Quantum Hall Interferometry: Status and Outlook

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Collaborators: **Woowon Kang, Bernd Rosenow**
+ Ady Stern, Bert Halperin, Eytan Grosfeld, Curt von Keyserlingk
Realization of interesting CFTs in condensed matter

(Moore-Read, Wen, …)

2+0 dimensional Bulk Wavefunction ↔ CFT Correlator
1+1 dimensional Edge Theory ↔ CFT
Many Theoretical Predictions (mainly regarding edge properties) Are based on CFT Calculations:

Kane and Fisher (Various Edge Transport Properties)
Ludwig, Fendley, Saleur (Noise at 1/3, Exact Calculation)
Chamon, et al (Interferometry at 1/3)
Stern Halperin; Shtengel, Bonderson, Kitaev (Interferometry at 5/2)
.... + many many more...

Experimental Situation:
Much less clear than one would hope

Experiments that seem to disagree with theory*
Experiments that are disputed by other experiment(alists)*

* = To a first approximation
Par Exemple :

1984: Prediction that quasiparticles of $\nu=1/3$
  have fractional statistics
  (Halperin; Arovas Schrieffer, Wilczek)

There is general agreement that as of today
no published experiment has ever demonstrated
fractional statistics.

How would you do it in principle?
The Quantum Hall Fabry-Perot Interferometer

*Hoping to prove fractional statistics*

Theory: Chamon, Wen, et al 1997 + many many others

Experiment: Goldman Group; Willett Group; Kang Group; Marcus Group; Heiblum Group

interference of two partial waves
Conventional Quantum Hall States ($\nu=1/3$)

Side gate changes phase

\[ B \Delta A = \frac{hc}{e^*} \]

Chamon, Wen, et al

\[ e^* = \frac{e}{3} \]

Conductance $G$

side gate voltage
Conventional Quantum Hall States ($\nu=1/3$)

Side gate changes phase

$$B \Delta A = \frac{hc}{e^*}$$

Chamon, Wen, et al

$$e^* = e/3$$

Adding 1 quasiparticle shifts interference pattern by $2\pi / 3$

**FRACTIONAL STATISTICS**
VERY LONG HISTORY TO EXPERIMENTAL EFFORTS TO DEMONSTRATE FRACTIONAL STATISTICS THIS WAY

Plagued with Complications and Confusion
Complication #1: How do you know when you added a qp?
(a) Addition of flux
(b) Change of voltage

Complication #2: Can you add a qp without deforming the “dot”

Area of dot changes to accommodate qps
Electrostatics of buried LL’s matter

*Aharonov-Bohm Regime vs. Coulomb Dominated Regime*
(Can be very complicated)

Theory: Rosenow, Halperin; Halperin, Stern, Neder, Rosenow
Exp: Y. Zhang et al (Marcus); N. Ofek et al (Heiblum); Godfrey et al (Kang)
Telegraph Noise

Slowish time scale = caused by glassy motion of dopant impurities
Telegraph Noise

Slowish time scale = caused by glassy motion of dopant impurities
That is all I have to say about Abelian Quantum Hall Effect

(take a deep breath)

... and on to the presumed Non-Abelian $\nu=5/2$
The Fundamental Principles of 5/2 Nonabelions
(Presumed Moore-Read or AntiPfaffian)

- For each pair of e/4 qps there is a single two state system. called: a “neutral (dirac) fermion” or a “qubit”
  (i.e., each qp associated with a majorana)

- Braiding a third qp through the two flips the state of the qubit

- A phase of $\pi$ is accumulated going around a neutral fermion
5/2 state interference experiment

Nayak, Wilczek; Stern, Halperin; Bonderson, Shtengel, Kitaev; Das Sarma, Nayak, Freedman

With even number of quasiparticles

Can get $\pi$ phase shift
Depending on even/odd neutral fermions

Conductance $G$

side gate voltage
5/2 state interference experiment

Nayak, Wilczek; Stern, Halperin; Bonderson, Shtengel, Kitaev; Das Sarma, Nayak, Freedman

With odd number of quasiparticles
5/2 state interference experiment

Nayak, Wilczek; Stern, Halperin; Bonderson, Shtengel, Kitaev; Das Sarma, Nayak, Freedman

With odd number of quasiparticles

No Interference!

Conductance $G$

side gate voltage
5/2 state interference experiment

Nayak, Wilczek; Stern, Halperin; Bonderson, Shtengel, Kitaev; Das Sarma, Nayak, Freedman

With odd number of quasiparticles

Partial Waves are Orthogonal ⇒ No Interference!
5/2 state interference experiment

Summary of Orthodox Theory:

- If an **odd # of qps** are in the interferometer, **no interference**
- If an **even # of qps** are in the interferometer, **yes interference**
  
  \[
  \text{Phase} = 0 \quad \text{if even # of neutral fermions} \\
  \text{Phase} = \pi \quad \text{if odd # of neutral fermions}
  \]

plus interference of e/2 particles occurs all the time
  - half gate-voltage period
  - expect lower amplitude
Weaker $e/2$ oscillations (double frequency) show up here instead.
R. L. Willett, L. N. Pfeiffer, and K. W. West
Adding 19 Gauss of Flux (presumed 1 qh) changes even to odd
Is this validation of the “Orthodox” theory?

(1) Is the data convincing?

(2) Why is e/2 so strong?  
   Why does it come and go?

(3) Why does this appear to be Aharonov-Bohm and not Coulomb Dominated?

(4) How reproducible is it?

(5) Why does no other group observe this.

....

LET’S BELIEVE THE EVEN-ODD EFFECT HAS BEEN SEEN

PROBLEM:

ORTHODOX THEORY SHOULD NOT HOLD!
PROBLEM = DEVICE IS SMALL….

qps (qubits) in the dot must be strongly coupled to each other, and to the edge… By majorana hopping!

VERY UNLIKE ORTHODOX THEORY
Energy Scales

(1) $T \approx V \approx 10 \text{ mK} \approx 200 \text{ MHz}$

(2) qp-qp majorana coupling

(3) qp-edge majorana coupling

All Potentially Similar Order
Estimate from Trial Wavefunction Monte-Carlo for tunneling
(Baraban, Zikos, Bonesteel, Simon):

Two qps a distance $d$ apart (4 qps in the calculation=2 fusion channels)

$1K \approx E_{gap}$

Assume Fairly Big Error Bars

$E_{splitting} = \text{Oscillatory} \times e^{-d/\xi}$

$\xi \approx 2.3\ell_0 \approx 230\text{Å}$
PROBLEM = DEVICE IS SMALL…. qps (qubits) in the dot must be strongly coupled to each other, and to the edge… By majorana hopping!

VERY UNLIKE ORTHODOX THEORY
**Energy Scales**

1. \( T \approx V \approx 10 \text{ mK} \approx 200 \text{ MHz} \)
2. qp-qp majorana coupling
3. qp-edge majorana coupling
4. \( 1/(\text{Time of Experiment}) = \text{Hz} = \text{Tiny.} \)

All Potentially Similar Order

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*For orthodox interpretation to hold, need*

qp-edge coupling \(<<\) \( 1 / (\text{Time Scale of Experiment}) \)
Why is edge-qp coupling a problem

Overbosch and Wen; Rosenow, Halperin, Simon, Stern; Bishara and Nayak

“Fast” tunneling of neutral fermions to the edge kills interference!!

Conductance $G$

| Even # of qps with even # of neutral fermions |
| Even # of qps with odd # of neutral fermions |

Path length (side gate voltage)
**Energy Scales**

1. $T \approx V \approx 10 \text{ mK} \approx 200 \text{ MHz}$

2. qp-qp majorana coupling

3. qp-edge majorana coupling

4. $1/(\text{Time of Experiment}) = \text{Hz} = \text{Tiny}.$

All Potentially Similar Order

_for orthodox interpretation to hold, need_

qp-edge coupling $<< 1 / (\text{Time Scale of Experiment})$

Modified (reform) interpretation, can save even-odd effect if

qp-qp coupling $> T$
Why does qp-qp coupling help?

Energy of $|0\rangle$ and $|1\rangle$ are split by $E$ (qp-qp coupling)

If $T < E$, qubit freezes into a single state. Does not fluctuate between two out of phase signals

Interference is then seen!...and even-odd effect!

... but not a good “qubit”

(actually need $T < E_{\text{min}}$ of band of majoranas)
**Energy Scales**

1. \( T \approx V \approx 10 \text{ mK} \approx 200 \text{ MHz} \)
2. qp-qp majorana coupling
3. qp-edge majorana coupling
4. \( 1/(\text{Time of Experiment}) = \text{Hz} = \text{Tiny} \)

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**For orthodox interpretation to hold, need**

\[ \text{qp-edge coupling} \ll \frac{1}{(\text{Time Scale of Experiment})} \]

---

**Modified (reform) interpretation, can save even-odd effect if**

\[ \text{qp-qp coupling} > T \]
How can qp-edge coupling stop even-odd?

For "odd" to kill interference, lone qp must be decoupled from edge

Need:

qp-edge coupling $\ll e^*V \approx T$

If a qp is coupled strongly to the edge,

it becomes part of the edge $\Rightarrow$ Nothing encircles it
\[ \mathcal{L}_{\text{charge}} = \frac{1}{4\pi\nu} \partial_x \varphi (v_c \partial_x \pm i \partial_\tau) \varphi \]
\[ \mathcal{L}_{\text{neutral}} = \psi (v_n \partial_x \pm \partial_t) \psi \]
\[ \mathcal{L}_{\text{qps}} = \Gamma_\alpha \partial_\tau \Gamma_\alpha \]
\[ \mathcal{L}_{\text{edge-qp}} = \lambda \Gamma_\alpha \psi (x_\alpha) \]
\[ \hat{T}(x) = t \sigma_u(x) \sigma_d(x) \left[ e^{\frac{i}{\sqrt{8}} (\phi_u - \phi_d)} + \text{h.c.} \right] \]

Edge charge mode (bosons)

Edge neutral (majorana) fermi mode

Vortex core (majorana) zero modes

Edge to bulk coupling

Point Contacts

Moves charge across

Interference Term \( = \) \( \text{Re} \int e^{iV_\tau} \langle \hat{T}_1(\tau) \hat{T}_2(0) \rangle \)

1. Perturbative
2. Exact
Rosenow, Halperin, Simon, Stern : Non-CFT Solution
(Majorana theories are quadratic Hamiltonians)
How can qp-edge coupling stop even-odd?

For “odd” to kill interference, lone qp must be decoupled from edge

Need: \[ \text{qp-edge coupling} \ll e^*V \approx T \]

If a qp is coupled strongly to the edge, it becomes part of the edge \( \Rightarrow \text{Nothing encircles it} \)

But also need to freeze qubit

\[ T < \text{qp-qp coupling} \]

probably impossible
Detailed Electrostatic Simulation (w/ von Keyserlingk)

15 qps in dot
- too far
- (\(E_{qp-qp}\) too low)

1 \(\mu m\)

20 qps in dot
- (bulk-edge too strong)
- too small

0.2 \(\mu m\)
**Energy Scales**

(1) \( T \approx V \approx 10 \text{ mK} \approx 200 \text{ MHz} \)

(2) qp-qp coupling

(3) qp-edge coupling

(4) \( 1/(\text{Time of Experiment}) = \text{Hz} = \text{Tiny} \)

All Potentially Similar Order

For orthodox interpretation to hold, need

\[
\text{qp-edge coupling} \ll 1 / (\text{Time Scale of Experiment})
\]

Modified (reform) interpretation, can save even-odd effect if

\[
\text{qp-qp coupling} > T \approx V \text{ and qp-edge coupling} \ll T \approx V
\]
**Energy Scales**

1. \( T \approx V \approx 10 \text{ mK} \approx 200 \text{ MHz} \)
2. qp-qp coupling
3. qp-edge coupling

**Prediction 1:**
No Even-Odd Effect.

Always Have Interference

Never Mind the Experimental Data:
Assume couplings are strong (small dot limit)

\[ \text{qp-qp coupling} \gg \text{qp-edge coupling} \gg T \approx V \]

*Modified (reform) interpretation, can save even-odd effect if*

\[ \text{qp-qp coupling} > T \approx V \]
and
\[ \text{qp-edge coupling} \ll T \approx V \]
Prediction 2

Expect \( \pi \) phase slips !!

– can occur without adding a qp

Which is lower energy (\(|0\rangle\) or \(|1\rangle\)) depends on the detailed configuration of qps in the dot.

Interference signals can flip by \( \pi \) if a qp moves

“Friedel” oscillations in splitting as a function of distance
Estimate from Trial Wavefunction Monte-Carlo for tunneling (Baraban, Zikos, Bonesteel, Simon):

Two qps a distance $d$ apart  (4 qps in the calculation=2 fusion channels)

\[ 1K \approx E_{gap} \]

\[ \approx 100 \text{mK at } d = 0.1 \mu \text{m} \]
**Prediction 3**

\[ \pm \pi/4 \text{ phase slips – occur with qp/qh addition} \]

Going from even to odd,

if \( E_{\text{edge-bulk}} >> e^*V \), zero-mode majorana absorbed into edge

Only see phase slip \((\pm \pi / 4)\) from abelian piece of the qp.

\[ \psi_{qh/qp} = \sigma e^{\pm i\phi/(2\sqrt{2})} \]

\( E_{\text{edge-bulk}} \sim e^*V, \ T \) gives not quite \( \pi / 4 \)

and less than full visibility of interference

Overbosch and Wen; Rosenow, Halperin, Simon, Stern; Bishara and Nayak
Expected signatures:

Phase slips of $\pi$, $\pi/4$, $5\pi/4$, 

Simulated data

$\pi/4$ slip

$\pi$ slip
Expected signatures:

Phase slips of $\pi$, $\pi/4$, $5\pi/4$,
With (in-) appropriate filtering, one might obtain multiple periods that look a bit like $e/2$ and $e/4$.

Willett’s low pass filter has a time constant of 100 seconds

(This is not a complete explanation of Willett’s data)
Summary

• “Orthodox” explanation of the even-odd effect for 5/2 interferometer seems impossible

• “Reformed” theory (freezing qubit state) still looks unlikely – coupling to edge too strong.

• Likely in a regime where all couplings are large
  Expect to always see interference (no even-odd).
  Expect slips of $\pi$ (qubit flips)
  Expect slips $\approx \pm \pi/4$ ($\pm 5 \pi/4$) for qp/qh addition

• Low pass filtering may obscure data

Phase slip measurements may be the cleanest way to demonstrate braiding statistics (7/3 and/or 5/2)
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Thank You For Listening

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