a different way to look at these problems in numerics and experiments, and a precise meaning to quantities computed in the field theory.

We consider the equation of motion for the overdamped dynamics of an elastic manifold parameterized by its time-dependent displacement field $u(x, t)$:

$$\begin{aligned} \eta \partial_t u(x, t) &= F_x[u(t); w(t)], \\ F_x[u; w] &= m^2(w - u(x)) + c \nabla_x^2 u(x) + F(x, u(x)) \end{aligned} \quad (1)$$

where $F_x[u(t); w(t)]$ is the total force exerted on the manifold (we note $u(t) = \{u(x, t)\}_{x \in \mathbb{R}^d}$ the manifold configuration, x being its d -dimensional internal coordinate); η is the friction coefficient and c the elastic constant. Here at the bare level, the random pinning force is $F(x, u) = -\partial_u V(x, u)$ and the random potential V has correlations $\overline{V(0, x)V(u, x')} = R_0(u)\delta^{(d)}(x - x')$. We consider first bare random bond disorder with a short-ranged $R_0(u)$. At non-zero temperature one adds the thermal noise $\langle \xi(x, t)\xi(x', t') \rangle = 2\eta T \delta(t - t')\delta^d(x - x')$. We have added a harmonic coupling to an external variable $w(t)$. It is useful to define the fixed- w energy

$$\mathcal{H}_w[u] = \int d^d x \frac{m^2}{2}(u(x) - w)^2 + \frac{c}{2}(\nabla_x u)^2 + V(x, u(x)) \quad (2)$$

associated to the force $F_x[u; w] = -\frac{\delta \mathcal{H}_w[u]}{\delta u(x)}$. If $w(t)$ increases with t , the model represents an elastic manifold “pulled” by a spring. Quasi-static depinning is studied for $v = \frac{dw}{dt} \rightarrow 0^+$.

We first describe qualitatively how to measure the FRG functions and later justify why the relation is expected to be exact. Consider the observable (extension of the spring) $w(t) - \langle \bar{u}(t) \rangle$, where $\bar{u}(t) = L^{-d} \int d^d x u(x, t)$ is the center of mass position, and $\langle \dots \rangle$ denotes thermal averages,

i.e. the ground state at zero temperature. Of particular interest are

$$\begin{aligned} \overline{w(t) - \langle \bar{u}(t) \rangle} &= m^{-2} f_{av}(t) \\ \overline{[w(t) - \langle \bar{u}(t) \rangle][w(t') - \langle \bar{u}(t') \rangle]} &= m^{-4} L^{-d} D_w(t, t'), \end{aligned} \quad (3)$$

where connected means with respect to the double average $\overline{\langle \dots \rangle}$. If $w(t)$ is such that $dw(t)/dt > 0$, one can write: $D_w(t, t') = \Delta_w(w(t), w(t'))$. In general, Δ_w may depend on the history $w(t)$. However we expect that for fixed L, m and slow enough $w(t)$, *e.g.* $w(t) = vt$ with $v \rightarrow 0^+$, one has $\Delta_w(w(t), w(t')) \rightarrow \Delta(w(t) - w(t'))$. This function $\Delta(w - w')$, which is independent of the process $w(t)$, is the one defined in the field theory. A detailed justification is given in [20,21].

Let us first consider a process $w(t)$ at $T > 0$ so slow that the system (with a finite number of degrees of freedom $(L/a)^d$) remains in equilibrium, *i.e.* with $\dot{w}t_L \ll u(L)$ where t_L is the largest relaxation time of the system, and $u(L)$ its width. The above definition is then consistent with the one in the statics, where it was shown that one can measure the equilibrium free energy in a harmonic well with fixed w (or its generalization to an arbitrary $w(x)$), defined through $e^{-\hat{V}(w)/T} = \int \mathcal{D}[u] e^{-\mathcal{H}_w[u]/T}$, and extract from it the pinning energy correlator $R(w)$. This is done by measuring the second cumulant $\overline{\hat{V}(w)\hat{V}(w')}^c = \hat{R}[w - w']$, with $\hat{R}[w] = L^d \hat{R}(w)$ for a uniform parabola $w(x) = w$, and using that $\hat{R} = R$ [16]. One equivalently obtains the force correlator $\Delta(w)$ via the equilibrium fluctuations of the center of mass $\langle \bar{u} \rangle_w$ at fixed w , *i.e.* $\overline{(w - \langle \bar{u} \rangle_w)(w' - \langle \bar{u} \rangle_{w'})}^c = m^{-4} L^{-d} \Delta(w - w')$. While in the statics one finds $\Delta(w) = -R''(w)$, the potentiality of this function breaks down in the driven dynamics, or at depinning, as discussed below.

Let us note at this stage that a second definition can be given using two “copies”. Consider two evolutions $u(x; w_1)$ and $u(x; w_2)$ driven by two (slow) processes $w_1(t) = w_2(t) + w$ of fixed separation, in the same disorder sample. Then define

$$\overline{(w_1(t) - \langle \bar{u}_1(t) \rangle)(w_2(t) - \langle \bar{u}_2(t) \rangle)}^c = m^{-4} L^{-d} \Delta_t(w) \quad (4)$$

which is now an equal-time correlation. For a slow equilibrated motion at $T > 0$, it identifies with the static definition. The general case is discussed below and in [20].

Let us now describe $T = 0$ quasi-static depinning for $N = 1$. It is studied as the limit where $dw/dt \rightarrow 0^+$. One starts in a metastable state $u_0(x)$ for a given $w = w_0$, *i.e.* a zero-force state $F_x(u_0(x); w) = 0$ which is a local minimum of $H_{w_0}[u]$ with a positive barrier. One then increases w . For smooth short-scale disorder, the resulting deformation of $u(x)$ is smooth. At $w = w_1$, the barrier vanishes. For $w = w_1^+$ the manifold moves downward in energy until it is blocked again in a metastable state $u_1(x)$ which again is a local minimum of $H_{w_1}[u]$. For the center of mass (*i.e.* translationally averaged) displacement $\bar{u} = L^{-d} \int d^d x u(x)$, this process defines a function

$\bar{u}(w)$ which exhibits jumps at the set w_i . Note that time has disappeared: evolution is only used to find the next location. The first two cumulants

$$\overline{w - u(w)} = m^{-2} f_c, \quad (5)$$

$$\overline{(w - u(w))(w' - u(w'))^c} = m^{-4} L^{-d} \Delta(w - w') \quad (6)$$

allow a direct determination (and definition) of the averaged (m -dependent) critical force f_c and of $\Delta(w)$, in analogy to the statics. As discussed below $u(w)$ depends *a priori* on the initial condition and its orbit but at fixed m one expects an averaging effect when w runs over a large region. Note that the definition of the critical force at large but finite size is delicate [22]. Here the quadratic well provides a way to obtain a stationary state.

Elastic systems driven by a spring and stick-slip-type motion were studied before, *e.g.* in the context of dry friction. The force fluctuations, and jump distribution were studied numerically [19]. However, the precise connection to quantities defined and computed in the field theory has to our knowledge not been made. The dependence on m for small m predicted by FRG, $\Delta(w) = m^{\epsilon-2\zeta} \tilde{\Delta}(wm^{-\zeta})$ is consistent with observations of [19] but the resulting $\tilde{\Delta}(w)$ has not been measured. Fully connected mean-field models of depinning also reduce to a particle pulled by a spring, together with a self-consistency condition, around which one can expand [10]. Our remarks here are much more general, independent of any approximation scheme, and provide a rather simple and transparent way to attack the problem. Indeed, as shown in [20,21], $\Delta(w)$ in (6) is —to all orders— the one defined in the field theory.

For the qualitative discussion it is useful to study the model in $d=0$, *i.e.* a particle with equation of motion

$$\eta \partial_t u = m^2(w - u) + F(u). \quad (7)$$

In the quasi-static limit where w is increased slower than any other time-scale, the zero force condition $F(u) = m^2(u - w)$ determines $u(w)$ for each w . The graphical construction of $u(w)$ is well known from studies of dry friction [5]. When there are several roots one must follow the root as indicated in fig. 1, where $F(u)$ is plotted *vs.* $m^2(u - w)$. This results in jumps and a different path for motion to the right and to the left. The area A of this hysteresis loop (the area of all colored/shaded regions in fig. 1) is the total work of the friction force when moving the center of the harmonic well quasi-statically once forth and back, *i.e.* the dissipated energy. For translationally invariant landscape statistics, the definition (5) of the averaged critical force (replacing disorder averages by translational ones over a large width M) gives

$$f_c = m^2 \overline{(w - u_w)}^{\text{tr}} = \frac{m^2}{M} \int_0^M dw (w - u_w) = \frac{A}{2M}, \quad (8)$$

using $\int u dw = \int w du$. As $m \rightarrow 0$ this definition of f_c becomes identical to the one on a cylinder, f_d , which

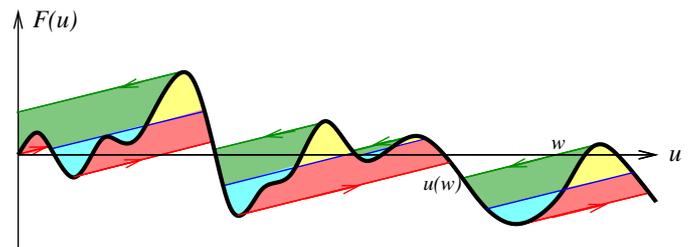


Fig. 1: (Color online) Construction of $u(w)$ in $d=0$. The pinning force $F(u)$ (bold black line). The two quasi-static motions driven right and left are indicated by red and green arrows, and exhibit jumps (“dynamical shocks”). The position of “static shocks” is shown from the Maxwell construction (equivalence of light blue and yellow areas, both bright in black and white). The critical force is $2/M$ times the area bounded by the hull of the construction.

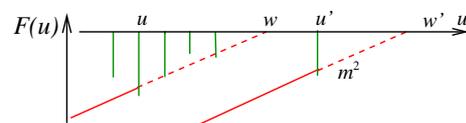


Fig. 2: Forward motion in the discrete model.

for a particle ($d=0$) is $2f_d = f_d^+ - f_d^- = \max_u F(u) - \min_u F(u)$ with $2f_d M = \lim_{m \rightarrow 0} A(m)$. (Since A depends on the starting point, this definition holds after a second tour, where the maximum (minimal) pinning force was selected). Finally, one can compare with the definition of shocks in the statics. There, the effective potential is a continuous function of w . Therefore, when making a jump, the integral over the force must be zero, which amounts to the Maxwell-construction of fig. 1.

One can compute f_c and $\Delta(w)$ in $d=0$ for a discrete force landscape, F_i , independently distributed with $P(F)$, and i integer. $u(w)$ is then integer and defined in fig. 2. The process admits a continuum limit for small m , which depends on the behaviour of $P(F)$ in its tails (negative tail for forward motion). One obtains [20] the distribution of $u(w)$, $P_w(u) du = e^{-a_w(u)} da_w(u)$, where $a'_w(u) = \int_{-\infty}^{m^2(u-w)} P(f) df$ and $a_w(-\infty) = 0$. One also obtains the joint distribution of $(u(w), u(w'))$, $P_{w;w'}(u, u') = (a'_w(u) - a'_{w'}(u)) a'_{w'}(u') e^{-a_w(u) - a_{w'}(u') + a_{w'}(u)} \theta(u' - u) + \delta(u' - u) a'_{w'}(u) e^{-a_w(u)}$ for $w > w'$. Define $\Delta(w) =: m^4 \rho_m^2 \tilde{\Delta}(w/\rho_m)$ and $f_c =: f_c^0 + cm^2 \rho_m$. This yields two main classes of universal behaviour at small m . The first contains i) exponential-like distributions with unbounded support *i.e.* $\ln P(f) \approx_{f \rightarrow -\infty} -A(-f)^\gamma$ (for which $f_c^0 = ((\ln m^{-2})/A)^{\frac{1}{\gamma}}$) and ii) distributions with exponential behaviour near an edge $P(f) \sim e^{-A(f+f_0)^\gamma} \theta(f+f_0)$ (with $\gamma < 0$ and $f_c^0 = f_0 - ((\ln m^{-2})/A)^{\frac{1}{\gamma}}$). For both i) and ii) the FP function is $\tilde{\Delta}(x) = \frac{x^2}{2} + \text{Li}_2(1 - e^x) + \frac{\pi^2}{6}$ and $\rho_m = \rho_m^\gamma := 1/(|\gamma| A^{1/\gamma} m^2 (\ln m^{-2})^{1-\frac{1}{\gamma}})$, $c = \gamma_E$ the Euler constant. The first class has $\zeta = 2$ up to log-corrections. The second class contains power law distributions near

an edge $P(f) = A\alpha(\alpha - 1)(f + f_0)^{\alpha-2}\theta(f + f_0)$, $\alpha > 1$, for which $c = -\Gamma(1 + \frac{1}{\alpha})$, $m^2\rho_m = (m^2/A)^{\frac{1}{\alpha}}$ and $f_c^0 = f_0$. The FP depends continuously on α with $\tilde{\Delta}(w) = -\Gamma(1 + \frac{1}{\gamma})\Gamma(1 + \frac{1}{\gamma}, w^\gamma) + w\Gamma(1 + \frac{1}{\gamma})e^{-w^\gamma} + \int_0^\infty dy e^{-(y+w)^\gamma + y^\gamma}\Gamma(1 + \frac{1}{\gamma}, y^\gamma)$ where $\Gamma(a, x) = \int_x^\infty dz z^{a-1}e^{-z}$; it has $\zeta = 2 - 2/\alpha$ (see footnote¹). Hence, despite the fact that $d=0$ is dominated by extreme statistics (*e.g.* the distribution of $\rho_m^{-1}(w - u(w))$ converges to the Gumbel and Weibul distributions for class I and II, respectively) it still exhibits some universality in cumulants and in all classes $\Delta(u)$ has a cusp non-analyticity at $u=0$. We have checked the above scaling functions *and* amplitudes numerically, with excellent agreement.

Returning to $d > 0$, we note that the interface in the harmonic well can be approximated by $(L/L_m)^d$ independent pieces with $L_m \sim 1/m$. The motion of each piece resembles the one of a particle, *i.e.* a $d=0$ model, but with a rescaled unit of distance in the u direction, $u_m \sim L_m^\zeta \sim m^{-\zeta}$. The “effective-force” landscape seen by each piece becomes uncorrelated on such distances, and its amplitude scales as $F_m \sim m^2 u_m$. Hence one is in a bulk regime not dominated by extremes, *i.e.* $\Delta(w)$ probes motion over about one unit. It is easy to check in fig. 1 that any initial condition joins the common unique orbit after about one correlation length. Hence the $d=0$ model suggests that starting the quasi-static motion in u_0 and driving the manifold over $w \sim L_m^\zeta$ should result in all orbits converging. Hence the definitions (4) and (6) are equivalent for $N=1$. A crossover to $d=0$ and extremal statistics occurs if $L < L_m$.

Note that the averaged critical force, defined in (5), should, for $d > 0$, go to a finite limit, with $f_c(m) = f_c^0 + Bm^{2-\zeta}$. Although f_c depends on short-scale details, one easily sees that $-m\partial_m f_c(m)$ depends only a single unknown scale. The definition (5) coincides with the one proposed recently as the maximum depinning force for all configurations having the same center of mass u_0 [22]. Since $\bar{u} - w$ is a fluctuating variable of order $(L/L_m)^{-d/2}$, the definition is the same as the above in the limit where $L \rightarrow \infty$, before $m \rightarrow 0$. The single w distribution is obtained from the distribution of $w - u(w)$ if all modes have a mass.

Measurements of $\Delta(w)$ reveal interesting features in any d . At the bare level, the disorder of the system is of random-bond type (*i.e.* potential). As the mass is decreased, one should observe a crossover from random-bond to random-field disorder. Also a finite velocity should round the cusp singularity. These features are well visible in the $d=0$ toy model as illustrated in the main plot of fig. 3 (quasi-static evolution in a model with $F_i = V_{i+1} - V_i$ and V_i uncorrelated) and the inset of fig. 3 (Langevin

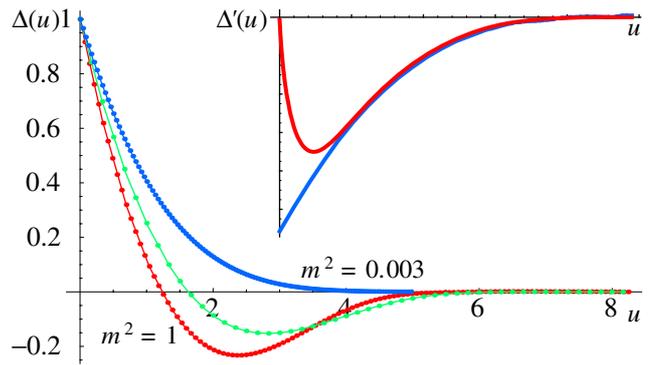


Fig. 3: Main plot. The measured $\Delta(u)$ for RB disorder distributed uniformly in $[0, 1]$, and rescaled such that $\Delta(0) = 1$ and $\int_0^\infty du |\Delta(u)| = 1$. From bottom (which has $\int_0^\infty du \Delta(u) \approx 0$) to top the mass decreases from $m^2 = 1$ to 0.5 to 0.003. One observes a clear crossover from RB to RF. Inset. $\Delta'(u)$ for a driven interface at vanishing (blue, $\Delta'(0) < 0$) and finite velocity (red, $\Delta'(0) = 0$).

dynamics at finite v) and can be obtained analytically in that case [20].

We have generalized [20] the above method to a manifold driven in N dimensions (*e.g.* a flux line in a 3D superconductor). For a particle, and fixed m , we have seen numerically that different initial conditions converge. The Middleton theorem [23] no longer holds, and particles can pass each other. To probe transverse motion and correlations $\Delta(w)$ in the transverse direction, we use the two-copy definition (4). Finally thermal rounding of depinning, creep and crossover from statics to depinning can be studied more precisely by this method.

To conclude we propose that $\Delta(w)$ be measured in experiments, an important test of the theory and the underlying assumptions. Creep and depinning of magnetic domains in films were investigated using imaging [1]: adding a magnetic field gradient confines the interface in a quadratic well, whose strength and position can be varied. The field gradient is proportional to our m^2 . The fluctuations of the translationally averaged position of the interface gives $\Delta(w)$, equation (6).

For contact lines of fluids, capillarity and gravity provide the quadratic well, and if large scale inhomogeneities can be controlled, $\Delta(w)$ could be measured from statistics on lengths larger than the capillary length (*i.e.* L_m here).

In vortex-lattice experiments, one may be able to provide a quadratic well for the vortices using a well-designed tip of a tunneling microscope.

We hope to stimulate numerical [24] and experimental [25] studies.

¹Distributions with unbounded tails $p(f) \sim A\alpha(\alpha + 1)f^{2+\alpha}$ at large negative f correspond to $\alpha \rightarrow -\alpha$, *i.e.* $\zeta = 2 + 2/\alpha$, $f_c^0 = 0$, $m^2\rho_m = (A/m^2)^{1/\alpha}$, $c = \Gamma(1 - \frac{1}{\alpha})$ and Fréchet class of single w statistics. They are dominated by rare events with no finite continuum limit for cumulants of order larger than α .

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