Loading atoms in
a bi-dimensional light trap

Pierre Desbiolles and Jean Dalibard
Laboratoire Kastler Brossel∗,
24 rue Lhomond, 75005 Paris, France

Abstract

We investigate theoretically a trap formed by two laser evanescent waves propagating at the surface of a dielectric prism, which confine the atoms in a Morse potential along the direction perpendicular to the prism. We consider a loading process based on the Sisyphus effect, in which a single spontaneous Raman transition is involved. We show that it is possible to achieve in this way an efficient loading of the ground state of the Morse potential, and to get thus a quasi bi-dimensional atomic gas at the surface of the dielectric.

Pacs 32.80.Pj, 42.50.Vk, 05.30.Jp

Using laser cooling techniques, it is now possible to prepare atomic samples cooled at the recoil limit, or even below, i.e. with a r.m.s. momentum of the order or smaller than the photon momentum \( \hbar k \) [1, 2, 3]. These techniques have allowed recently a major breakthrough in the physics of quantum degenerate gas, i.e. the observation of Bose-Einstein condensation [4, 5, 6]. One of the key elements for this realization is the possibility to confine the atomic gas without heating, which was achieved in [4, 5, 6] using a magnetic trap. This allows a further cooling of the atoms using evaporation, leading to a spectacular increase of the phase space density of the gas.

Another way for confining an atomic gas without heating is to use a laser dipole trap [7]. In this case, the confining potential is the light-shift,
or AC Stark shift, of the atomic internal ground state \[8\]. Consequently, depending on the sign of the detuning between the laser frequency and the atomic resonance frequency, the atoms are stored in locations where the light intensity is maximal \([9, 10, 11]\) or minimal \([12, 13, 14]\). This detuning is any case chosen large enough so that photon scattering processes, which cause some heating of the trapped atoms, occur only with a small rate.

In the present paper, we focus on a particular type of laser dipole trap, which provides a strong confinement of the atoms along the vertical direction \(z\), while the atoms remain quasi free in the horizontal plane. We show that it is possible, using Sisyphus cooling, to prepare an appreciable fraction of atoms in the ground state of the motion along the \(z\) direction. The resulting atomic sample is a 2D gas; as it is well known from statistical physics, this gas should have remarkable properties, quite different from 3D ensembles, when the quantum degeneracy factor \(\xi_{2D} = \sigma \Lambda_{dB}^2\) reaches unity \([15, 16, 17, 18, 19]\).

We have introduced here the surfacic density of the gas \(\sigma\) and the de Broglie thermal wavelength \(\Lambda_{dB} = h/\bar{p} \sqrt{2\pi}\), where \(\bar{p}\) is the r.m.s. momentum along a given axis of the transverse motion.

The trap that we investigate is formed with two evanescent waves \(A\) and \(B\), obtained from the total internal reflection of two incident Gaussian laser waves at the surface of a dielectric prism with index \(n\) (fig. 1). This surface is supposed to be plane and horizontal. The wavelengths \(\lambda_A\) and \(\lambda_B\) of the two waves are of the same order and we put \(k = 2\pi/\lambda_A \simeq 2\pi/\lambda_B\). The incidence angles \(\theta_i (i = A, B)\) are different so that the decay lengths \(\kappa_i^{-1}\) of the evanescent electric fields \((\kappa_i = k(n^2 \sin^2 \theta_i - 1)^{1/2})\) are also different. We choose \(\theta_A < \theta_B\) so that \(\kappa_A^{-1} > \kappa_B^{-1}\); in addition the wave \(A\) has a negative detuning with respect to the atomic transition of interest \(b - e\), so that it attracts towards the prism the atoms in the ground state \(b\), while the wave \(B\) has a positive detuning and repels those atoms from the prism. Consequently the total light force acting on the atoms is attractive on the long range and repulsive on the short range. There exists a plane parallel to the surface of the prism, where the force is zero, and which is located a fraction of optical wavelength above it.

The principle of this trap using a double evanescent wave has been first proposed in \([20]\); in that work, the authors have analyzed classically the motion of the atoms in the trap. Here, in section 1, we focus on the quantum aspect of the vertical motion of the atoms. We consider the particular case \(\kappa_B = 2\kappa_A\), for which the potential created by the light is a Morse poten-
tial; analytical results can then be obtained concerning the number and the position of the bound states.

We then address the central point of this paper which concerns the loading of the trap. The scheme that we investigate is based on a Sisyphus effect [21, 22] and it is represented in fig.2. We consider atoms with two internal ground states $a$ and $b$, corresponding for instance to the two hyperfine states of an alkali atom. We assume that the frequencies $\omega_A$ and $\omega_B$ have been chosen such that atoms in level $a$ see only a strong repulsive potential. On the contrary, atoms in $b$ see the Morse potential and they can be trapped just above the surface of the prism. The loading procedure that we study theoretically works as follows. Atoms are prepared in state $a$, and they are dropped onto the prism on which they bounce if the repulsive potential barrier created by the lasers is larger than their initial energy. The turning point $z_0$ in the repulsive potential is adjusted at the same altitude as the minimum of the Morse potential. Around this turning point, a Raman process from $a$ to $b$ can occur, involving the absorption of a laser photon, and a spontaneous emission. This happens to be an efficient way for populating the ground state of the Morse potential.

We analyze this process using two different theoretical approaches. The first one is based on the Born approximation (section 2). The second approach, presented in section 3, consists in a numerical integration of the Schrödinger equation describing the atomic motion, the spontaneous emission processes being taken into account using the Monte-Carlo wave function approach [23, 24, 25]. The two approaches are found to be in good agreement within their validity domain. They indicate that an appreciable trapping probability into the ground state of the Morse potential is achievable using this Sisyphus process.

To end this presentation, we note that other schemes have been proposed and/or demonstrated to take benefit of the short spatial scale of an evanescent laser field and to achieve an efficient cooling process. A simple Sisyphus cooling process can occur in a single evanescent wave for 3-level atoms $a, b, e$ such as those considered here. The evanescent wave has to be blue detuned with respect to both transitions $a \rightarrow e$ and $b \rightarrow e$, and the repulsive potential is then stronger for atoms in level $b$, since the detuning for the $b \rightarrow e$ transition is smaller than for the $a \rightarrow e$ transition. Atoms incident in level $b$ may undergo a spontaneous Raman transition towards level $a$ close to the turning point $z_0$; they loose in this case an appreciable fraction of their initial kinetic
energy, although the final motion is still asymptotically free, contrarily to the situation considered in the present paper. This process has been studied both theoretically [26, 27] and experimentally [28, 29]. It has also been proposed to achieve a trap with a Morse potential similar to the one considered here, and to load it directly from a magneto-optical trap located at the contact of the dielectric prism [30]. Finally, it has been suggested recently [31] to use an evanescent wave to decelerate incident atoms, which are then transferred through a spontaneous Raman transition at the antinode of a laser standing wave located in the vicinity of the dielectric, achieving therefore a quasi bi-dimensional gas similar to the one considered here.

1 Trapping in the Morse potential

In this section, we derive the potential created by the pair of evanescent waves shown in fig.2 onto the atoms, depending on their internal state $a$ or $b$. For state $b$, this corresponds to a Morse potential, and we briefly recall the number and position of the corresponding bound states. We assume that the evanescent laser fields $E_i \exp(-\kappa_i z) \ (i = A, B)$ do not saturate the atomic transitions, i.e. the maximal Rabi frequency along an atomic trajectory $\Omega_i(z_0) = dE_i \exp(-\kappa_i z_0)/2\hbar$ is small compared to the detuning of the wave $i$ with respect to the transitions $a - e$ and $b - e$. We suppose for simplicity that the reduced atomic dipole $d$ is the same for both transitions. This is the case for alkali atoms provided the evanescent field is linearly polarized and provided the detunings are large compared to the hyperfine excited state structure. The action of the lasers onto the atomic motion can then be described by a potential equal to the total light-shift of the occupied level $\eta = a, b$ [8]:

$$U_\eta(z) = \sum_{i=A,B} \frac{\hbar \Omega_i^2(z)}{4(\omega_i - \omega_{\eta e})}$$

(1)

where $\omega_i$ is the frequency of the wave $i = A, B$, and $\omega_{\eta e}$ is the frequency of the atomic transition $e \leftrightarrow \eta = a, b$.

We choose here

$$\omega_{ae} < \omega_A < \omega_{be} \quad \text{and} \quad \omega_{ae} < \omega_{be} < \omega_B$$

(2)

Therefore both waves tend to repel the atoms in state $a$. We assume that
the effect of the wave $A$ is dominant for those atoms so that:

$$U_a(z) \simeq \frac{\hbar \Omega_A^2(0)}{4(\omega_A - \omega_{ae})} e^{-2\kappa_A z}$$

For level $b$, the two waves $A$ and $B$ have opposite effects, $A$ attracts the atoms towards the prism and $B$ repells them. For $\kappa_B = 2\kappa_A$, $U_b(z)$ is a Morse potential. With a prism of index $n = 1.5$, this is achieved taking for instance $\theta_A = 48^\circ$ and $\theta_B = 70^\circ$; we get in this case $\kappa_B = 2\kappa_A = k$. The depth $U_0$ and the position of the minimum $z_0$ of the Morse potential are given by:

$$U_0 = \frac{\hbar \Omega_B^2(0)}{4(\omega_B - \omega_{be})} e^{-4\kappa z_0} = \frac{1}{2} \frac{\hbar \Omega_A^2(0)}{4(\omega_{be} - \omega_A)} e^{-2\kappa z_0}$$

where we have put $\kappa = \kappa_A$.

The number of bound states in the Morse potential is given by the integral part of $\sqrt{mU_0/(2\hbar^2\kappa^2)} + 1/2$, where $m$ is the atomic mass [32]. We choose in the following:

$$U_0 = 50\frac{\hbar^2\kappa^2}{m}$$

which corresponds to 5 bound states. Each eigenstate $|n\rangle$ ($n = 0, 1, \ldots$) of the Hamiltonian describing the motion in the Morse potential corresponds to a wave function $\sqrt{\kappa}X_n(\kappa(z - z_0))$, which can be expressed in terms of the degenerate hypergeometric function. The energies associated with these bound states are given by [32]:

$$E_n = -U_0 \left( 1 - \sqrt{\frac{2\hbar^2\kappa^2}{mU_0}} \left( n + \frac{1}{2} \right) \right)^2$$

Using (5) and $\kappa = k/2 = 3.7 \times 10^6 m^{-1}$ for cesium atoms, we find that the positions $E_n/h$ of the five bound states are $-41.6$ kHz, $-25.1$ kHz, $-12.8$ kHz, $-4.6$ kHz and $-0.5$ kHz.

The choice (5) associated with the relations (4) determines the two ratios $\Omega_A^2(z_0)/(\omega_{be} - \omega_A)$ and $\Omega_B^2(z_0)/(\omega_B - \omega_{be})$. The final constraint on our parameters originates from the requirement that the turning point of the trajectories in $U_a(z)$, for a given asymptotic momentum $p_i$, should coincide with $z_0$. This determines the values of $\Omega_A(z_0)$ and $\omega_A$, in terms of the two
frequency scales of the problem, $\hbar\kappa^2/m$ and $\Delta = \omega_{be} - \omega_{ae}$. For instance, for a typical incident momentum $p_i = 25\hbar\kappa$, we get:

$$\Omega_A(z_0) = 18 \left( \frac{\hbar\kappa^2}{m\Delta} \right)^{1/2} \omega_A - \omega_{ae} = \Delta/4$$

(7)

For sodium and cesium atoms, this value for $p_i$ corresponds to atoms dropped respectively from a height of 7 mm and 100 µm.

We have neglected in the present reasoning the effect of the van der Waals interaction between the atoms and the dielectric prism, varying as $1/z^3$. This approximation is valid provided the minimum of the Morse potential $z_0$ is far enough from the dielectric surface. This is illustrated in fig.3 where we have taken $\kappa z_0 = 2$ and plotted the potential for cesium atoms, with and without the van der Waals interaction [33]. We have also neglected in our analysis the effect of gravity. This is valid since the variation of the gravitational potential $mgz$ is small compared to that of the Morse potential on the length scale $\kappa^{-1}$. The modification of the position of the bound states of the Morse potential can therefore be neglected. One should keep in mind however that the atoms are not completely free when they emerge from the evanescent wave region and that they will eventually slow down and turn back towards the dielectric prism.

2 The trapping probability in the Born approximation

We now turn to the calculation of the probability for an atom incident in state $a$, with asymptotic momentum $p_i$, to be transferred into a bound state $|b, n\rangle$. The incident state $|p_i\rangle$ corresponds to the eigenfunction of the Hamiltonian $H = (p^2/2m) + U_a \exp(-2\kappa z)$ [34]:

$$\frac{1}{\sqrt{L}} \Psi_i(\kappa(z - z_0)) = \left( \frac{2P_i}{\pi L} \sinh(\pi P_i) \right)^{1/2} K_{iP_i} \left( P_i e^{-\kappa(z-z_0)} \right)$$

(8)

where $P_i = p_i/\hbar\kappa$ and where $K_{iP}$ is the Bessel K-function of imaginary parameter $iP$. These wave functions are normalized in a box between $z = 0$ and $z = L \gg \kappa^{-1}$. 

6
This calculation is done using the second order time perturbation theory and dividing the transition rate by the incident flux $\frac{p_i}{2ML}$, which corresponds to the Born approximation for this problem. The transition rate for the absorption of a laser photon and the spontaneous emission of a Raman photon with wave vector $\vec{k}_f$ and polarization $\vec{\epsilon}_f$, can be written in this approximation:

$$W_n = \frac{2\pi}{\hbar} \left| \sum_e \frac{\langle b, n; \vec{k}_f, \vec{\epsilon}_f | V | e \rangle \langle e | V | a, p_i \rangle}{E_a + \hbar\omega_A - E_e} \right|^2 \rho(E_f)$$  \hspace{1cm} \text{(9)}$$

The operator $V$ describes the atom-field coupling, responsible for absorption, spontaneous and stimulated emission processes; $\rho(E_f)$ corresponds to the final density of states for the fluorescence photon.

The sum (9) runs on all possible excited states $e$, with energy $E_e$. In the following, we assume that $\omega_A$ is chosen sufficiently close to an atomic resonant frequency so that only a single excited state $e$ contributes\(^1\). After summation over $\vec{k}_f$ and $\vec{\epsilon}_f$, the equation (9) can then be simplified to give:

$$W_n = \Gamma_{eb} \frac{\Omega_A^2(z_0)}{4\delta_A^2} \frac{1}{\kappa L} |\mathcal{O}_n(p_i)|^2$$  \hspace{1cm} \text{(10)}$$

where $\Gamma_{eb}$ is the decay rate from $e$ to $b$, $\delta_A = \omega_A - \omega_{ae}$, and where $\mathcal{O}_n(p_i)$ describes the overlap between the incident wave function and the final one, taking into account the exponential variation of the field:

$$\mathcal{O}_n(p_i) = \int \chi_n(u) e^{-u\Psi_i(u)} \, du$$  \hspace{1cm} \text{(11)}$$

Dividing the transition rate $W_n$ by the incident flux $\frac{p_i}{2mL}$, we obtain the trapping probability $P_n$:

$$P_n = \Gamma_{eb} \frac{\Omega_A^2(z_0)}{4\delta_A^2} \tau |\mathcal{O}_n(p_i)|^2$$  \hspace{1cm} \text{(12)}$$

where $\tau = 2m/\kappa p_i$ is the typical bouncing time, i.e. the time spent by an atom with velocity $p_i/m$ in the region of extension $\kappa^{-1}$ where the evanescent

\(^1\)For the specific case of an alkali atom, Cesium for instance, this means that the detuning between $\omega_A$ and the $D_2$ (or $D_1$) resonance line is small compared to the fine structure splitting, but large compared to the hyperfine structure of the $P_{3/2}$ (or $P_{1/2}$) multiplet.
field is significant. As expected, this probability does not depend of the length \( L \) of the quantization box.

The result (12) can also be written:

\[
P_n = n_{\text{scat}} |O_n(p_i)|^2 \tag{13}
\]

where:

\[
n_{\text{scat}} = \frac{\Omega_A^2(z_0)}{4\delta_A^2} \Gamma_{eb} \tau = \frac{\Gamma_{eb}}{\delta_A} P_i \tag{14}
\]

is the mean number of photons which would be scattered per bounce for a closed \( a - e \) transition and with a decay rate for \( e \) equal to \( \Gamma_{eb} \) [35, 12]. Actually in the situation considered here, the number of photons emitted on the \( e - b \) transition by a given atom cannot exceed 1, and the result (13) is meaningful only when \( n_{\text{scat}} \) is small compared to 1. Otherwise, the atom has a large probability to decay to \( b \) before it reaches the location of the Morse potential well. In the next section, we will show how to get rid of this hypothesis using the Monte Carlo wave function analysis.

For a given \( n_{\text{scat}} \ll 1 \), we now consider the overlap factor \( O_n(p_i) \). On fig.4, we have plotted the spatial variations of the function \( \Psi_i(z) \) for \( p_i = 25\hbar \kappa \) (fig.4a). We have also plotted (fig.4b) the spatial variations of the two lower states of the Morse potential (\( n = 0 \) and 1), adjusted as explained in section 1 so that the minimum of the Morse potential coincides with the turning point \( z_0 \). Since the first lobe of the function \( \Psi_i \) has a size similar to the extension of the ground state of the Morse potential, this leads to a large overlap factor \( O_0(p_i) = 0.55 \). The overlap with the state \( \chi_1 \) is smaller \( O_1(p_i) = -0.25 \).

The variations of the square of the overlap factor \( |O_0(p_i)|^2 \) with the incident momentum \( p_i \) are given in fig.5.

In a real experiment the atoms are not necessarily prepared with a well defined momentum \( p_i \). This causes a variation of the position of the turning point \( z_0 \), which does not coincide anymore with the bottom of the Morse potential. To estimate the corresponding reduction of the transfer efficiency, we have plotted in fig.6 the variations of \( |O_0(p)|^2 \), when the incident momentum \( p \) is varied around \( 25 \hbar \kappa \). This shows that a dispersion of a few \( \hbar \kappa \), which is typical of a laser cooled atomic source, does not dramatically change the conclusions drawn above concerning the trapping probability.
3 Quantum Monte-Carlo analysis of the Sisyphus process

The Born approximation presented in the previous section is valid only when the predicted number of scattered Raman photons per bounce $n_{\text{scat}}$ (eq.14) is small compared to 1. Actually, we expect that the loading of the Morse trap is optimum when $n_{\text{scat}}$ is above 1, so that most atoms “are given a chance” to decay during their bounce. On the other hand, for an efficient loading of the $\chi_0$ state, the decay should not occur before the atom has reached the turning point region, which imposes that $n_{\text{scat}}$ should not be too large compared to 1. In order to determine the optimal value for this number, we have developed a Monte-Carlo wave function analysis of the bouncing process [23, 24, 25].

We consider incident atoms prepared in state $a$ far from the surface. The initial state is a Gaussian wave packet, with an average momentum $p_i = 25\hbar \kappa$, an average position $z_i = 12.5\kappa^{-1}$ and a momentum dispersion $0.5 \hbar \kappa$. This wave packet is propagated according to the Schrödinger equation with the non-Hermitian Hamiltonian:

$$H = \frac{p^2}{2m} + U_a(z) - \frac{i\hbar}{2}C^\dagger C$$

(15)

where the jump operator $C$ is defined by

$$C = \sqrt{\Gamma_{eb}} \frac{\Omega_A(0)e^{-\kappa z}}{2\delta_A} |b\rangle \langle a|$$

(16)

The non-Hermitian part describes the dissipation occurring because of photon scattering processes. We consider here only Raman processes, which pump the atoms from $a$ to $b$, and we neglect the atomic recoil. To be rigorous, one should also take into account the Rayleigh processes, in which the atom ends up in state $a$ after the scattering event. However we do not expect that those processes will play a significant role, expect for a slight heating due to photon recoil.

The wave function $\Psi$ is propagated using a 4th order Runge-Kutta algorithm. At each time step, we evaluate the decrease $dP$ of the norm of the wave function ($dP \ll 1$), which gives the probability for a scattering event. We compare $dP$ with a random number $\epsilon$ equally distributed between 0 and 1. If no scattering occurs ($\epsilon > dP$), the wave function is just renormalized and the evolution continues. If a scattering occurs, we apply onto
the wave function the jump operator $C$, we renormalize the result and we project it onto the states $|n\rangle$. We store the coefficients $|\langle n|\Psi\rangle|^2$, which give the probability for the atom to end up in the $n$’th bound state of the Morse potential.

This time evolution of the wave function is illustrated in fig.7. As long as the atom is localized far from the evanescent wave region (fig.7a), the probability for a Raman scattering process is very low, because of the exponential factor $\exp(-\kappa z)$ appearing in the jump operator $C$. When the wave function is localized close to the classical turning point (fig.7b), a scattering event may occur (fig.7c) projecting the atom in the $b$ internal state. If no scattering occurs, the atom is reflected back (fig.7d); we then stop the time evolution when the center of the wave packet reaches the starting point of the evolution.

This procedure is repeated 100 times, for a given value of $n_{\text{scat}}$ (eq.14). The probabilities for ending into the two lower states of the Morse potential are given in fig.8 as a function of this number. The error bars indicate the fluctuations of the results due to the noise inherent in the Monte-Carlo method. The maximal loading probability of the ground state is 21%, obtained for $n_{\text{scat}} = 2.5$. At this value, the probability for ending in the low excited states of the Morse potential are 4.5% for $n = 1$ and 0.73% for $n = 2$.

We have also indicated in fig.8 the prediction derived using the Born approximation: $n_{\text{scat}}|\mathcal{O}_n(p_{\text{e}})|^2$ for $n = 0$ and 1. As expected, we find a good agreement between the Monte-Carlo method and this prediction for low values of $n_{\text{scat}}$, and we confirm that the optimal value for $n_{\text{scat}}$ is outside the range of validity of the Born approximation.

Finally we note that when $\Omega_A(z_0)$ and $\omega_A$ have been adjusted according to the condition (7), the parameter $n_{\text{scat}}$ is fixed to the value $100\Gamma_{\text{eb}}/\Delta$. For alkali atoms, this quantity is much smaller than the optimum $n_{\text{scat}} \simeq 2.5$. In practice, to circumvent this problem, one can mix with $A$ another weak laser wave, resonant with the $a-e$ transition, whose intensity is adjusted so that $n_{\text{scat}}$ equals the desired value, without perturbing significantly the light-shift potential.
4 Conclusion

We have presented in this paper an efficient way for loading an atom trap which consists in two evanescent waves propagating at the surface of a dielectric and which confines the atoms via a Morse potential. We have shown that a noticeable fraction of the incident atoms (21 % for $p_i = 25\hbar\kappa$) ends up into the ground state of the Morse potential. The loading probability of the excited states of the Morse potential is much smaller, which leads to a quasi-bidimensionnal gas.

We have restricted our treatment to the atomic motion perpendicular to the plane of the evanescent waves. The confinement in this plane can be achieved using the dipole force resulting from the Gaussian character of the laser beams forming the evanescent waves. It can also be provided using an additional quadrupole static magnetic field.

The lifetime of the trapped atoms may be limited by the off-resonant photon scattering from the evanescent fields. This scattering can be made arbitrarily low for the wave $B$, by choosing a large detuning $\delta_B = \omega_B - \omega_{be}$, and an accordingly large intensity $\Omega_B^2$. One then takes advantage of the fact that the photon scattering rate varies like $\Omega_B^2/\delta_B^2$ while the confining potential varies like $\Omega_B^2/\delta_B$. For the wave $A$ on the contrary, the detuning with respect to the transition $b - e$ has to be chosen smaller than the splitting $a - b$. In the case of Cesium atoms, this splitting corresponds to the hyperfine structure of the ground level (9.2 GHz); to achieve the Morse potential studied in this paper (5 bound states), one has to take an intensity for $A$ which leads to a photon scattering rate of $\sim 500$ photons per second\(^2\). If needed one can reduce this rate by changing rapidly the parameters of $A$ once the loading sequence is over.

The detection of the trapped gas can be made using a pump-probe technique, comparable to the one used to study optical lattices [36]. The pump and the probe should be directed along the $z$ axis and amplification or absorption of the probe should occur when its frequency differs from the pump frequency by an amount equal to $|E_0 - E_n|$, which is the range of a few tens of kHz for Cesium atoms.

\(^2\)These scattering processes essentially heat the transverse atomic motion. The longitudinal motion on the contrary is much less affected: due to the Lamb Dicke effect [36], each scattering event has a small probability to eject a trapped atom out of the $n=0$ state.
As pointed out in the introduction, this scheme provides an efficient way to obtain a degenerate bi-dimensionnal gas, even when starting from a non-degenerate 3D source. Indeed the surface density $\sigma_s$ after loading is given by $\sigma_s = \rho H P_0$ where $\rho$ is the volume density of the source, $H$ its vertical size, and $P_0$ the trapping probability in the ground state of the Morse potential. Even if $\xi_{3D} = \rho A_{dB}^3 \ll 1$ (non degenerate 3D gas), we can reach degeneracy for the 2D gas ($\xi_{2D} = \sigma_s A_{dB}^2 \geq 1$) if $H P_0 \xi_{3D} \geq \lambda_{dB}$. This should be feasible if $\xi_{3D}$ is not too small, since $H$ is a macroscopic length, much larger than $\lambda_{dB}$. In this reasoning we have omitted the fact that when an atom is loaded into the 2D gas, a photon is emitted which can be reabsorbed by another atom already present in the gas; this second atom may then be ejected from the 2D gas. This problem of reabsorption, which puts a limit onto the achievable phase space density by radiative cooling techniques, is essential in a 3D geometry [37]. For the present 2D geometry however, this problem is much less severe since most photons are emitted with an appreciable momentum along the $z$ axis, in a direction where no atom is present.

We thank C. Cohen-Tannoudji, C. Salomon, Y. Castin and the ENS cooling group for several useful discussions. We also acknowledge a very helpful discussion with T. Hijmans, M. Reynolds, G. Schlyapnikov and J. Walraven. This work is partially supported by DRET, CNRS, Collège de France and DRED.

* Unité de Recherche de l’Ecole normale supérieure et de l’Université Pierre et Marie Curie, associée au CNRS.

References


Figure 1: Geometrical configuration studied in this paper. (a) Atoms are confined in the $z$ direction by a Morse potential realized using two evanescent waves $A$ and $B$ propagating at the surface of a dielectric, with decay lengths $\kappa_{A}^{-1} = 2\kappa_{B}^{-1}$. (b) The waves $A$ and $B$ are detuned respectively below and above the atomic resonance transition $b-e$.

Figure 2: Loading scheme based on a Sisyphus process. Incident atoms are prepared in state $a$. They are repelled by the two evanescent waves $A$ and $B$ which are both detuned blue from the resonance $a-e$. The atom-laser parameters are chosen such that the classical turning point is just located above the minimum of the Morse potential for state $b$. A Raman scattering process occurring in the vicinity of the turning point can transfer the atom to the ground state of this Morse potential.

Figure 3: Spatial variations of the potential $U_b(z)$ without (continuous line) and with (broken line) the van der Waals interaction between the atoms and the dielectric prism. This figure corresponds to the case of Cesium atoms, with a refractive index $n = 1.5$ for the dielectric material.

Figure 4: Spatial variations of the eigenstates of the Hamiltonian $H_a$ corresponding to an asymptotic momentum $25\hbar \kappa$ (a). (b): spatial variations of the two lower states of the Morse potential $\chi_0$ (continuous line) and $\chi_1$ (broken line), adjusted such that the minimum of the Morse potential coincides with the classical turning point for state $a$.

Figure 5: Square of the overlap factor $O_0(p_i)$ between the incident wave function $\Psi_i$ corresponding to the asymptotic momentum $p_i$ and the ground state of the Morse potential $\chi_0$. The minimum of the Morse potential is adjusted in order to coincide with the classical turning point for $p_i$. The trapping probability derived from the Born approximation is equal to this quantity times the average number of scattered photons $n_{\text{scat}}$. 

16
Figure 6: Variations of the trapping probability with the incident momentum $p$, when the Morse potential has been adjusted such that its minimum coincides with the classical turning point for state $a$ and for incident momentum $p_i = 25\hbar \kappa$.

Figure 7: Time evolution of the atomic density probability, obtained using a Monte-Carlo wave function approach. (a) incident Gaussian wave packet (internal state $a$), (b) wave packet around the classical turning point (internal state $a$), (c) wave packet after a spontaneous Raman process (internal state $b$), (d) outcoming wave packet if no spontaneous Raman process has occurred during the bounce. The potential $U_a(z)$ is plotted in broken line.

Figure 8: Variations of the trapping probability in state $\chi_0$ (●) and $\chi_1$ (○), for an incident momentum $p_i = 25\hbar \kappa$, calculated from a Monte-Carlo wave function analysis. The error bars indicate the statistical fluctuations of the results. The dotted lines represent the prediction of the approach based on the Born approximation.