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Continuous loading of a non-dissipative atom trap

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Abstract. – We study theoretically a scheme in which particles from an incident beam are trapped in a potential well when colliding with particles already present in the well. The balance between the arrival of new particles and the evaporation of particles from the trapped cloud leads to a steady state that we characterize in terms of particle number and temperature. This scheme is particularly interesting for a cigar-shaped potential where different longitudinal and transverse evaporation thresholds can be chosen. We show that a resonance occurs when the transverse evaporation threshold coincides with the energy of the incident particles. It leads to a dramatic increase in phase space density with respect to the incident beam.

The trapping of atomic particles has been an invaluable tool for recent developments in atomic physics and quantum optics. It can be performed by suddenly switching on a confining potential when the particles are in the vicinity of its minimum. This method is used successfully to trap ions in electromagnetic traps [1], and neutrons [2] or atoms [3] in magnetostatic traps. Another way of trapping particles is to take advantage of a dissipative mechanism, such as in a magneto-optical trap [4]. In this way, the trap can be loaded continuously since a friction force dissipates the excess energy of the particles and prevents them from escaping. The loading of neutral atoms into a trap using collisions with a buffer gas also belongs to the dissipative category [5].

In this paper we investigate a different loading mechanism, in which particles are injected into a potential well, and can be trapped by undergoing an elastic collision with one of the particles already present in the well. After the collision, the incident particle has an energy below the depth U of the potential well and gets trapped. The excess energy is then redistributed to the whole trapped sample via elastic collisions and it is subsequently released by the evaporation of a trapped particle. We show that it is possible to accumulate in this way particles with an equilibrium temperature $T \ll U/k_{\mathrm{B}}$. We also show that, for experimentally feasible conditions, a large increase in phase space density can be achieved with respect to the one of the incident beam of particles. Our study constitutes a realistic description of the continuous loading of a trap, consistent with Liouville's theorem [6]. It has to be contrasted with the elegant, but simplified model of [7], where new atoms are injected into a trap at a

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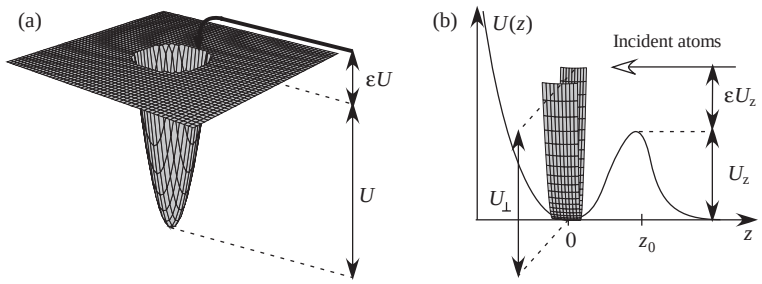

Fig. 1 – (a) First model: atoms with an energy $(1+\epsilon)U$ are injected into a harmonic trap and can be captured by collisions with particles already present in the trap. Trapped atoms with an energy larger than U are evaporated. (b) Second model: incoming atoms undergo collisions with trapped particles or with those reflected on the potential barrier at $z < 0$. Evaporation occurs if the longitudinal or transverse energy of a particle exceeds the evaporation threshold U_z or $U_\perp$. The phase space density of the trapped particles can be orders of magnitude larger than the phase space density of the incoming beam.

given energy, but where evaporation of previously trapped atoms at the same energy is not taken into account; in that case, the gain in phase space is only limited by *ad hoc* losses, such as three-body collisions.

We develop two models in this paper. The first model deals with an isotropic harmonic trap, assuming an evaporation criterion based on the total energy of a particle. This allows to use simple analytical expressions for the evaporation rate and it leads to a proof-of-principle of the process, as long as the energy of the injected particles is small with respect to the trap depth. The second model is more elaborate and leads to much more spectacular predictions for a realistic situation. It assumes an anisotropic trap (cigar-shaped potential) and it takes advantage of different evaporation rates along the long axis of the cigar and in the transverse directions. One can then accumulate particles in the well even if their incident energy notably exceeds the well depth. We conclude the paper by giving some indications on the dynamics of the loading process.

For our first model, we consider an isotropic harmonic potential with frequency ω. An atomic beam with flux Φ and a mean energy $U(1+\epsilon)$ per particle is injected into a potential well (fig. 1a). We assume that those atoms undergoing collisions are trapped. Atoms are evaporated as soon as their total (kinetic+potential) energy after a collision exceeds the threshold U. The steady-state energy distribution $P(E)$ of the trapped gas is approximated by a truncated Boltzmann distribution with temperature T [8]:

$$P(E) \propto \rho(E) e^{-E/T} \theta(U - E),$$

where $\rho(E)$ is the density of states and θ the Heaviside step function. For simplicity, we set the Boltzmann constant $k_{\mathrm{B}} = 1$.

In order to determine T and the number N of trapped atoms, we equalize the incoming and outgoing fluxes of energy and particles. An incoming atom brings the energy $U(1+\epsilon)$. The average energy carried away by an evaporated atom can be written as $U + \kappa T$, where κ is a dimensionless coefficient depending on the ratio $\eta = U/T$. It can obtained from the collision kernel of the Boltzmann equation, by calculating the probability that an atom emerges from a binary elastic collision with a kinetic energy larger than a given threshold [8, 9]. From the equation

$$(1+\epsilon)U = U + \kappa T \Longrightarrow \frac{\kappa(\eta)}{\eta} = \epsilon, \tag{1}$$

we deduce η, hence the temperature. The function $\kappa(\eta)/\eta$ equals 1/6 for $\eta = 0$ and decreases when η increases. Therefore, a steady state exists only for values of ϵ smaller than 1/6. For large η, the function $\kappa(\eta)$ tends to 1 and we obtain $\eta \simeq \epsilon^{-1}$, *i.e.*

$$\epsilon \ll 1 \Longrightarrow T \simeq \epsilon U. \tag{2}$$

In other words, the equilibrium temperature is equal to the excess energy of the incoming atoms.

We now determine the number of atoms in steady state by comparing the flux of atoms injected into and evaporated from the trap. We first note that only a fraction of the incident flux Φ contributes to the feeding of the trap. Indeed, the above description of the evaporation is only valid in the collisionless (or Knudsen) regime, where the mean free path is much larger than the size of the atom cloud. Therefore, most of the incoming atoms cross the trap without any collision. To calculate the fraction f of incoming atoms which collide with a trapped atom, we assume that all incoming trajectories pass through the center of the trap. For the low temperatures considered here, the elastic collisions are isotropic (s-wave regime) and they are characterized by the total elastic cross-section σ. We find $f = \sqrt{2\pi}\bar{r}n_0\sigma$, where $\bar{r} = \sqrt{kT/m\omega^2}$ is the size of the trapped cloud and n_0 the density at the center of the harmonic trap. The outgoing flux can be calculated using classical kinetic equations. The average collision rate is $\gamma = \sqrt{2/\pi}n_0\sigma\Delta v$, where $\Delta v = \sqrt{kT/m}$. In the collisionless regime $\gamma \ll \omega$, the probability that an atom is evaporated after a collision is $p \simeq 2\eta e^{-\eta}$ if $\eta \gg 1$ [8]. The balance between the incoming flux $\Phi_{\rm in} = f\Phi$ and the outgoing flux $\Phi_{\rm out} = p\gamma N$ then gives

$$\Phi_{\rm in} = p\gamma N \Longrightarrow N = \epsilon\frac{\Phi}{\omega}e^{1/\epsilon}\,. \tag{3}$$

This result is intuitive. In the regime $\epsilon^{-1} \sim \eta \gg 1$, the number of atoms in the trap scales as $e^{1/\epsilon} \sim e^{U/T}$, which is a direct consequence of Boltzmann's law. The number N can be in principle very large. However, one should keep in mind the validity criterion for the Knudsen regime $f < 1$, which imposes $e^{1/\epsilon} < U/(\Phi m\sigma\omega)$. This puts an upper bound $N_{\rm max} \sim \epsilon U/(m\omega^2\sigma)$ on the number of trapped atoms.

Qualitatively new features arise in the second model that we now discuss and which constitutes the main subject of this paper. Here, atoms are injected across the plane $z = z_0$ with a negative velocity along the z-axis, and with some transverse potential and kinetic energy (fig. 1b). For such a geometry, the evaporation can be either longitudinal or transverse. The longitudinal evaporation is a direct consequence of the loading mechanism: an atom which crosses the plane $z = z_0$ with a positive velocity is evaporated. Furthermore, one can experimentally arrange that an atom is also evaporated when its distance $r = (x^2 + y^2)^{1/2}$ to the z-axis exceeds a threshold value r_0. For instance, one can use a magnetic gradient to confine the atoms transversally and use a radio-frequency wave to flip their magnetic moment when they cross the surface of the cylinder $r = r_0$ [11].

The possibility to control independently the transverse and longitudinal evaporation thresholds is of particular interest if one considers an anisotropic trap with $\omega_z \ll \omega_\perp$, where $\omega_\perp$ and ω_z stand for the oscillation frequencies in the xy plane and along the z-axis, respectively. In this case, the trapped gas is cigar-shaped and one can reach a regime where the z motion is in the hydrodynamic regime ($\gamma \gg \omega_z$), while the transverse motion is still collisionless ($\gamma \ll \omega_\perp$). In other words, the mean free path of an atom is supposed to be larger than the radial size of the cigar, but smaller than its length. All incident atoms undergo a collision with the trapped atoms and are captured. We now study how the simple approach presented above is modified and we derive the expected gain in phase space density for a realistic situation.

The evaporation rate along the z-axis is notably reduced as compared with the rate derived in the collisionless regime for the same ratio $\eta_z = U_z/T$. Indeed most atoms which emerge from a collision with an energy E_z larger than the evaporation energy $U_z = m\omega_z^2 z_0^2/2$ undergo, before reaching the point z_0, another collision which can bring their energy E_z below U_z. Using a molecular-dynamics simulation [12–14], we have calculated i) the probability p_z that an atom reaches $z = z_0$ after a collision and is evaporated, ii) the average energy $U_z + \kappa_z T$ carried away by this atom. For η_z in the range from 4 to 7, we find

$$\omega_z \ll \gamma: \quad p_z \simeq 0.14 e^{-\eta_z} \omega_z/\gamma, \quad \kappa_z \simeq 2.9. \tag{4}$$

Under these operating conditions, the evaporation threshold $U_\perp = m\omega_\perp^2 r_0^2/2$ in the xy plane is a crucial control parameter. One can set $U_\perp$ to a value larger than U_z, so that the particles evaporated transversely carry an energy $U_\perp + \kappa_\perp T$ notably larger than the energy $U_z + \kappa_z T$ for a particle evaporated along z. If the trap were operated in the collisionless regime for the z motion, this would make the transverse evaporation unlikely and inefficient. However, thanks to the reduction of p_z by a factor ω_z/γ with respect to the collisionless case, we can raise $U_\perp$ to a value larger than U_z and still obtain similar probabilities p_z and $p_\perp$ for longitudinal and transverse evaporation.

The equilibrium temperature T is now obtained by generalizing (1):

$$(1+\epsilon)U_z = \frac{p_z}{p_z + p_\perp}(U_z + \kappa_z T) + \frac{p_\perp}{p_z + p_\perp}(U_\perp + \kappa_\perp T). \tag{5}$$

Similarly, the number of atoms in steady state is obtained from the generalization of (3):

$$\Phi_{\text{in}} = (p_\perp + p_z)\gamma N. \tag{6}$$

The values of p_z and κ_z have been given in (4). We have also derived the corresponding values for the transverse motion from a molecular-dynamics simulation. For collision dynamics ranging from the Knudsen regime ($\gamma \ll \omega_\perp$) to the hydrodynamic regime (up to $\gamma = 5\omega_\perp$), we fit our results for $\eta_\perp = 8$ to 13 by the formula

$$p_\perp \simeq 2.0 e^{-\eta_\perp} \frac{\omega_\perp}{\omega_\perp + 1.4\gamma}, \quad \kappa_\perp \simeq 2.0. \tag{7}$$

The set of equations (4)-(7) now allows us to determine T and N for a given experimental situation.

As a concrete example, we consider a beam of rubidium atoms ($\sigma = 7.1 \times 10^{-16}\,\text{m}^2$) injected in a trap with $\omega_\perp = 100\omega_z$ and $\omega_z/2\pi = 10\,\text{Hz}$. The flux is $\Phi = 10^7$ atoms/s, with an average velocity $v_i = 20\,\text{cm/s}$ and a velocity dispersion $\Delta v_i = 4\,\text{cm/s}$ along each of the three axes x, y, z (temperature $T_i \sim 17\,\mu\text{K}$) [15]. This beam is confined transversely by a harmonic potential with the same frequency $\omega_\perp$ as in the trap. In these conditions, the initial phase space density is $\mathcal{D}_i \sim 2 \times 10^{-5}$. The barrier height U_z in z_0 (see fig. 1b) is chosen such that only atoms with an incident velocity larger than $v_i - \Delta v_i$ are transmitted ($U_z = m(v_i - \Delta v_i)^2/2 \sim 135\,\mu\text{K}$). The flux passing in $z = z_0$ is then $\Phi_{\text{in}} = 0.84\Phi$ and the average excess energy ϵU_z of the incoming atoms is $\sim 140\,\mu\text{K}$ (*i.e.* $\epsilon \sim 1$).

The temperature and the number of atoms are shown in fig. 2 as a function of the transverse evaporation threshold. We also give the phase space density $\mathcal{D}$ in the trap calculated using Boltzmann statistics. It shows a sharp maximum $\mathcal{D} \sim 10^{-2}$ when $U_\perp$ is close to $270\,\mu\text{K}$ (*i.e.* $U_\perp \sim (1+\epsilon)U_z$), in which case $p_\perp/p_z \sim 2$. The gain in phase space density with respect to the incident beam is then ~ 500, while it is only 10 if one restricts evaporation to the longitudinal

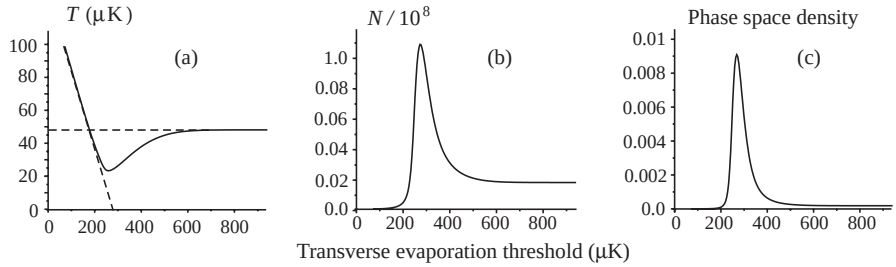

Fig. 2 – Variation of the temperature (a), of the number of trapped atoms (b), and of the phase space density (c), as a function of the transverse evaporation threshold $U_\perp$. The characteristics of the injected atomic beam are given in the text. The dashed lines in (a) indicate the variation of the temperature if one neglects either longitudinal evaporation (line with negative slope) or transverse evaporation (horizontal line).

direction ($U_\perp = \infty$). In this optimum regime, we find in steady state $\gamma \sim \omega_\perp$, $N \sim 10^8$ atoms and $T = 24\,\mu$K ($\eta_z \sim 5.7$, $\eta_\perp \sim 11.4$). Note that the temperature is now much lower than the excess energy ϵU_z. This has to be contrasted with the result of the first model where one always gets $T \geq \epsilon U$.

The spectacular resonance in the steady-state phase space density shown in fig. 2 is a general feature of our second model. It occurs when the transverse evaporation threshold $U_\perp$ is set to a value near the total incident energy $(1+\epsilon)U_z$. This resonance, which allows to increase by orders of magnitude the phase space density of the trapped cloud with respect to that of the incoming beam, constitutes the main result of this letter.

We now discuss the characteristics of the system on the two sides of the resonance. When $U_\perp$ is lower than the optimum value, the evaporation is essentially transverse since $p_\perp \gg p_z$. In this case, (5) simplifies to give

$$p_\perp \gg p_z : T = \frac{1}{\kappa_\perp}((1+\epsilon)U_z - U_\perp). \tag{8}$$

We recover in this way the linear variation of T for a small $U_\perp$ (see fig. 2). The number of atoms, given by $\Phi_{\rm in} \sim p_\perp \gamma N$, increases as $U_\perp$ increases. The approximation (8) is valid until $U_\perp$ approaches $(1+\epsilon)U_z$, in which case it would lead to $T \sim 0$. At this point the longitudinal evaporation cannot be neglected any more, and p_z and $p_\perp$ are of the same order.

When $U_\perp$ is larger than the optimal value, the transverse evaporation becomes inefficient and one is left with a purely longitudinal evaporation. The temperature and the number of trapped atoms are then independent of $U_\perp$. From (5)-(6), one gets $T = \epsilon U_z/\kappa_z$ (horizontal dashed line in fig. 2a) and $N = \Phi_{\rm in} e^{\eta_z}/(0.14\omega_z)$, which is very reminiscent of the results of the first model.

The gain G in phase space is an important result of our study. It depends on the incident flux Φ for a given loading and trapping configuration (parameters ϵ and $\omega_\perp/\omega_z$). For relatively small incident flux, we find that G increases with increasing Φ. We consider again the above example and we suppose that we increase Φ from 10^7 to 10^8 atoms/s. Due to an increase of the collision rate and thus of the evaporation efficiency, we find that the steady-state atom number N is multiplied by 15, and that the temperature decreases by 10%. The phase space gain then passes from 500 to 900.

The phase space gain G as a function of the flux Φ saturates when the transverse motion enters the hydrodynamic regime ($\gamma \gg \omega_\perp$). In this case, we obtain a "universal" result for

the gain G, which does not depend on the details of the loading process, but only on the two dimensionless parameters ϵ and $\omega_\perp/\omega_z$. Indeed the ratio $p_\perp/p_z$ obtained from (4)-(7) is a function of $\eta_\perp - \eta_z$ and ϵ only. The temperature and the ratio N/Φ are independent of the flux Φ, hence a gain G also independent of Φ. Assuming $\epsilon \sim 1$ as in the example above, this maximal gain is $G \sim 10^3$ for $\omega_\perp/\omega_z = 10^2$, and $G \sim 10^4$ for $\omega_\perp/\omega_z = 10^3$.

We now investigate the dynamics of the system and the time necessary to reach the steady state of the system around the resonance. The time evolution of the atom number N and of the total energy $3NT$ is given by

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \Phi_{\mathrm{in}} - (p_z + p_\perp)\gamma N, \tag{9}$$

$$\frac{\mathrm{d}(3NT)}{\mathrm{d}t} = \Phi_{\mathrm{in}}(1+\epsilon)U_z - p_z\gamma N(U_z + \kappa_z T) - p_\perp \gamma N(U_\perp + \kappa_\perp T). \tag{10}$$

The steady state of this set of non-linear equations corresponds to the solution of (5)-(6). Initially, the description in terms of a thermal equilibrium fails since no atoms are present. The loading of the trap is initiated by collisions between atoms of the incoming and the reflected outgoing beams. In the above example, the collision rate γ between the two beams is of the order of ω_z, which ensures an efficient start of the whole process.

The detailed study of the non-linear dynamics involved in (9)-(10) is outside the scope of this letter. A simple hint on the behavior of the solutions consists in linearizing these equations around the steady state discussed above. Suppose, for simplicity, that evaporation occurs essentially in the longitudinal direction. The two time constants of the corresponding system are (within a numerical coefficient) $\tau_1 = \tau\eta_z^2$ and $\tau_2 = \tau/\eta_z^2$, where $\tau = N_{\mathrm{s}}/\Phi_{\mathrm{in}}$ is the time required to send the number of atoms corresponding to the steady-state number N_{s} into the trap. Since $\eta_z \gg 1$, these two time constants are very different from each other and the time τ_1 needed to reach equilibrium exceeds by a factor η_z^2 the "natural" time scale τ. For the same experimental conditions as in fig. 2, we find a slow time evolution of $N(t)$, with a time constant $\tau_1 \simeq 160\,\mathrm{s}$ (to be compared with $\tau = N_{\mathrm{s}}/\Phi_{\mathrm{in}} \sim 10\,\mathrm{s}$). By contrast, the temperature $T(t)$ reaches rapidly a value close to steady state.

In order to confirm the above analytical treatment, we have run *ab initio* a numerical simulation based on molecular dynamics. We have obtained values for the evolution of $N(t)$ and $T(t)$ which differ by no more than 20% from those deduced from the set of equations (9)-(10). It is worth noting that for such long time scales, losses, *e.g.* due to 3-body collisions, become important and will limit in practice the gain in phase space density as in [7].

Our analysis has been carried out for a gas in the non-degenerate regime. Quantum effects could be taken into account using a generalization of the Boltzmann equation which includes bosonic stimulation or fermionic inhibition [16]. In particular for bosons, "stimulated emission" processes will speed up the dynamics of the whole scheme. Such a detailed analysis is outside the scope of the present paper. For this reason, we have deliberately chosen in the example above a very small initial phase space density ($\mathcal{D}_i \sim 2 \times 10^{-5}$) so that the trapped gas always remains in the classical regime, even after a gain $G \sim 10^3$. A transposition of this result to a denser beam ($\mathcal{D}_i \geq 10^{-3}$) would eventually bring the trapped gas into the degenerate regime.

To summarize, we have shown that a continuous loading of a non-dissipative trap could be achieved with the help of evaporation. This process can be remarkably efficient in the case of an anisotropic, cigar-shaped, geometry. In this case, the incident particles can have an incident energy ϵU_z of the order of the entrance barrier U_z and still reach a temperature $T \ll U_z$. For a realistic configuration, a gain in phase space of several orders of magnitude is possible between the incident beam and the trapped cloud. This method constitutes an

alternative to the proposal of [17] based on a continuous evaporative cooling of a guided beam. It is also complementary of the scheme recently achieved at MIT, where a succession of condensates was merged in the same trap, producing thus a steady-state BEC in a given location [18].

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