PhD thesis / Master-2 internship project

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Polarized superfluid phases of the attractive Hubbard model

Strongly correlated fermions are ubiquitous in various contexts: electrons in solids or molecules, nucleons in nuclei or neutron stars, quarks in QCD. Our understanding of such systems is limited by the difficulty to compute their properties in a reliable and unbiased way. For conventional quantum Monte Carlo methods, the computational time generically grows exponentially with the number of fermions (due to the "fermion sign problem"). The situation is fundamentally different with "diagrammatic Monte Carlo" approaches that work directly in the thermodynamic limit. The idea is to expand physical quantities into connected Feynman diagrams, to evaluate all diagrams up to a maximal expansion order using an efficient Monte Carlo algorithm, and to extrapolate to the infinite-order limit (if necessary after applying a divergent-series resummation method).

This project focuses on the Hubbard model, a textbook model which accurately describes cold atoms in optical lattices and is also relevant to materials. We have recently reported the first diagrammatic Monte Carlo results in the conventional superfluid phase (i.e. the superconducting phase in the solid-state context) of the Hubbard model with attractive interactions, implementing spontaneous symmetry breaking by expanding around a BCS Hamiltonian [1]. Thanks to the connected determinant algorithm [2] generalized to anomalous propagators, we could sum all connected Feynman diagrams up to 12 loops. Our study includes the case of a polarizing Zeeman field, where unbiased benchmarks are unavailable due to the fermion sign problem. At low temperature, we observe a first-order superfluid-to-normal transition; compared to mean-field theory, our results agree qualitatively, but quantitatively the deviations can be very large.

The main goal of the project is to extend the study [1] to superfluid phases with a spatially modulated order parameter. These "FFLO" phases occupy a large part of the phase diagram according to mean-field theory, but these predictions cannot be trusted outside of the weak-coupling regime. Our approach offers the opportunity to provide reliable answers to this open problem, which is of direct relevance to cold-atom and solid-state experiments. The project is also relevant to the striped phases of the repulsive Hubbard model, which are related to FFLO phases of the attractive model via particle-hole transformation, and are now believed to win over d-wave superconductivity in the doped repulsive Hubbard model on the square lattice in the parameter regime relevant to cuprates.

References:

[1] G. Spada, R. Rossi, F. Simkovic, R. Garioud, M. Ferrero, K. Van Houcke, F. Werner, arXiv:2103.12038

[2] R. Rossi, PRL **119**, 045701 (2017) [arXiv:1612.05184]