Dressed-atom approach to atomic motion in laser light: the dipole force revisited

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We show that the dressed-atom approach provides a quantitative understanding of the main features of radiative dipole forces (mean value, fluctuations, velocity dependence) in the high-intensity limit where perturbative treatments are no longer valid. In an inhomogeneous laser beam, the energies of the dressed states vary in space, and this gives rise to dressed-state-dependent forces. Spontaneous transitions between dressed states lead to a multivalued instantaneous force fluctuating around a mean value. The velocity dependence of the mean force is related to the modification, induced by the atomic motion, of the population balance between the different dressed states. The corresponding modification of the atomic energy is associated with a change of the fluorescence spectrum emitted by the atom. The particular case of atomic motion in a standing wave is investigated, and two regimes are identified in which the mean dipole force averaged over a wavelength exhibits a simple velocity dependence. The large values of this force achievable with reasonable laser powers are pointed out with view to slowing down atoms with dipole forces.

INTRODUCTION

Absorption and emission of photons by an atom irradiated by a resonant or quasi-resonant laser beam give rise to a variation of the atomic momentum that can be analyzed, for time scales longer than the radiative lifetime, in terms of radiative forces fluctuating around a mean value.[1–3]

For an atom at rest or slowly moving, the mean radiative force is usually split into two parts[1,2]: The first part is related to the phase gradient of the laser wave (and to the quadrature part of the atomic dipole) and is called radiation pressure. The second one, related to the intensity gradient of the laser (and to the in-phase atomic dipole), is called dipole force.

The radiation-pressure force is now well understood, and its various features, such as velocity dependence and momentum diffusion, have been analyzed in detail in terms of cycles involving absorption of laser photons and spontaneous emission of fluorescence photons.[1,2,4–6]

The dipole force, on the other hand, is due to redistribution of photons among the various plane waves forming the light wave, using absorption-stimulated emission cycles.[1,2,7,8] Unfortunately, this interpretation does not give a physical account for some characteristics of the dipole force, especially at high intensity. Consider, for example, the problem of atomic motion in a standing wave formed by two counterpropagating plane waves. At low intensity, one finds that the dipole force averaged over a wavelength is just the sum of the radiation pressures of the two running waves.[5,9] In particular, the force is a damping one for negative detuning (laser frequency lower than atomic frequency), and this is easily understood when one considers that, owing to the Doppler effect, a moving atom "sees" the counterpropagating running wave more than the other one; it therefore experiences a force opposed to its velocity (usual radiative cooling). At high intensity, however, this conclusion is reversed[1,10,11]: One finds that the force heats the atoms for a negative detuning and cools them for a positive one. Such a surprising result has not yet been interpreted physically.

The purpose of this paper is to present precisely for the high-intensity domain a new theoretical treatment of the dipole force that will allow us to give a physical interpretation for such unexpected features. This treatment is based on the dressed-atom approach that has been already applied with success to the physical interpretation of resonance fluorescence in the saturation regime.[14–16]

The fact that the dressed-atom approach is well adapted to the high-intensity limit can be easily understood. When the Rabi frequency ω_1 characterizing the strength of the laser atom coupling is large compared with the spontaneous damping rate Γ, it is a good approximation to consider first the energy levels of the combined system: atom and laser photons interacting together (dressed states). Then, in a second step, we can take into account the effect of spontaneous emission (coupling with all other empty modes of the radiation field), which can be described as a radiative-relaxation mechanism inducing, for example, population transfers between the dressed states with well-defined rates.

The case of an atom moving in an inhomogeneous laser beam raises new interesting questions. The Rabi frequency ω_1 then varies in space, since it is proportional to the position-dependent laser amplitude. It follows that the energy of the dressed states is now a function of $\mathbf{r}$, so that it is possible to define for each dressed state a force equal to minus the gradient of the energy of this state. As in the Stern–Gerlach effect, we can introduce a force that depends on the internal state of the dressed atom. Is the dipole force

connected with such dressed-state-dependent forces? Is it possible with such an approach to build a clear physical picture of dipole forces and to derive quantitative expressions for their main features: mean values, fluctuations, velocity dependence? This is the problem that we want to address in this paper.

The paper is organized as follows: In Section 1, we present our notations. The dressed-atom approach is introduced in Section 2. Section 3 is devoted to the calculation of the mean dipole force and of its velocity dependence. We then calculate in Section 4 the momentum diffusion coefficient induced by the fluctuations of the dipole force. Finally, in Section 5, we investigate the particular case of atoms moving in a standing wave, and we compare our results with previous ones.

1. NOTATIONS AND ASSUMPTIONS

The total Hamiltonian is the sum of three parts:

$$H = H_A + H_R + V, \tag{1.1}$$

where H_A is the atomic Hamiltonian, H_R the Hamiltonian of the radiation field, and V the atom–field coupling. H_A is the sum of the kinetic and the internal energies of the atom considered here as a two-level system:

$$H_A = \frac{\mathbf{P}^2}{2m} + \hbar\omega_0 b^+ b. \tag{1.2}$$

ω_0 is the atomic resonance frequency and b and b^+ the lowering and raising operators:

$$b = |g\rangle\langle e|, \qquad b^+ = |e\rangle\langle g| \tag{1.3}$$

($|g\rangle$ is the atomic ground level and $|e\rangle$ the excited one). The electromagnetic field is quantized on a complete set of orthonormal field distributions $\mathscr{E}_\lambda(\mathbf{r})$, one among which corresponds to the laser field $\mathscr{E}_L(\mathbf{r})$. The Hamiltonian H_R of the free radiation field is then

$$H_R = \sum_\lambda \hbar\omega_\lambda a_\lambda^+ a_\lambda, \tag{1.4}$$

where a_λ and a_λ^+ are, respectively, the destruction and the creation operators of a photon in the mode λ. The atom–field coupling V can be written in the electric-dipole and rotating-wave approximations as

$$V = -\mathbf{d} \cdot [b^+ \mathbf{E}^+(\mathbf{R}) + b\mathbf{E}^-(\mathbf{R})], \tag{1.5}$$

where $\mathbf{d}$ is the atomic electric-dipole moment and $\mathbf{E}^+(\mathbf{R})$ and $\mathbf{E}^-(\mathbf{R})$ the positive and the negative frequency components of the electric field taken for the atomic position operator $\mathbf{R}$:

$$\mathbf{E}^+(\mathbf{R}) = \sum_\lambda \mathscr{E}_\lambda(\mathbf{R})a_\lambda,$$

$$\mathbf{E}^-(\mathbf{R}) = \sum_\lambda \mathscr{E}_\lambda{}^*(\mathbf{R})a_\lambda^+. \tag{1.6}$$

We shall use here a semiclassical approximation in the treatment of the atomic motion, by replacing the atomic position operator $\mathbf{R}$ by its average value $\langle\mathbf{R}\rangle = \mathbf{r}$ in expressions such as $\mathscr{E}_L(\mathbf{R})$. This is valid as soon as the extension Δr of the atomic wave packet is small compared with the laser wavelength λ, scale of variation of $\mathscr{E}_L$:

$$\Delta r \ll \lambda. \tag{1.7}$$

On the other hand, we shall also require the atomic velocity to be known with a good accuracy, such that the Doppler-effect dispersion $k\Delta v$ is small compared with the natural linewidth Γ of the excited level:

$$k\Delta v \ll \Gamma. \tag{1.8}$$

Note that, because of Heisenberg inequality:

$$m\Delta r\Delta v \gtrsim \hbar, \tag{1.9}$$

inequalities (1.7) and (1.8) are compatible only if

$$\frac{\hbar^2 k^2}{2m} \ll \hbar\Gamma. \tag{1.10}$$

We shall suppose that inequality (1.10) is satisfied throughout this paper. Note that it is then possible to show[12] that the forces and the diffusion coefficients calculated semiclassically, as is done here, are identical to the ones appearing in a fully quantum treatment of the atomic motion.[3,4,12,13]

2. DRESSED-ATOM APPROACH

The semiclassical approximation leads to expressions for the forces and the diffusion coefficients that are average values of products involving internal atomic operators and field operators taken at the center of the atomic wave packet $\mathbf{r}$ (see Section 3). The usual method for calculating these average values is to start from optical Bloch equations (OBE's) for an atom at point $\mathbf{r}$ and to extract from their steady-state solution the required quantities. This method is in theory simple, but it does not lead to any physical picture concerning the dipole force and its velocity dependence.

The dressed-atom approach used in this paper treats the atom–field coupling in a different way.[14–16] We first diagonalize the Hamiltonian of the coupled system (atom + laser photons) and obtain in this way the so-called dressed states. We then take into account the coupling of the dressed atom with the empty modes of the electromagnetic field that is responsible for spontaneous emission of fluorescence photons. Note that, as when one uses OBE's, this treatment is done at a given point $\mathbf{r}$, i.e., we omit in this section the kinetic energy term $\mathbf{P}^2/2m$ in H_A [Eq. (1.2)].

A. Position-Dependent Dressed States

Let us start with the dressed-atom Hamiltonian at point $\mathbf{r}$, $H_{DA}(\mathbf{r})$, which is the sum of the atomic internal energy, the laser mode energy, and the atom–laser mode coupling:

$$H_{DA}(\mathbf{r}) = \hbar(\omega_L - \delta)b^+ b + \hbar\omega_L a_L{}^+ a_L$$
$$- [\mathbf{d} \cdot \mathscr{E}_L(\mathbf{r})b^+ a_L + \mathbf{d} \cdot \mathscr{E}_L{}^*(\mathbf{r})ba_L{}^+], \tag{2.1}$$

where we have introduced the detuning δ between the laser and the atomic frequencies:

$$\delta = \omega_L - \omega_0 \ll \omega_L, \omega_0. \tag{2.2}$$

If the atom–laser mode coupling is not taken into account ($\mathbf{d} = \mathbf{0}$), the eigenstates of the dressed Hamiltonian are bunched in manifolds $\mathscr{E}_n$ (Fig. 1a), n integer, separated by the energy $\hbar\omega_L$, each manifold consisting of the two states $|g, n + 1\rangle$ and $|e, n\rangle$ (atom in the internal state g or e in the
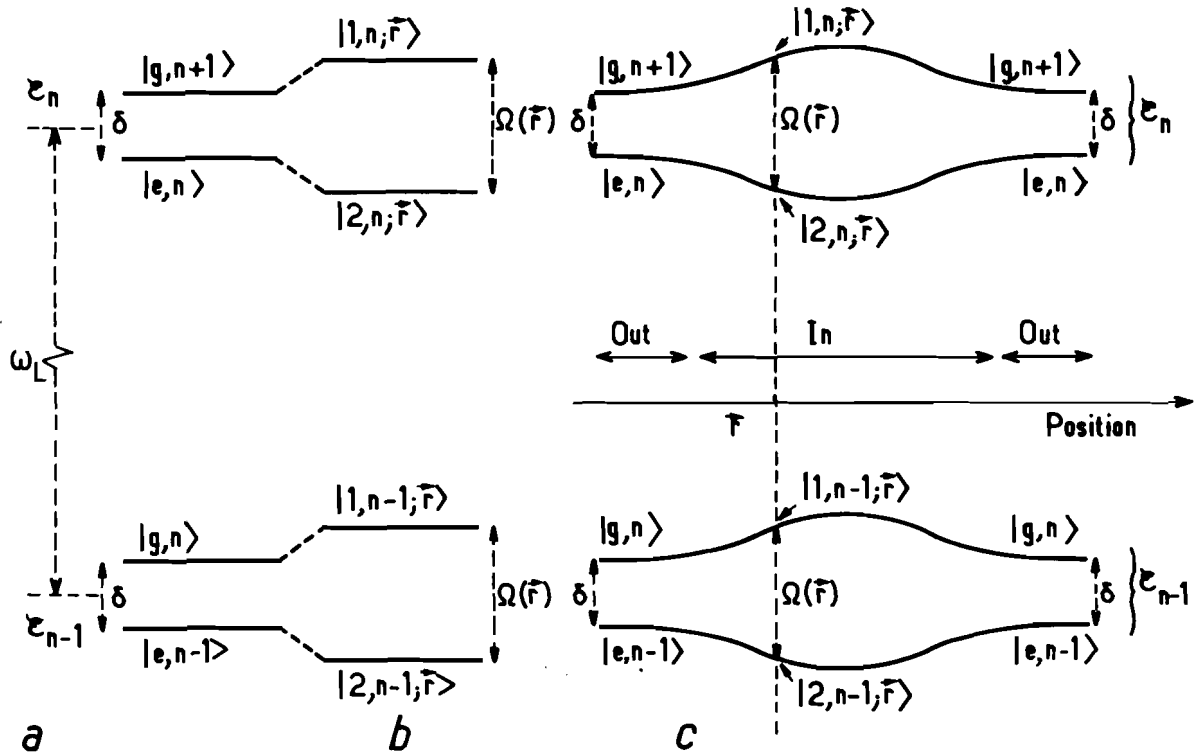
Fig. 1. Dressed-atom energy diagram. a, States of the combined atom–laser mode system without coupling, bunched in well-separated two-dimensional manifolds. b, Dressed states in a given point **r**. The laser–atom coupling produces a **r**-dependent splitting $\Omega(\mathbf{r})$ between the two dressed states of a given manifold. c, Variation across the laser beam of the dressed-atom energy levels. The energy splitting and the wave functions both depend on **r**. Out of the laser beam, the energy levels connect with the uncoupled states of a.

presence of $n + 1$ or n laser photons). The atom–laser coupling connects only the two states of a given manifold (transition from g to e with the absorption of one laser photon), and it can be characterized by the phase $\varphi(\mathbf{r})$ and the Rabi frequency $\omega_1(\mathbf{r})$ (real) defined by

$$\frac{2}{\hbar}\langle e, n|V|g, n + 1\rangle = -2\sqrt{n + 1}\,\frac{\mathbf{d}\cdot\boldsymbol{\mathcal{E}}_L(\mathbf{r})}{\hbar} = \omega_1(\mathbf{r})\exp[i\varphi(\mathbf{r})].$$

$$(2.3)$$

Note that $\omega_1(\mathbf{r})$ actually depends on the number n of photons in the manifold, but we shall neglect this dependence by supposing that the laser beam is initially excited in a coherent state with a Poisson distribution for n, the width Δn of which is very small compared with the average number of photons.

At this approximation, we then find a periodic energy diagram for H_{DA} when we take the atom–laser coupling into account (Fig. 1b). The new eigenenergies for the manifold $\mathcal{E}_n$ are

$$E_{1n}(\mathbf{r}) = (n + 1)\hbar\omega_L - \frac{\hbar\delta}{2} + \frac{\hbar\Omega(\mathbf{r})}{2},$$

$$E_{2n}(\mathbf{r}) = (n + 1)\hbar\omega_L - \frac{\hbar\delta}{2} - \frac{\hbar\Omega(\mathbf{r})}{2},$$

$$(2.4)$$

with

$$\Omega(\mathbf{r}) = [\omega_1{}^2(\mathbf{r}) + \delta^2]^{1/2},$$

$$(2.5)$$

and the corresponding eigenvectors (dressed states) can be written as

$$|1, n; \mathbf{r}\rangle = \exp[i\varphi(\mathbf{r})/2]\cos\theta(\mathbf{r})|e, n\rangle$$
$$+ \exp[-i\varphi(\mathbf{r})/2]\sin\theta(\mathbf{r})|g, n + 1\rangle,$$
$$|2, n; \mathbf{r}\rangle = -\exp[i\varphi(\mathbf{r})/2]\sin\theta(\mathbf{r})|e, n\rangle$$
$$+ \exp[-i\varphi(\mathbf{r})/2]\cos\theta(\mathbf{r})|g, n + 1\rangle, \qquad (2.6)$$

where the angle $\theta(\mathbf{r})$ is defined by

$$\cos 2\theta(\mathbf{r}) = -\delta/\Omega(\mathbf{r}), \qquad \sin 2\theta(\mathbf{r}) = \omega_1(\mathbf{r})/\Omega(\mathbf{r}). \quad (2.7)$$

The important point is that, in an inhomogeneous laser beam, these energies and eigenstates will vary with the position **r**. In Fig. 1c, we have represented the variation of the energy levels across a Gaussian laser beam: Out of the beam, the dressed levels coincide with the bare ones, and their splitting in a manifold is just $\hbar\delta$. Inside the beam, each dressed level is a linear superposition of $|g, n + 1\rangle$ and $|e, n\rangle$, and the splitting between the two dressed states of a given manifold is now $\hbar\Omega(\mathbf{r})$, larger than $\hbar\delta$.

B. Effect of Spontaneous Emission

We now take into account the coupling of the dressed atom with the empty modes, responsible for spontaneous emission of fluorescence photons. The emission frequencies correspond to transitions allowed between dressed levels, i.e., to transitions between states connected by a nonzero matrix element of the atomic dipole. In the uncoupled basis, the only transition allowed is from $|e, n\rangle$ to $|g, n\rangle$. In the coupled basis, we find transitions from the two dressed states of $\mathcal{E}_n$ [both contaminated by $|e, n\rangle$, see Eqs. (2.6)] to the two dressed states of $\mathcal{E}_{n-1}$ (both contaminated by $|g, n\rangle$), and the dipole matrix element $d_{ij}(\mathbf{r})$ between $|j, n; \mathbf{r}\rangle$ and $|i, n - 1; \mathbf{r}\rangle$ is given by [cf Eqs. (2.6)]
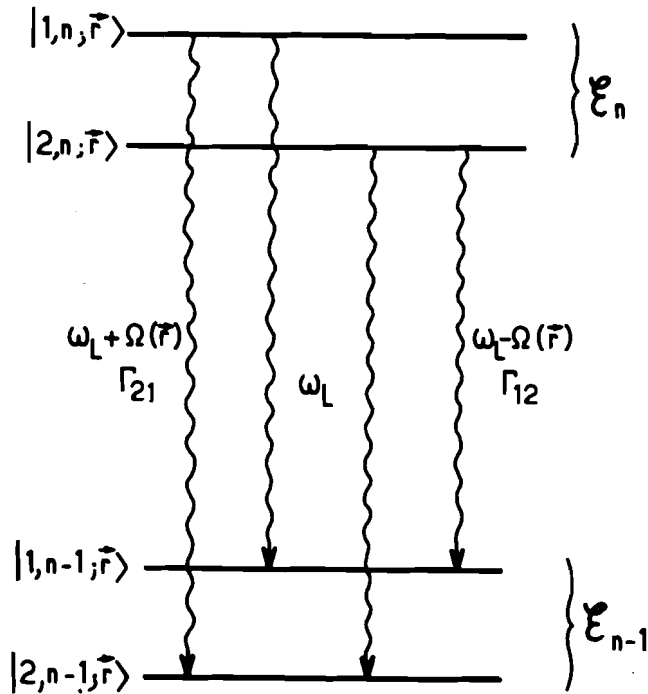
Fig. 2. Spontaneous radiative transitions between two adjacent manifolds, giving rise to the three components of the fluorescence spectrum with frequencies $\omega_L + \Omega$, ω_L, $\omega_L - \Omega$. Γ_{ji} is the transition rate from a level i to a level j.

$$\mathbf{d}_{ij}(\mathbf{r}) = \langle i, n - 1; \mathbf{r} | \mathbf{d}(b + b^+) | j, n; \mathbf{r} \rangle,$$

$$\mathbf{d}_{11} = -\mathbf{d}_{22} = \mathbf{d} \cos \theta \sin \theta e^{i\varphi},$$

$$\mathbf{d}_{12} = -\mathbf{d} e^{i\varphi} \sin^2 \theta, \qquad \mathbf{d}_{21} = \mathbf{d} e^{i\varphi} \cos^2 \theta. \qquad (2.8)$$

Three different frequencies correspond to these four transitions allowed (Fig. 2): $\omega_L + \Omega(\mathbf{r})$ for $|1, n; \mathbf{r}\rangle$ to $|2, n - 1; \mathbf{r}\rangle$, $\omega_L - \Omega(\mathbf{r})$ for $|2, n; \mathbf{r}\rangle$ to $|1, n - 1; \mathbf{r}\rangle$, and ω_L for $|1, n; \mathbf{r}\rangle$ to $|1, n - 1; \mathbf{r}\rangle$ and $|2, n; \mathbf{r}\rangle$ to $|2, n - 1; \mathbf{r}\rangle$. We interpret simply in this way the triplet structure of the fluorescence spectrum.[17]

It is now possible to write a master equation[14–16] for the density matrix ρ of the dressed atom at a given point $\mathbf{r}$. This equation describes both the internal free evolution of the dressed atom (atom–laser coupling) and the relaxation that is due to the atom–vacuum coupling, which causes, through the transitions $\mathscr{E}_n \rightarrow \mathscr{E}_{n-1} \rightarrow \ldots$, a cascade of the dressed atom down its energy diagram. Actually, in this paper, we shall need not this complete master equation but only the evolution of the four reduced populations and coherences $\Pi_i(\mathbf{r})$ and $\rho_{ij}(\mathbf{r})$ defined by

$$\Pi_i(\mathbf{r}) = \sum_n \langle i, n; \mathbf{r} | \rho | i, n; \mathbf{r} \rangle, \qquad (2.9a)$$

$$\rho_{ij}(\mathbf{r}) = \sum_n \langle i, n; \mathbf{r} | \rho | j, n; \mathbf{r} \rangle. \qquad (2.9b)$$

Furthermore, we shall restrict ourselves in the following to the limit of well-resolved lines:

$$\Omega(\mathbf{r}) \gg \Gamma, \qquad (2.10)$$

which means either intense fields ($|\omega_1(\mathbf{r})| \gg \Gamma$) or large detunings ($|\delta| \gg \Gamma$). In such a case, a secular approximation can be made that greatly simplifies the relaxation part of the

master equation. For a fixed point $\mathbf{r}$, one then finds for the evolution of Π_1 and Π_2 that

$$\dot{\Pi}_1(\mathbf{r}) = -\Gamma_{21}(\mathbf{r})\Pi_1(\mathbf{r}) + \Gamma_{12}(\mathbf{r})\Pi_2(\mathbf{r}),$$

$$\overline{\dot{\Pi}_2(\mathbf{r}) = \Gamma_{21}(\mathbf{r})\Pi_1(\mathbf{r}) - \Gamma_{12}(\mathbf{r})\Pi_2(\mathbf{r}),} \qquad (2.11)$$

where the rates of transfer Γ_{ij} from j to i are proportional to the squares of the dipole matrix elements d_{ij}'s:

$$\Gamma_{12}(\mathbf{r}) = \Gamma \sin^4 \Theta(\mathbf{r}), \qquad \Gamma_{21}(\mathbf{r}) = \Gamma \cos^4 \Theta(\mathbf{r}). \qquad (2.12)$$

Since each transition $1 \rightarrow 2$ or $2 \rightarrow 1$ corresponds to the emission of a photon $\omega_L + \Omega$ or $\omega_L - \Omega$, $\Gamma_{21}\Pi_1$ and $\Gamma_{12}\Pi_2$ represent, respectively, the number of photons emitted per unit time in the sidebands $\omega_L + \Omega$ and $\omega_L - \Omega$. For a fixed point $\mathbf{r}$, the evolution of ρ_{12} and ρ_{21} is given by

$$\dot{\rho}_{12}(\mathbf{r}) = [-i\Omega(\mathbf{r}) - \Gamma_{\mathrm{coh}}(\mathbf{r})]\rho_{12}(\mathbf{r}),$$

$$\dot{\rho}_{21}(\mathbf{r}) = [i\Omega(\mathbf{r}) - \Gamma_{\mathrm{coh}}(\mathbf{r})]\rho_{21}(\mathbf{r}), \qquad (2.13)$$

with

$$\Gamma_{\mathrm{coh}}(\mathbf{r}) = \Gamma\left[\frac{1}{2} + \cos^2 \Theta(\mathbf{r}) \cdot \sin^2 \Theta(\mathbf{r})\right]. \qquad (2.14)$$

The steady-state solution of Eqs. (2.11) and (2.13) is

$$\Pi_1{}^{\mathrm{st}}(\mathbf{r}) = \frac{\Gamma_{12}(\mathbf{r})}{\Gamma_{12}(\mathbf{r}) + \Gamma_{21}(\mathbf{r})} = \frac{\sin^4 \Theta(\mathbf{r})}{\sin^4 \Theta(\mathbf{r}) + \cos^4 \Theta(\mathbf{r})},$$

$$\Pi_2{}^{\mathrm{st}}(\mathbf{r}) = \frac{\Gamma_{21}(\mathbf{r})}{\Gamma_{12}(\mathbf{r}) + \Gamma_{21}(\mathbf{r})} = \frac{\cos^4 \Theta(\mathbf{r})}{\sin^4 \Theta(\mathbf{r}) + \cos^4 \Theta(\mathbf{r})},$$

$$\rho_{12}{}^{\mathrm{st}}(\mathbf{r}) = \rho_{21}{}^{\mathrm{st}}(\mathbf{r}) = 0. \qquad (2.15)$$

It is reached with a rate $\Gamma_{\mathrm{coh}}(\mathbf{r})$ for the coherences ρ_{ij} and with a rate

$$\Gamma_{\mathrm{pop}}(\mathbf{r}) = \Gamma_{12}(\mathbf{r}) + \Gamma_{21}(\mathbf{r}) = \Gamma[\cos^4 \Theta(\mathbf{r}) + \sin^4 \Theta(\mathbf{r})]$$

$$(2.16)$$

for the populations Π_i, since Eqs. (2.11) can be rewritten, using Eqs. (2.15) and $\Pi_1 + \Pi_2 = 1$, as

$$\dot{\Pi}_i(\mathbf{r}) = -\Gamma_{\mathrm{pop}}(\mathbf{r})[\Pi_i(\mathbf{r}) - \Pi_i{}^{\mathrm{st}}(\mathbf{r})]. \qquad (2.17)$$

Note, finally, that in steady state we have, according to Eqs. (2.11), $\Gamma_{21}\Pi_1{}^{\mathrm{st}} = \Gamma_{12}\Pi_2{}^{\mathrm{st}}$, so that equal numbers of photons are emitted in the two sidebands $\omega_L \pm \Omega$. Furthermore, one can show that the two sidebands have the same width, given by Eq. (2.14). It follows that, in steady state, the fluorescence spectrum is symmetric.[17]

C. Effect of Atomic Motion

If the point $\mathbf{r}$ varies with time (case of a moving atom), it is still possible to obtain the evolution of the Π_i's and ρ_{ij}'s. The master equation must be modified to take into account the time dependence of $|i, n; \mathbf{r}(t)\rangle$ and $|j, n; \mathbf{r}(t)\rangle$ in Eqs. (2.9). Using

$$\dot{\Pi}_i(\mathbf{r}) = \sum_n \langle i, n; \mathbf{r} | \dot{\rho} | i, n; \mathbf{r} \rangle + \overline{\langle \dot{i, n; \mathbf{r}} | \rho | i, n; \mathbf{r} \rangle}$$

$$+ \langle i, n; \mathbf{r} | \rho | \dot{\overline{i, n; \mathbf{r}}} \rangle \qquad (2.18)$$

with

$$|\dot{\overline{i, n; \mathbf{r}}}\rangle = \mathbf{v} \cdot \nabla |i, n; \mathbf{r}\rangle \qquad (2.19)$$

and coming back to the definitions (2.6) of $|i, n; \mathbf{r}\rangle$, one obtains, for example,

$$\dot{\Pi}_1 = -\Gamma_{\text{pop}}(\Pi_1 - \Pi_1^{\text{st}}) + \mathbf{v} \cdot \nabla\Theta(\rho_{12} + \rho_{21})$$
$$+ i\mathbf{v} \cdot \nabla\varphi \sin\Theta \cos\Theta(\rho_{21} - \rho_{12}), \qquad (2.20a)$$

$$\dot{\rho}_{12} = -[i(\Omega + \mathbf{v} \cdot \nabla\varphi \cos 2\Theta) + \Gamma_{\text{coh}}]\rho_{12}$$
$$+ [\mathbf{v} \cdot \nabla\Theta + i\mathbf{v} \cdot \nabla\varphi \sin\Theta \cos\Theta](\Pi_2 - \Pi_1). (2.20b)$$

The variation $d\Pi_i = \dot{\Pi}_i dt$ of Π_i during the time dt now appears from Eq. (2.20a) as the sum of two terms:

$$d\Pi_i = (d\Pi_i)_{\text{Rad}} + (d\Pi_i)_{\text{NA}}. \qquad (2.21)$$

$(d\Pi_i)_{\text{Rad}}$ corresponds to the modification of Π_i that is due to radiative relaxation through spontaneous emission:

$$(d\Pi_i)_{\text{Rad}} = -\Gamma_{\text{pop}}(\Pi_i - \Pi_i^{\text{st}}). \qquad (2.22)$$

Such a term already appeared for an atom at rest [cf. Eq. (2.17)]. $(d\Pi_i)_{\text{NA}}$, on the other hand, represents the contribution of the two terms proportional to the atomic velocity:

$$(d\Pi_1)_{\text{NA}} = \mathbf{v}dt \cdot [\nabla\Theta(\rho_{12} + \rho_{21}) + i \sin\Theta \cos\Theta\nabla\varphi(\rho_{21} - \rho_{12})]$$
$$= -(d\Pi_2)_{\text{NA}}. \qquad (2.23)$$

This modification of the dressed populations, induced by the atomic motion, is due to the spatial variation of the dressed levels: We shall call this term nonadiabatic (NA) kinetic coupling since it describes the possibility for a moving atom initially on a level $|i, n; \mathbf{r}(0)\rangle$ to reach, in the absence of spontaneous emission, the other level $|j, n; \mathbf{r}(t)\rangle$ ($j \neq i$), instead of following adiabatically $|i, n; \mathbf{r}(t)\rangle$. Let us take, for example, the case of an atom that goes from the antinode to the node of a standing wave and that thus experiences a rapid change of the dressed levels, and let us give an order of magnitude of the velocity v for which this NA coupling becomes important. The probability $p_{i \to j}$ for a NA transition from $|i, n; \mathbf{r}\rangle$ to $|j, n; \mathbf{r}\rangle$ is overestimated by (see Ref. 18)

$$p_{i\to j} \lesssim \sup\left\{ \frac{|\langle j, n; \mathbf{r}(t)|\dot{\overline{i, n; \mathbf{r}(t)}}\rangle|^2}{|\omega_{ij}(t)|^2} \right\}. \qquad (2.24)$$

$\omega_{ij}(t)$ is the Bohr frequency between levels i and j at time t equal to $\pm\Omega[\mathbf{r}(t)]$. On the other hand, using expression (2.6) for the dressed levels, we obtain (φ constant)

$$|\dot{\overline{i, n; \mathbf{r}(t)}}\rangle = \pm\mathbf{v} \cdot \nabla\Theta|j, n; \mathbf{r}(t)\rangle, \qquad (2.25)$$

and Eqs. (2.7) give for $\nabla\Theta$

$$\nabla\Theta = \frac{-\delta\nabla\omega_1}{2(\delta^2 + \omega_1^2)}. \qquad (2.26)$$

In a standing wave, ω_1 varies as $\omega_1(\mathbf{r}) = \tilde{\omega}_1 \cos \mathbf{kr}$ so that

$$|\nabla\Theta| = \frac{k}{2}\left| \frac{\delta\tilde{\omega}_1 \sin \mathbf{k} \cdot \mathbf{r}}{\delta^2 + \tilde{\omega}_1^2 \cos^2 \mathbf{k} \cdot \mathbf{r}} \right| \lesssim \frac{k}{2}\left| \frac{\tilde{\omega}_1}{\delta} \right| \qquad (2.27)$$

and

$$p_{i\to j} \lesssim \left(\frac{k\dot{v}}{2}\right)^2 \frac{|\delta\tilde{\omega}_1 \sin \mathbf{kr}|^2}{|\delta^2 + \tilde{\omega}_1^2 \cos^2 \mathbf{kr}|^3} \lesssim \left| \frac{k\dot{v}}{2} \cdot \frac{\tilde{\omega}_1}{\delta^2} \right|^2. \qquad (2.28)$$

The effect of this NA coupling, which is maximal at the nodes of the standing wave, is negligible compared with the transfer by spontaneous emission (smaller than $\Gamma\lambda/2v$) when

$$kv \ll kv_{cr} = (2\Pi\Gamma\delta^4/\tilde{\omega}_1^2)^{1/3}. \qquad (2.29)$$

Note that for a resonant wave ($\delta = 0$), $\nabla\Theta$ is zero [see Eq. (2.26)] so that the adiabatic approximation holds for any velocity.

3. MEAN DIPOLE FORCE f_{dip}

A. Expression of f_{dip} in Terms of Dressed States

As usual in semiclassical theory, we start, according to the Heisenberg point of view, from the equation of motion of the atomic momentum $\mathbf{P}$. The force operator $\mathbf{F}$ is defined as the time derivative of $\mathbf{P}$:

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = \frac{i}{\hbar}[H, \mathbf{P}] = -\nabla_{\mathbf{R}}H = -\nabla_{\mathbf{R}}V. \qquad (3.1)$$

In the semiclassical treatment followed here, we replace in $\nabla_R H$ the position operator $\mathbf{R}$ by its average value $\mathbf{r}$. Furthermore, we are interested in the average $\mathbf{f}$ of $\mathbf{F}$ over both field and internal atomic states:

$$\mathbf{f}(\mathbf{r}) = \langle \mathbf{F}(\langle \mathbf{R}\rangle)\rangle. \qquad (3.2)$$

In such an average, the contribution of empty modes in Eq. (3.1) vanishes, and the average force is related only to the gradient of the atom–laser mode coupling:

$$\mathbf{f}(\mathbf{r}) = \langle b^+a_L\nabla[\mathbf{d} \cdot \mathscr{E}_L(\mathbf{r})] + ba_L^+\nabla[\mathbf{d} \cdot \mathscr{E}_L^*(\mathbf{r})]\rangle. \qquad (3.3)$$

Using Eq. (2.3), this can be written as

$$\mathbf{f}(\mathbf{r}) = \frac{\hbar\omega_1}{2}i\nabla\varphi(\rho_{eg}e^{-i\varphi} - \rho_{ge}e^{i\varphi}) - \frac{\hbar\nabla\omega_1}{2}(\rho_{eg}e^{-i\varphi} + \rho_{ge}e^{i\varphi}), \qquad (3.4)$$

where we have put

$$\rho_{eg} = \sum_n \langle e, n|\rho|g, n + 1\rangle,$$

$$\rho_{ge} = \sum_n \langle g, n + 1|\rho|e, n\rangle. \qquad (3.5)$$

Expression (3.4) for $\mathbf{f}(\mathbf{r})$ is well known. Its two parts are, respectively, the radiation-pressure term proportional to the gradient of the phase $\varphi(\mathbf{r})$ and the dipole force proportional to the gradient of the Rabi frequency $\omega_1(\mathbf{r})$. As indicated in the Introduction, we are interested here in the dipole force, so we shall focus on this second term in all the following, and we shall omit all terms proportional to $\nabla\varphi$. Using expressions (2.6) for the dressed levels, the mean dipole force f_{dip} can be written in the dressed-atom basis as

$$f_{\text{dip}} = \frac{\hbar\nabla\Omega}{2}(\Pi_2 - \Pi_1) - \hbar\Omega\nabla\Theta(\rho_{12} + \rho_{21}). \qquad (3.6)$$

B. Energy Balance in a Small Displacement

In order to get some physical insight into expression (3.6) of f_{dip}, we now calculate the work dW that has to be provided for moving the dressed atom by a quantity $d\mathbf{r}$:

Mechanical Effects of Light

$$dW = -\mathbf{f}_{\text{dip}} \cdot d\mathbf{r}$$

$$= -\hbar \frac{\nabla \Omega}{2} \cdot d\mathbf{r}(\Pi_2 - \Pi_1) + \hbar \Omega \nabla \Theta \cdot d\mathbf{r}(\rho_{12} + \rho_{21}). \tag{3.7}$$

The first term of this expression can be written simply as

$$-\frac{\hbar d\Omega}{2} (\Pi_2 - \Pi_1) = \sum_{i=1,2} \Pi_i dE_i, \tag{3.8}$$

where dE_i represents the change of energy of the dressed level $|i, n; \mathbf{r}\rangle$ in the manifold $\mathcal{E}_n$:

$$E_1(\mathbf{r}) = \frac{1}{2} \hbar \Omega(\mathbf{r}),$$

$$E_2(\mathbf{r}) = -\frac{1}{2} \hbar \Omega(\mathbf{r}) = -E_1(\mathbf{r}). \tag{3.9}$$

[E_1 and E_2 are the deviations of E_{1n} and E_{2n} from (E_{1n} + E_{2n})/2, which is independent of $\mathbf{r}$—see Eqs. (2.4).] Using Eq. (2.23), we can reexpress the second term of Eq. (3.7) in terms of the NA change of Π_i in the displacement $d\mathbf{r} = \mathbf{v}dt$:

$$\hbar \Omega \nabla \Theta \cdot \mathbf{v}dt(\rho_{12} + \rho_{21}) = \hbar \Omega (d\Pi_1)_{\text{NA}} = -\hbar \Omega(d\Pi_2)_{\text{NA}}$$

$$= \frac{\hbar \Omega}{2} [(d\Pi_1)_{\text{NA}} - (d\Pi_2)_{\text{NA}}] = \sum_i E_i(d\Pi_i)_{\text{NA}}. \tag{3.10}$$

Finally, dW can be written as

$$dW = \sum_{i=1,2} [\Pi_i dE_i + E_i(d\Pi_i)_{\text{NA}}]. \tag{3.11}$$

A first comment that can be made about Eq. (3.11) is that only NA changes of populations $(d\Pi_i)_{\text{NA}}$ appear in dW. It follows that, if v is low enough, NA effects can be neglected [(see expression 2.29)], and we can keep only the first term of Eq. (3.11):

$$dW = \sum_i \Pi_i dE_i \quad \text{if} \quad v \ll v_{cr}. \tag{3.12}$$

Since $(d\Pi_i)_{\text{NA}}$ is not equal to $(d\Pi_i)$, it is clear also from Eq. (3.11) that dW is *not* equal to the change of $\sum_i E_i \Pi_i$, which can be interpreted as the mean potential energy of the atom in the field. In other words, if we put

$$U_A = \sum_i E_i \Pi_i, \tag{3.13}$$

then

$$dW \neq dU_A. \tag{3.14}$$

In order to understand the physical meaning of the difference between dW and dU_A, we now add and substract $\sum_i E_i(d\Pi_i)_{\text{rad}}$ to the right-hand side of Eq. (3.11). Using Eqs. (2.21) and (3.13), we first obtain

$$dW = dU_A - \sum_i E_i(d\Pi_i)_{\text{Rad}}. \tag{3.15}$$

Then, we use Eqs. (2.11) and (3.9) to transform the last term of Eq. (3.15), and, finally, we obtain

$$dW = dU_A + (\Gamma_{21}\Pi_1 \hbar \Omega - \Gamma_{12}\Pi_2 \hbar \Omega)dt. \tag{3.16}$$

We claim now that the last term of Eq. (3.16) is the energy change dU_F of the electromagnetic field (laser + fluores-

cence photons) during the time dt of the displacement $d\mathbf{r}$:

$$dU_F = (\Gamma_{21}\Pi_1 \hbar \Omega - \Gamma_{12}\Pi_2 \hbar \Omega)dt. \tag{3.17}$$

Such a result can be understood by considering that, during the time dt, dn laser photons (energy $\hbar \omega_L$) disappear, and dn fluorescence photons are emitted. Photons emitted on transitions $|i, n; \mathbf{r}\rangle \rightarrow |i, n - 1; \mathbf{r}\rangle$ can be omitted in the energy balance since they have the same energy $\hbar \omega_L$ as laser photons. By contrast, photons emitted on transitions $|1, n; \mathbf{r}\rangle \rightarrow |2, n - 1; \mathbf{r}\rangle$ or $|2, n; \mathbf{r}\rangle \rightarrow |1, n - 1; \mathbf{r}\rangle$ have an energy $\hbar \omega_L + \hbar \Omega$ or $\hbar \omega_L - \hbar \Omega$, and the emission of such photons changes the energy of the field by a quantity $\hbar \Omega$ or $-\hbar \Omega$. [Actually, because of the Doppler effect, the mean frequencies of the three components of the fluorescence spectrum are slightly shifted from the values ω_L, $\omega_L + \Omega$, $\omega_L - \Omega$. We neglect these shifts for the moment, and we shall discuss their physical consequences later on. See remark (2) at the end of this subsection.] Since there are, respectively, $\Gamma_{21}\Pi_1 dt$ and $\Gamma_{12}\Pi_2 dt$ transitions during dt, we understand why the energy change dU_F of the field is given by Eq. (3.17). Note that dU_F different from zero implies that $\Gamma_{21}\Pi_1 dt \neq \Gamma_{12}\Pi_2 dt$, i.e., that the numbers of photons dn_+ and dn_- emitted in the two sidebands $\omega_L + \Omega$ and $\omega_L - \Omega$ are not the same. In other words, the energy change dU_F of the field is associated with an *asymmetry* between the two sidebands of the fluorescence spectrum emitted by the atom during the displacement $d\mathbf{r}$.

Finally, we can write

$$dW = -\mathbf{f}_{\text{dip}} \cdot d\mathbf{r} = dU_A + dU_F, \tag{3.18}$$

which shows that the work done against the dipole force for moving the atom from $\mathbf{r}$ to $\mathbf{r} + d\mathbf{r}$ is transformed into a variation of atomic and field energies. Note that Eq. (3.18) is valid for any velocity (since NA effects are included in dU_A).

Remarks

(1) The energy dU_F taken by the field is different from zero even in the quasi-static limit, i.e., when the velocity $\mathbf{v}$ between $\mathbf{r}$ and $\mathbf{r} + d\mathbf{r}$ is extremely low. This is because the steady-state populations $\Pi_i^{\text{st}}(\mathbf{r})$ depend on $\mathbf{r}$. If, for example, $\Pi_1^{\text{st}}(\mathbf{r} + d\mathbf{r}) < \Pi_1^{\text{st}}(\mathbf{r})$ and, consequently, $\Pi_2^{\text{st}}(\mathbf{r} + d\mathbf{r}) > \Pi_2^{\text{st}}(\mathbf{r})$, there are necessarily more transitions from 1 to 2 than from 2 to 1 during the displacement $d\mathbf{r}$. This means that, in the quasi-static limit, the difference $dn_+ - dn_-$ between the number of photons emitted in the two sidebands depends only on $d\mathbf{r}$ and not on the velocity. On the other hand, when v decreases, the time dt required for going from $\mathbf{r}$ to $\mathbf{r} + d\mathbf{r}$ increases, and the number of photons dn_+ and dn_- increases. This shows that the *relative* asymmetry of the fluorescence spectrum $(dn_+ - dn_-)/(dn_+ + dn_-)$ tends to zero when v tends to zero, whereas the *absolute* asymmetry $dn_+ - dn_-$ remains constant and proportional to dU_F.

(2) We come back now to the mean Doppler shift of the fluorescence spectrum mentioned above. Consider the simple case of an atom moving with velocity $\mathbf{v}$ in a plane wave with wave vector $\mathbf{k}$. In the rest frame of the atom, the laser photons have a frequency $\omega_L - \mathbf{k} \cdot \mathbf{v}$, and the fluorescence spectrum is centered on $\omega_L - \mathbf{k} \cdot \mathbf{v}$. Coming back to the laboratory frame and averaging over the direction of the spontaneously emitted photons, one finds that the fluorescence spectrum is still centered, on the average, on $\omega_L - \mathbf{k} \cdot \mathbf{v}$.

If n fluorescence cycles occur per unit time, the energy balance dU_F of the field calculated above in Eq. (3.17) must be corrected in the laboratory frame by an amount $-ndt\hbar\mathbf{k}\cdot\mathbf{v}$ equal to $-n\hbar\mathbf{k}\cdot d\mathbf{r}$. Such a correction corresponds to the work done against the *radiation-pressure force* $n\hbar\mathbf{k}$. This shows that taking into account the Doppler shift in the energy balance is equivalent to including the radiation-pressure force in this energy balance. Since this paper is devoted to dipole forces, we shall ignore these corrections in what follows.

C. Mean Dipole Force for an Atom at Rest in r

If the atom is at rest in $\mathbf{r}$, we can ignore NA effects and replace in Eq. (3.12) the populations Π_i by their steady-state values Π_i^{st}, which gives

$$\mathbf{f}_{\text{dip}}^{\text{st}} = -\sum_i \Pi_i^{st}\nabla E_i = -\Pi_1^{st}\nabla E_1 - \Pi_2^{st}\nabla E_2. \quad (3.19)$$

The physical meaning of this expression is clear. The mean dipole force $\mathbf{f}_{\text{dip}}$ is the average of the forces $-\nabla E_1$ and $-\nabla E_2$ "seen," respectively, on levels $|1, n; \mathbf{r}\rangle$ and $|2, n; \mathbf{r}\rangle$ and weighted by the probabilities of occupation Π_1^{st} and Π_2^{st} of these two types of states.

Using Eqs. (3.9) and expressions (2.5) and (2.15) of Ω and Π_i^{st}, we can write Eq. (3.19) as

$$\mathbf{f}_{\text{dip}}^{\text{st}} = -\hbar\delta\frac{\omega_1^2}{\omega_1^2 + 2\delta^2}\alpha = -\nabla\left[\frac{\hbar\delta}{2}\log\left(1 + \frac{\omega_1^2}{2\delta^2}\right)\right], \quad (3.20)$$

where we have put

$$\alpha = \frac{\nabla\omega_1}{\omega_1} = \frac{\Omega}{\omega_1^2}\nabla\Omega. \quad (3.21)$$

Expression (3.20) of $\mathbf{f}_{\text{dip}}^{\text{st}}$ coincides exactly with the value derived from OBE's in the limit (2.10) of well-separated lines.[1,2,4]

We would like, finally, to show how the dressed-atom approach gives a simple understanding of the connection between the sign of the dipole force and the sign of the detuning $\delta = \omega_L - \omega_0$ between the laser and atomic frequencies. If the detuning δ is positive (Fig. 3a), the levels 1 are
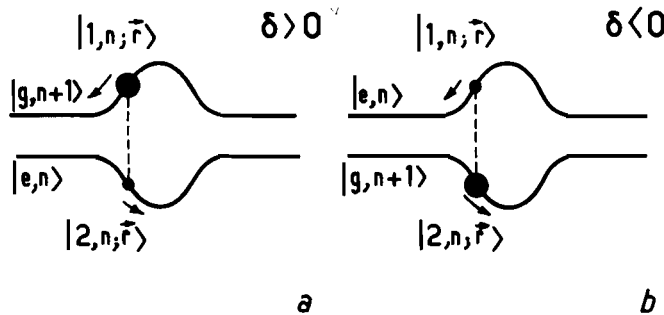

$$a \qquad\qquad\qquad b$$

Fig. 3. Interpretation of the sign of the mean dipole force, which is the average of the two dressed-state-dependent forces weighted by the steady-state populations of these states represented by the filled circles. a, When $\delta = \omega_L - \omega_0 > 0$, the state $|1, n; \mathbf{r}\rangle$, which connects to $|g, n+1\rangle$ out of the laser beam, is less contaminated by $|e; n\rangle$ than $|2, n; \mathbf{r}\rangle$ and therefore more populated. The mean dipole force then expels the atom out of the laser beam. b, When $\delta = \omega_L - \omega_0 < 0$, the state $|2, n; \mathbf{r}\rangle$ is more populated than $|1, n; \mathbf{r}\rangle$, and the mean dipole force attracts the atom toward the high-intensity region.
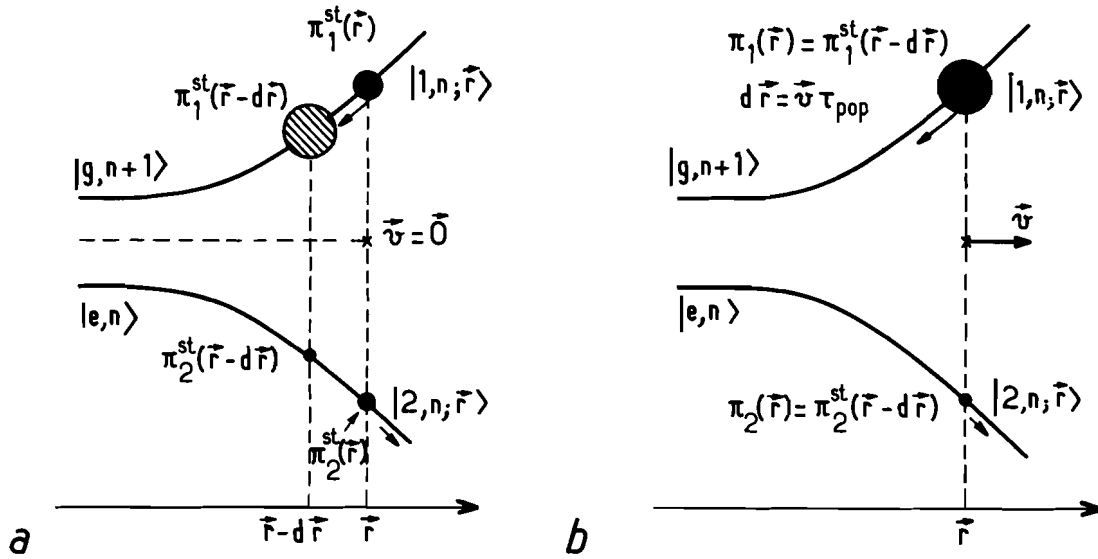
those that coincide with $|g, n+1\rangle$ outside the laser beam. It follows that they are less contaminated by $|e, n\rangle$ than levels 2 and that fewer spontaneous transitions start from 1 than from 2. This shows that levels 1 are more populated than levels 2 ($\Pi_1^{st} > \Pi_2^{st}$). The force resulting from levels 1 is therefore dominant, and the atom is expelled from high-intensity regions. If the detuning is negative (Fig. 3b), the conclusions are reversed: Levels 2 are more populated, and the atom is attracted toward high-intensity regions. Finally, if $\omega_L = \omega_0$, both states 1 and 2 contain the same admixture of e and g, they are equally populated ($\Pi_1^{st} = \Pi_2^{st}$), and the mean dipole force vanishes.

D. Mean Dipole Force for a Slowly Moving Atom

We consider now a slowly moving atom for which NA effects are negligible, so that we can use Eq. (3.12) and write

$$\mathbf{f}_{\text{dip}} = -\Pi_1\nabla E_1 - \Pi_2\nabla E_2. \quad (3.22)$$

Furthermore, we limit ourselves in this subsection to extremely small velocities such that

$$\frac{kv}{\Gamma} \ll 1 \qquad \Leftrightarrow \qquad v\Gamma^{-1} \ll \lambda. \quad (3.23)$$

Condition (3.23) means that the Doppler effect is very small compared with the natural width or, equivalently, that the atom travels over a small distance (compared with the laser wavelength λ) during the radiative-relaxation time. It follows that the populations $\Pi_i(\mathbf{r})$ for the moving atom are very close to the steady-state values $\Pi_i^{st}(\mathbf{r})$, the difference between $\Pi_i(\mathbf{r})$ and $\Pi_i^{st}(\mathbf{r})$ being of the order of kv/Γ. We therefore expect that Eq. (3.22) differs from the steady-state value (3.20) by a term linear in kv/Γ, which we now want to evaluate. Before doing this calculation, let us emphasize that the NA terms neglected in Eq. (3.22) would also give rise to velocity-dependent dipole forces if they were taken into account, but we shall see at the end of this section that their contribution to $\mathbf{f}_{\text{dip}} - \mathbf{f}_{\text{dip}}^{\text{st}}$ is much smaller than that of $\pi_i - \pi_i^{st}$.

In order to obtain the first-order correction (in kv/Γ) to $\Pi_i - \Pi_i^{st}$, we go back to Eq. (2.17) [since $(\dot{\Pi}_i)_{NA}$ is negligible], and we replace $\dot{\Pi}_i(\mathbf{r})$ by $\mathbf{v}\cdot\nabla\Pi_i(\mathbf{r})$ in the left-hand-side member of this equation:

$$\mathbf{v}\cdot\nabla\Pi_i(\mathbf{r}) = -\Gamma_{\text{pop}}(\mathbf{r})[\Pi_i(\mathbf{r}) - \Pi_i^{st}(\mathbf{r})]. \quad (3.24)$$

Since the left-hand-side member of Eq. (3.24) already contains v, we can replace in this term $\Pi_i(\mathbf{r})$ by $\Pi_i^{st}(\mathbf{r})$. Equation (3.24) then gives

$$\Pi_i(\mathbf{r}) \simeq \Pi_i^{st}(\mathbf{r}) - \mathbf{v}\tau_{\text{pop}}\cdot\nabla\Pi_i^{st}(\mathbf{r}), \quad (3.25)$$

where

$$\tau_{\text{pop}}(\mathbf{r}) = 1/\Gamma_{\text{pop}}(\mathbf{r}). \quad (3.26)$$

Expression (3.25), which can be also written as

$$\Pi_i(\mathbf{r}) \simeq \Pi_i^{st}(\mathbf{r} - \mathbf{v}\tau_{\text{pop}}), \quad (3.27)$$

has a clear physical meaning. The radiative relaxation between the two states 1 and 2 takes place with a certain time constant τ_{pop}. Since the atom is moving, and since the steady-state population $\Pi_i^{st}(\mathbf{r})$ generally depends on $\mathbf{r}$, the radiative relaxation cannot instantaneously adjust the population $\Pi_i(\mathbf{r})$ to the steady-state value $\Pi_i^{st}(\mathbf{r})$ that would be obtained if the atom were staying in $\mathbf{r}$. There is a certain

Fig. 4.   Interpretation of the sign of the velocity-dependent dipole force ($\delta > 0$) for a slowly moving atom. a, Level $|1, n; r\rangle$ connects with $|g, n +$ 1⟩ out of the laser beam, so that the steady-state population $\Pi_1{}^{st}$ decreases as the atom is put in higher-intensity regions: $\Pi_1{}^{st}$ ($r - dr$) (hatched circle) is larger than $\Pi_1{}^{st}$ (r) (filled circle), and consequently, $\Pi_2{}^{st}$ ($r - dr$) $< \Pi_2{}^{st}(r)$. b, For an atom entering with a velocity v in the laser beam, because of the time lag τ_{pop} of radiative relaxation the instantaneous population in r, Π_1 (r), is the steady population at point $r - v\,\tau_{pop}$, which is larger than $\Pi_1{}^{st}$ (r). The mean dipole force then expels the moving atom out of the laser beam more than if it were at rest. The component of the force linear in v is therefore a damping term.

lag, characterized by the time constant τ_{pop}, so that the population $\Pi_i(r)$ for an atom passing in r at t is the steady-state population corresponding to a previous time $t - \tau_{pop}$, i.e., corresponding to the position $r - v\tau_{pop}$.

Such a simple idea provides a straightforward interpretation of the unusual sign of the velocity-dependent dipole force, which we have already mentioned in the Introduction. The problem is to understand why the dipole force is a friction force when the detuning δ is positive, rather than a heating force, as is the case for radiation pressure. Consider an atom entering into a laser beam and suppose that $\delta = \omega_L - \omega_0$ is positive. We have represented in Fig. 4a the steady-state populations in r, $\Pi_1{}^{st}(r)$, and $\Pi_2{}^{st}(r)$ by a circle with a diameter proportional to $\Pi_1{}^{st}(r)$ and $\Pi_2{}^{st}(r)$, respectively. Since, for $\delta > 0$, the state $|1, n; r\rangle$ is transformed continuously into $|g, n + 1\rangle$ out of the laser beam, it is more contaminated by $|g, n + 1\rangle$ than $|2, n; r\rangle$, and it is more populated: $\Pi_1{}^{st}(r) > \Pi_2{}^{st}(r)$. Furthermore, the contamination of $|1, n; r\rangle$ by $|e, n\rangle$ increases when the Rabi frequency $\omega_1(r)$ increases. It follows that $\Pi_1{}^{st}(r)$ decreases when r is shifted toward the laser beam: $\Pi_1{}^{st}(r - dr) > \Pi_1{}^{st}(r)$. Consider now an atom moving with the velocity v (Fig. 4b). According to Eq. (3.27), the population $\Pi_1(r)$ in r is not $\Pi_1{}^{st}(r)$ but $\Pi_1{}^{st}(r - dr)$ with $dr = v\tau_{pop}$. It follows that

$$\Pi_1(r) = \Pi_1{}^{st}(r - v\tau_{pop}) > \Pi_1{}^{st}(r). \qquad (3.28)$$

(The filled circles in Fig. 4b have the same diameters as the hatched circles of Fig. 4a.) The same argument gives

$$\Pi_2(r) = \Pi_2{}^{st}(r - v\tau_{pop}) < \Pi_2{}^{st}(r). \qquad (3.29)$$

Finally, the moving atom is expelled from the high-intensity region with the force

$$f_{dip} = -\frac{\hbar\nabla\Omega}{2} [\Pi_1(r) - \Pi_2(r)], \qquad (3.30)$$

which is *larger* than the steady-state force that it would experience if it were at rest in r: The extra velocity-dependent force is therefore a *damping* force. With a negative detuning, the conclusions would be reversed: The velocity-dependent dipole force would then be a *heating* force.

We can also derive an explicit expression for the velocity-dependent force by inserting expression (3.25) into Eq. (3.22) and by using expressions (2.15) and (2.16) of $\Pi_i{}^{st}$ and $\Gamma_{pop} = \tau_{pop}{}^{-1}$. We obtain in this way

$$f_{dip}(r, v) = f_{dip}{}^{st}(r) - \frac{2\hbar\delta}{\Gamma}\left(\frac{\omega_1{}^2(r)}{\omega_1{}^2(r) + 2\delta^2}\right)^3 (\alpha \cdot v)\alpha, \qquad (3.31)$$

where $f_{dip}{}^{st}$ is the dipole force (3.20) for an atom at rest in r and where α is given in Eq. (3.21). Such an expression coincides exactly [in the limit (2.10)] with the one derived by Gordon and Ashkin.[1] In Section 5, we shall come back to Eq. (3.31) for the particular case of a standing wave, and we shall calculate the average of $f_{dip}(r, v)$ over a wavelength. We shall see that the contribution of $f_{dip}{}^{st}(r)$ to this average is zero so that one is left only with the contribution of the second part, proportional to the velocity, which is a damping or a heating force, depending on whether the detuning δ is positive or negative.

Remark

In this section we have neglected NA effects coming from $(d\Pi_i)_{NA}$. If they were taken into account, they would give rise also to velocity-dependent forces, since $(d\Pi_i)_{NA}$ is proportional to v [see Eq. (2.23)]. To estimate the order of magnitude of these NA forces, we come back to Eq. (3.6), and we try to overestimate the last term of this equation that has been neglected here. From Eq. (2.20b), we obtain

$$|\rho_{12}|, |\rho_{21}| \leqslant \mathrm{Max}\left\{\frac{kv}{\Gamma} \times \frac{\Gamma}{\Omega}, \frac{kv}{\Gamma} \times \frac{\Gamma\omega_1}{\Omega\delta}\right\}, \qquad (3.32)$$

since $|\nabla\Theta|$ and $|\nabla\varphi|$ are, respectively, smaller than or equal to $k\omega_1/\delta$ and k [see Eq. (2.27)]. This has to be compared with the v-dependent contribution of the first term of Eq. (3.6), which is of the order of kv/Γ since it comes from the difference between Π_i and Π_i^{st}. This shows that NA forces are Ω/Γ or $\Omega\delta/\omega_1\Gamma$ smaller than the velocity-dependent forces studied in this section, which come from the lag of radiative relaxation for a moving atom. It is therefore correct to neglect them in the limit (2.10) of well-resolved lines and for δ not too small compared with ω_1. (Note, finally, that, if Γ were not small compared with Ω, it would be also necessary to include nonsecular couplings between diagonal and off-diagonal elements of the density matrix in the master equation describing radiative relaxation).

4. ATOMIC-MOMENTUM DIFFUSION DUE TO THE FLUCTUATIONS OF DIPOLE FORCES

Because of the random character of spontaneous emission, the forces acting upon an atom in a laser beam fluctuate around their mean value, and these fluctuations produce a diffusion of the atomic momentum **P**, characterized by a diffusion coefficient D. The calculation of D exhibits several contributions, most of which are now well understood.[6] For example, there is a contribution that is proportional to the square of the phase gradient and that is associated with the fluctuations in the number of fluorescence cycles occuring in a given time interval (fluctuations of radiation pressure). There is also another term describing the fluctuations of the recoil momentum transferred by the fluorescence photons that are emitted in random directions. In this section we shall focus on the fluctuations of dipole forces that give rise to a contribution D_{dip} proportional to the square of the intensity gradient. Gordon and Ashkin have already pointed out that the dressed-atom approach provides a simple physical picture for the fluctuations of dipole forces.[1] We shall show here that such an approach can lead also to a quantitative evaluation of D_{dip} for an atom at rest.

A. Fluctuations of Dipole Forces
The picture of the dressed atom cascading down its energy diagram leads to a simple interpretation of the mean value and fluctuations of dipole forces for an atom initially at rest in **r**. If the dressed atom is in a dressed state of type 1, it undergoes a force $-\nabla E_1$ equal to minus the gradient of the energy of this state (Fig. 5). Then, by spontaneous emission of a photon $\hbar(\omega_L + \Omega)$, which occurs at a *random* time, it jumps into a state of type 2, where the force $-\nabla E_2 = +\nabla E_1$ has a value opposite the previous one. A subsequent emission of a photon $\hbar(\omega_L - \Omega)$ puts it back in a state of type 1 and again changes the sign of the force. And so on .... It thus appears that the instantaneous force experienced by the atom switches back and forth between two opposite values after random time intervals $\tau_1, \tau_2, \ldots$. If $\bar\tau_1$ and $\bar\tau_2$ are the mean values of these successive time intervals spent by the dressed atom in levels of type 1 and 2, it is clear that the mean dipole force is the average value of $-\nabla E_1$ and $-\nabla E_2$, respectively, weighted by $\bar\tau_1/(\bar\tau_1 + \bar\tau_2)$ and $\bar\tau_2/(\bar\tau_1 + \bar\tau_2)$, which are actually just the steady-state probabilities of occupation
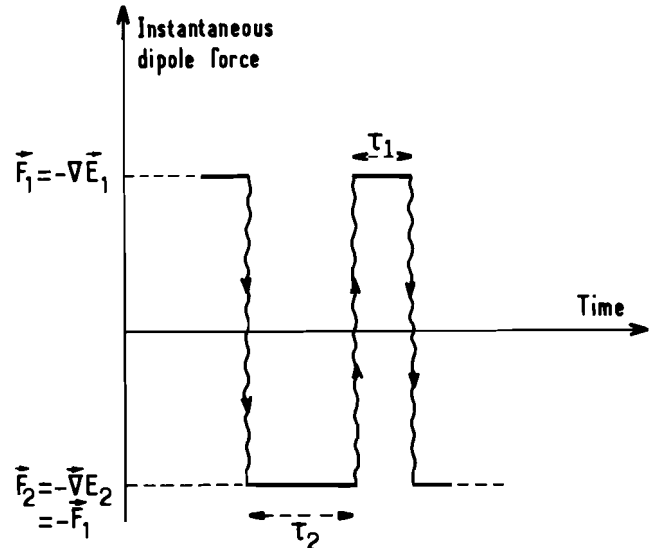

Fig. 5. The instantaneous dipole force switches back and forth between the two dressed-state-dependent forces $-\nabla E_1$ and $-\nabla E_2$. The intervals of time τ_1 and τ_2 spent in each dressed state between two successive jumps are random variables.

Π_1^{st} and Π_2^{st} of levels 1 and 2. We obtain in this way the result already derived in Subsection 3.B for the mean dipole force. But the present discussion also provides a good insight into the random nature of the two-valued instantaneous dipole force.

B. Evaluation of the Momentum Diffusion Coefficient
The fluctuations of dipole forces are responsible for a diffusion of atomic momentum described by a diffusion coefficient D_{dip}, which is given (in the semiclassical treatment followed here for an atom at rest) by

$$D_{\mathrm{dip}} = \int_0^\infty \mathrm{d}t\,[\langle \mathbf{F}(t) \cdot \mathbf{F}(t+\tau)\rangle - \mathbf{f}_{\mathrm{dip}}^2], \qquad (4.1)$$

where **F** is the two-valued instantaneous dipole force with mean value $\mathbf{f}_{\mathrm{dip}}$.

The expression of the correlation function $\langle \mathbf{F}(t) \cdot \mathbf{F}(t + \tau)\rangle$ results from the physical picture given above:

$$\langle \mathbf{F}(t) \cdot \mathbf{F}(t+\tau)\rangle = \sum_{i=1,2}\sum_{j=1,2} (-\nabla E_i)(-\nabla E_j)P(i, t; j, t+\tau).$$
$$(4.2)$$

It is equal to the product of the two instantaneous forces $-\nabla E_i$ and $-\nabla E_j$ weighted by the probability to be in a state of type i at time t *and* in a state of type j at time $t + \tau$ and summed over i and j. In the steady-state regime, $P(i, t; j, t + \tau)$ depends only on τ and can be written as

$$P(i, t; j, t + \tau) = P_i P(j, \tau/i, 0), \qquad (4.3)$$

where P_i is the steady-state probability to be in i, which is just Π_i^{st}:

$$P_i = \Pi_i^{\mathrm{st}}, \qquad (4.4)$$

and $P(j, \tau/i, 0)$ is the *conditional* probability to be in j at τ, *knowing* that the atom is in i at $t = 0$. Solving Eq. (2.17) with the initial condition $\Pi_i(0) = 1$ gives

$$P(i, \tau/i, 0) = \Pi_i^{st} + \Pi_j^{st} \exp(-\Gamma_{pop}\tau), \qquad (4.5a)$$

$$P(j, \tau/i, 0) = 1 - P(i, \tau/i, 0). \qquad (4.5b)$$

Finally, inserting Eqs. (4.3)–(4.5) into Eqs. (4.2) and (4.1) and using the expressions given above for E_i, Π_i^{st}, Γ_{pop}, one obtains

$$D_{dip} = \frac{\hbar^2}{2\Gamma} \left(\frac{\omega_1^2}{\omega_1^2 + 2\delta^2} \right)^3 (\nabla\omega_1)^2. \qquad (4.6)$$

C. Discussion

Expression (4.6) for D_{dip} coincides exactly with the one derived from OBE's[1] in the limit (2.10) of well-resolved lines. Actually, Gordon and Ashkin have used a dressed-atom picture, similar to the one presented here in Subsection 4.A, for evaluating the order of magnitude of D_{dip} at resonance. Our result (4.6), which is valid for any detuning δ, shows that the dressed-atom picture is useful not only for an understanding of the physical mechanisms but also for a quantitative calculation of D_{dip}.

The most important feature of Eq. (4.6) is that D_{dip} increases as $(\nabla\omega_1)^2$ and does not saturate when the light intensity increases. Such a behavior is quite different from the one exhibited by the diffusion coefficient of radiation pressure that saturates to values of the order of $(\hbar k)^2\Gamma$. The large value of D_{dip} is due to the fact that, between two spontaneous emission processes, the atomic momentum increases as ∇E_i and is therefore not limited when ω_1 increases. This introduces severe limitations for the realization of radiative traps using standing waves: The diffusion coefficient increases more rapidly with ω_1 (as ω_1^2) than the depth of the potential well, which varies only as $\log(1 + \omega_1^2/2\delta^2)$ [see Eq. (3.20)].

5. ATOMIC MOTION IN A STANDING WAVE

This last section is devoted to the application of the dressed-atom approach to the particular case of atomic motion in a standing wave. We are interested here in the average $\bar{f}$ of the dipole force over a wavelength and in the variations of $\bar{f}$ with the atomic velocity. We first relate $\bar{f}$ to the rate of variation of the field energy (Subsection 5.A); we then restrict ourselves to the adiabatic limit [velocities smaller than v_{cr}; see inequality (2.29)], and we investigate in detail the two velocity ranges $kv \ll \Gamma$ (Subsection 5.B) and $kv \gtrsim \Gamma$ (Subsection 5.C). Finally (Subsection 5.D), we compare our results with the ones obtained with a continued-fraction expansion of the dipole force.

A. Dipole Force Averaged on a Wavelength

We consider in the following a one-dimensional standing wave:

$$\omega_1(z) = \bar{\omega}_1 \cos kz, \qquad (5.1)$$

and we restrict ourselves to a motion along the z direction. With view to using this standing wave for slowing down (or accelerating) atoms, it is interesting to consider the kinetic energy ΔW lost or gained by the atom over a wavelength and equal to the work of the dipole force along a wavelength[19]:

$$\Delta W = \int_z^{z+\lambda} f_{dip}dz = \lambda\bar{f}. \qquad (5.2)$$

$\bar{f}$ is the average of the dipole force over a wavelength. Because of the spatial periodicity of the system, ΔW depends only on the atomic velocity v and not on z. Now, using Eq. (3.18), $\bar{f}$ can be related to the change of the atomic potential energy U_A and of the field energy U_F over one wavelength:

$$\bar{f} = -\frac{1}{\lambda} \int_z^{z+\lambda} dU_A - \frac{1}{\lambda} \int_z^{z+\lambda} dU_F. \qquad (5.3)$$

Since Π_i and E_i are periodic functions of z, the atomic potential energy $U_A(z)$ given in Eq. (3.13) is also periodic in z, and the contribution of the first term of Eq. (5.3) vanishes:

$$-\frac{1}{\lambda} \int_z^{z+\lambda} dU_A = -\frac{1}{\lambda} [U_A(z+\lambda) - U_A(z)] = 0. \quad (5.4)$$

Equation (5.3) then reduces to

$$\bar{f} = -\frac{1}{\lambda} \int_z^{z+\lambda} dU_F. \qquad (5.5)$$

The integral of dU_F over one wavelength, contrary to dU_A, is not necessarily zero. The atomic motion can indeed induce an asymmetry of the fluorescence triplet, so that the energy radiated in the two sidebands of this triplet, at $\omega_L + \Omega$ and $\omega_L - \Omega$, can be larger or smaller than the energy of the photons ω_L absorbed in the laser wave. This gain (or loss) of energy by the electromagnetic field is then of course compensated by a loss (or gain) of kinetic energy of the atom, by the action of the dipole force upon this atom.

To go further in the calculation of $\bar{f}$, we now need to evaluate f_{dip} [Eq. (5.2)] or dU_F [Eq. (5.5)] as functions of the position and the velocity of the atom. From Eq. (3.6) for f_{dip} or Eq. (3.17) for dU_F, we see that this amounts to determining the populations Π_i and the coherences ρ_{ij} at a given point and for a given velocity. A general determination of the Π_i's and the ρ_{ij}'s (i.e., for any velocity) would require that the set of equations (2.20) be solved. Actually, we shall restrict ourselves to velocities such as kv small compared with kv_{cr} so that only the Π_i's are needed and NA terms can be neglected in Eqs. (2.20). The equation of evolution for the Π_i's is then Eq. (2.17), which we are now going to solve for the following two velocity ranges: very low velocities $kv \ll \Gamma$ (which have already been explored in Subsection 3.D) and intermediate velocities $\Gamma \lesssim kv \ll kv_{cr}$.

B. Case of Very Low Atomic Velocities

For very low velocities such that kv is small compared with Γ, the principle of the resolution of the equation of evolution of Π_i [Eq. (2.17)] has already been given in Subsection 3.D: One can expand the solution of Eq. (2.17) in powers of kv/Γ, the zeroth-order solution being simply the steady-state value Π_i^{st}. The simplest way to obtain $\bar{f}$ is then to start from the expression of $f_{dip}(z, v)$ [Eq. (3.31)] obtained in Subsection 3.D and to average it over a wavelength:

$$\bar{f} = \frac{1}{\lambda} \int_{-\lambda/2}^{\lambda/2} f_{dip}(z, v)dz$$

$$= \frac{1}{\lambda} \int_{-\lambda/2}^{\lambda/2} f_{dip}^{st}(z)dz - \frac{1}{\lambda} \int_{-\lambda/2}^{\lambda/2} \frac{2\hbar\delta}{\Gamma} \left(\frac{\omega_1^2}{\omega_1^2 + 2\delta^2} \right)^3 \alpha^2 v dz. \qquad (5.6)$$

$f_{dip}{}^{st}(z)$ is an odd function of z so that its contribution to Eq. (5.6) vanishes. One is then left only with the second term of Eq. (5.6) that can be written as

$$\bar{f} = -\frac{m}{\tau} v, \qquad (5.7)$$

provided that the dipole force is sufficiently small so that the velocity v can be considered as constant over a wavelength. Depending on the sign of the characteristic time τ, $\bar{f}$ is either a damping or an accelerating force. The explicit calculation of τ from Eq. (5.6) gives

$$\frac{1}{\tau} = \frac{1}{\tau_0}\frac{\delta}{\Gamma}\left\{\frac{3}{4}\sqrt{1+s} + \frac{3}{2\sqrt{1+s}} - \frac{1}{4(1+s)^{3/2}} - 2\right\}, \qquad (5.8)$$

with

$$\tau_0 = m/\hbar k^2, \qquad s = \tilde{\omega}_1{}^2/2\delta^2. \qquad (5.9)$$

Several remarks can be made about Eq. (5.8). First, we note, as in Subsection 3.D, that the force damps the motion (τ positive) for a positive detuning δ and accelerates it (τ negative) for a negative detuning. Second, Eq. (5.8) permits a comparison of τ with τ_0 [given in Eqs. (5.9)], which is the characteristic time of radiation-pressure cooling. We see that τ can be much shorter than τ_0, which indicates that the dipole force in a strong standing wave can be much more efficient for radiative cooling than radiation pressure.[20] Let us mention, finally, that Eq. (5.8) is equal, in the limit of well-separated lines, to the result found by Minogin and Serimaa[10] by a continued-fraction expansion of the dipole force, calculated from OBE's.

C. Case of Intermediate Velocities

We now turn to the case of intermediate velocities $\Gamma \lesssim kv \ll kv_{cr}$, for which NA terms can still be neglected in the evolution of the Π_i's:

$$v\frac{d\Pi_i}{dz} = -\Gamma_{pop}(\Pi_i - \Pi_i{}^{st}) \qquad (5.10)$$

but for which an expansion of Π_i in powers of kv/Γ is no longer possible. We then have to start with the general solution of Eq. (5.10):

$$\Pi_i(z) = e(z_0, z)\Pi_i(z_0) + \int_{z_0}^{z} dz' \frac{\Gamma_{pop}\Pi_i{}^{st}}{v} e(z', z),$$

$$e(z_1, z_2) = \exp\left\{-\int_{z_1}^{z_2} dz'' \frac{\Gamma_{pop}}{v}\right\}. \qquad (5.11)$$

We now take advantage of the periodicity of $\Pi_i(z)$ and write Eqs. (5.11) with $z_0 = z - \lambda$. This gives

$$\Pi_i(z) = \frac{\int_{z-\lambda}^{z} dz' \frac{\Gamma_{pop}\Pi_i{}^{st}}{v} e(z', z)}{1 - e(z - \lambda, z)}. \qquad (5.12)$$

This last expression is valid for any velocity satisfying the adiabatic approximation. It would now be possible to insert this value into the dipole force (3.22) and to obtain in this way a general result for the dipole force at the adiabatic approximation. A similar result has already been obtained by Fiordilino and Mittleman[21] by a method using a Fourier expansion of the OBE solution in the standing wave. We shall not perform this general calculation here, since we are

rather interested in $\bar{f}$, and, furthermore, we shall restrict ourselves to the high-velocity side: $kv \gg \Gamma$. For such velocities, the function $e(z_1, z_2)$ is close to 1 if $z_1 - z_2$ is smaller than or of the order of λ, so that we can write

$$e(z_1, z_2) \simeq 1 - \int_{z_1}^{z_2} dz'' \frac{\Gamma_{pop}}{v}. \qquad (5.13)$$

Assuming that the velocity v does not change much on a wavelength, this gives the approximate value for Π_i:

$$\Pi_i(z) \simeq \frac{\overline{\Gamma_{pop}\Pi_i}}{\overline{\Gamma_{pop}}}, \qquad (5.14)$$

where $\bar{A}$ stands for the average of a quantity $A(z)$ over a wavelength. We have neglected the second term of expression (5.13) in the numerator of Eq. (5.12) since this numerator is already in $1/v$. Using Eqs. (2.15) and (2.16) for Π_i and Γ_{pop}, expression (5.14) can be written as

$$\Pi_1 \simeq \frac{\bar{\Gamma}_{12}}{\bar{\Gamma}_{12} + \bar{\Gamma}_{21}}, \qquad \Pi_2 \simeq \frac{\bar{\Gamma}_{21}}{\bar{\Gamma}_{12} + \bar{\Gamma}_{21}}. \qquad (5.15)$$

These two last expressions have a straightforward interpretation: When kv is large compared with Γ, the atom has a small probability of emitting a fluorescence photon when it moves over a single wavelength. It is therefore not sensitive to the local Γ_{ij}'s, but rather it averages them over a wavelength λ; the populations then become nearly independent of z, and they are determined by the balance of transfers between the dressed levels with these averaged rates $\bar{\Gamma}_{ij}$.

The final step of the calculation of $\bar{f}$ is now to insert the Π_i's into expression (3.17) of dU_F and integrate the result over one wavelength to get $\bar{f}$ by using Eq. (5.5). We find that

$$dU_F = \left(\hbar\Omega\Gamma_{21}\frac{\bar{\Gamma}_{12}}{\bar{\Gamma}_{12} + \bar{\Gamma}_{21}} - \hbar\Omega\Gamma_{12}\frac{\bar{\Gamma}_{21}}{\bar{\Gamma}_{12} + \bar{\Gamma}_{21}}\right)dt \qquad (5.16)$$

so that, by putting $dt = dz/v$,

$$\bar{f} = -\frac{\hbar}{v}\frac{\overline{\Omega\Gamma_{21}} \times \bar{\Gamma}_{12} - \overline{\Omega\Gamma_{12}} \times \bar{\Gamma}_{21}}{\bar{\Gamma}_{12} + \bar{\Gamma}_{21}}. \qquad (5.17)$$

For this second velocity range ($\Gamma \ll kv$), it appears that the variation of $\bar{f}$ with the velocity is completely different from what has been found for the first range $kv \ll \Gamma$: Instead of being proportional to the velocity (5.7), $\bar{f}$ is now inversely proportional to v, thus indicating that the power dissipated by $\bar{f}$ and transferred to the field is independent of the velocity.

Before giving the explicit results for $\bar{\Gamma}_{ij}$ and $\overline{\Omega\bar{\Gamma}_{ij}}$, let us transform Eq. (5.17) in order to get some physical insight for $\bar{f}$:

$$-\bar{f}v = \frac{dU_F}{dt} = \frac{\dfrac{\overline{\hbar(\omega_L + \Omega)\Gamma_{21}}}{\bar{\Gamma}_{21}} + \dfrac{\overline{\hbar(\omega_L - \Omega)\Gamma_{12}}}{\bar{\Gamma}_{12}} - 2\hbar\omega_L}{\dfrac{1}{\bar{\Gamma}_{12}} + \dfrac{1}{\bar{\Gamma}_{21}}}. \qquad (5.18)$$

The two quantities $\overline{\hbar(\omega_L + \Omega)\Gamma_{21}}/\bar{\Gamma}_{21}$ and $\overline{\hbar(\omega_L + \Omega)\Gamma_{12}}/\bar{\Gamma}_{12}$ are, respectively, the average energies of photons emitted in the transitions $|1, n\rangle \to |2, n - 1\rangle$ and $|2, n'\rangle \to |1, n' - 1\rangle$, whereas $-2\hbar\omega_L$ is the energy lost by the laser field when these two transitions occur. The numerator of Eq. (5.8) is then the total variation of the field energy in a cycle $1 \to 2 \to 1$ (or $2 \to 1 \to 2$). On the other hand, the denominator is just
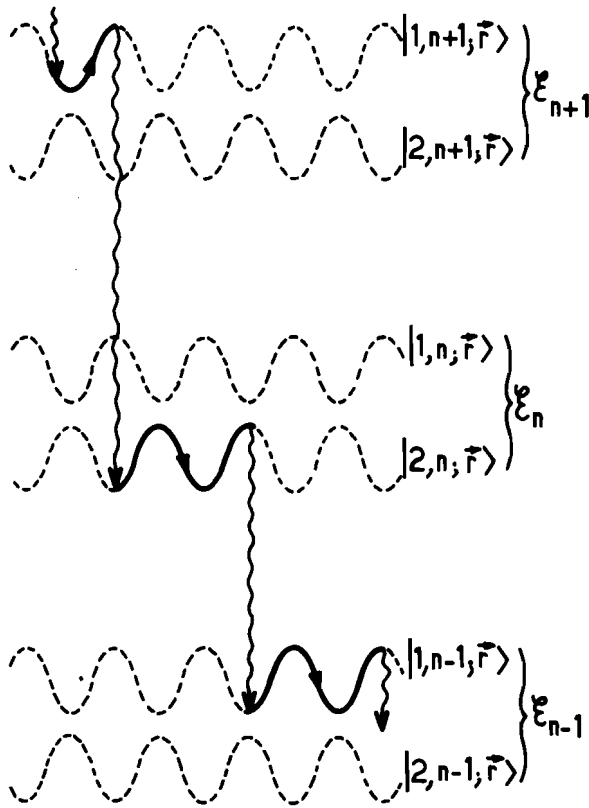
Fig. 6. Interpretation of the slowing down for $\delta > 0$ of an atom moving in a standing wave with intermediate velocities. The emission of the upper sideband of the fluorescence spectrum (transitions $1 \rightarrow 2$) occurs preferentially at the antinodes of the standing wave where the contaminations of $|1, n + 1; \mathbf{r}\rangle$ by $|e, n + 1\rangle$ and $|2, n, \mathbf{r}\rangle$ by $|g, n + 1\rangle$ are the largest. By contrast, for the lower sideband, the transitions $2 \rightarrow 1$ occur preferentially at the nodes where $|2, n; \mathbf{r}\rangle$ and $|1, n - 1; \mathbf{r}\rangle$ coincide, respectively, with $|e, n\rangle$ and $|g, n\rangle$. Consequently, between two transitions $1 \rightarrow 2$ or $2 \rightarrow 1$, the moving atom "sees" on the average more "uphill" parts that "downhill" ones in the dressed-atom energy diagram and is therefore slowed down.

the duration of such a cycle so that the right-hand side of Eq. (5.18) is the variation of the field energy per unit time, as expected. The main interest of Eq. (5.18) is to indicate clearly the dependence of the sign of $\bar{f}$ with the detuning δ. Take, for example, $\delta > 0$ (Fig. 6). Photons emitted on the lines $1, n \rightarrow 2, n - 1(\omega_L + \Omega)$ will be preferentially emitted at the *antinodes* of the standing wave, since it is at these points that the level $|1, n; \mathbf{r}\rangle$ (essentially $|g, n + 1\rangle$ for $\delta > 0$) contains the largest admixture of $|e, n\rangle$ and is therefore the most unstable. We then obtain

$$\frac{\overline{\hbar(\omega_L + \Omega)\Gamma_{21}}}{\bar{\Gamma}_{21}} \approx \hbar[\omega_L + (\tilde{\omega}_1{}^2 + \delta^2)^{1/2}]. \qquad (5.19)$$

By contrast, photons emitted on the line $2, n \rightarrow 1, n - 1$ will be preferentially emitted at the *nodes* of the standing wave since, at these points, $|2, n; \mathbf{r}\rangle$ is equal to $|e, n\rangle$ and is therefore the most unstable. This gives

$$\frac{\overline{\hbar(\omega_L - \Omega)\Gamma_{12}}}{\bar{\Gamma}_{12}} \approx \hbar(\omega_L - \delta). \qquad (5.20)$$

Putting approximations (5.19) and (5.20) into expression (5.18), we obtain

$$-\bar{f}v \approx \frac{\hbar\overline{(\omega_1{}^2 + \delta^2)^{1/2}} - \hbar\delta}{\dfrac{1}{\bar{\Gamma}_{12}} + \dfrac{1}{\bar{\Gamma}_{21}}}, \qquad (5.21)$$

which shows that $\bar{f}$ will be a damping force for δ positive. Another way of expressing this result is to say that the moving atom "sees" more "uphill" parts than "downhill" ones, since transitions preferentially occur for $\delta > 0$ from the "top" of a given dressed energy level to the "bottom" of the other one (see Fig. 6). For $\delta < 0$, the conclusion is of course reversed so that, finally, it appears that the respective signs of $\bar{f}$ and δ are the same here as for very slow atoms (Subsection 5.B).

An explicit calculation of $\bar{\Gamma}_{ij}$ and $\overline{\Omega\Gamma}_{ij}$ is possible from expressions (2.5) and (2.12) of Ω and Γ_{ij}, where ω_1 is replaced by Eq. (5.1). One obtains

$$\bar{\Gamma}_{\substack{12 \\ 21}} = \frac{\Gamma}{4}\left\{1 + \frac{1}{4(1 + 2s)^{1/2}}\left[1 \pm \frac{4\epsilon}{\pi}K\left(\frac{2s}{1 + 2s}\right)^{1/2}\right]\right\},$$

$$\overline{\Omega\Gamma}_{\substack{12 \\ 21}} = \frac{\Gamma|\delta|}{2\pi}\left\{(1 + 2s)^{1/2}E\left(\frac{2s}{1 + 2s}\right)^{1/2}\right\}$$

$$+ \frac{1}{(1 + 2s)^{1/2}}K\left(\frac{2s}{1 + 2s}\right)^{1/2} \pm \pi\epsilon\right\}, \qquad (5.22)$$

where K and E are the elliptic integrals of the first and the second kind[22]:

$$K(k) = \int_0^{\pi/2} \frac{d\alpha}{(1 - k^2 \sin^2 \alpha)^{1/2}},$$

$$E(k) = \int_0^{\pi/2} d\alpha\, (1 - k^2 \sin^2 \alpha)^{1/2}, \qquad (5.23)$$

where s is given by Eq. (5.9) and where ϵ is the sign of $\delta(\epsilon = \delta/|\delta|)$.

Remark

It is interesting to come back to the problem of the asymmetry of the fluorescence spectrum emitted by the moving atom. In Subsection 2.B we have seen that, in a small displacement $d\mathbf{r} = \mathbf{v}\, dt$, a change dU_F of the field energy reflects that different numbers of photons dn_+ and dn_- are emitted in the two sidebands $\omega_L \pm \Omega(\mathbf{r})$. On the other hand, because of the periodic variation of the populations $\Pi_i(z)$ for an atom moving in a standing wave, the averages over a distance λ of the numbers of photons emitted in the two sidebands $\bar{N}_+$ and $\bar{N}_-$ are necessarily equal. This equality $\bar{N}_+ = \bar{N}_-$ does not imply, however, that the field energy does not change when the atom travels over λ but only that the total weights of the two sidebands of the fluorescence spectrum emitted during this interval are equal. Actually the field energy *does* vary, because the average energies of the $\bar{N}_+$ and $\bar{N}_-$ photons emited in the upper and lower sidebands are larger (smaller) than $\omega_L + \bar{\Omega}$ and $\omega_L - \bar{\Omega}$ if δ is positive (negative) (see Fig. 6). In other words, even if the two sidebands have the same weights, their centers of gravity are not symmetric with respect to ω_L.

D. Connection with Previous Works

In the low-intensity limit ($\tilde{\omega}_1 < \Gamma$), atomic motion in a standing wave can be analyzed in perturbation terms. At the lowest order (in $\tilde{\omega}_1{}^2/\Gamma^2$), the mean dipole force averaged over λ appears as the difference between the radiation pressures of the two counterpropagating waves, "seen" with different Doppler shifts by the moving atom.[9] At higher orders, resonant multiphoton processes appear that involve absorption of photons from one wave and stimulated emission of photons in the opposite wave ("Doppleron" resonances).[7]

At higher intensities ($\tilde{\omega}_1 \gg \Gamma$), such a perturbative approach is no longer valid. Most previous works use then an exact solution of OBE's in terms of continued fractions or Fourier-series expansion.[10,11,21] These methods provide an exact solution, but, unfortunately, they do not give any physical picture for the new features that appear in this intensity range.

The dressed-atom approach followed in this paper is precisely well adapted to the limit $\tilde{\omega}_1 \gg \Gamma$. It has the advantage of providing not only simple physical pictures but also tractable analytical expressions in different velocity ranges. In order to demonstrate the accuracy of expressions (5.7) and (5.17)–(5.22) given above, we compare now their predictions with those of an exact calculation.

For example, the solid curve of Fig. 7 represents the result obtained by the continued-fraction method for the variations with kv/Γ of the mean dipole $\bar{f}$ when $\tilde{\omega}_1 = 1000\Gamma$ and $\delta = 200\Gamma$, whereas the two dashed lines represent the dressed-atom predictions (5.7) and (5.17)–(5.22):

$$kv \ll \Gamma, \qquad \bar{f} = -460\left(\frac{\hbar k\Gamma}{2}\right)\left(\frac{kv}{\Gamma}\right), \quad (5.24a)$$

$$\Gamma \ll kv \ll kv_{cr} \simeq 20\Gamma, \qquad \bar{f} = -40\left(\frac{\hbar k\Gamma}{2}\right)\left(\frac{\Gamma}{kv}\right). \quad (5.24b)$$
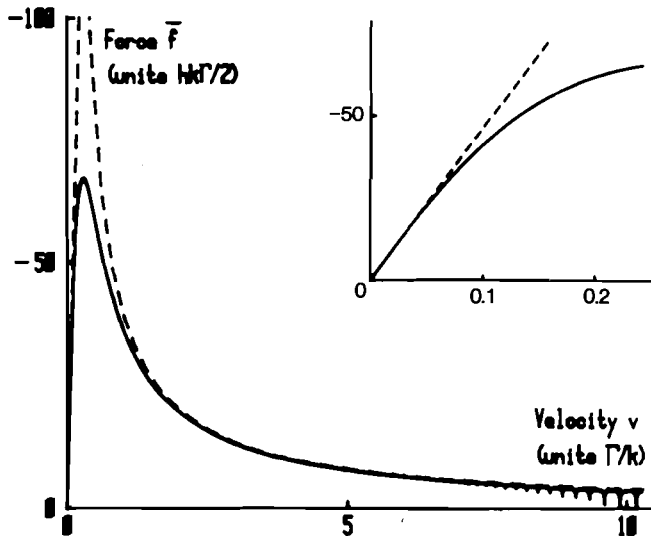

Fig. 7. Mean dipole force $\bar{f}$ (in units of $\hbar k\Gamma/2$) versus velocity (in units of Γ/k) for an atom moving in a standing wave ($\tilde{\omega}_1 = 1000\Gamma$, $\delta = 200\Gamma$). The solid curve is an exact numerical solution obtained by the method of continued fractions. The two dashed lines represent the analytical predictions of the dressed-atom approach for very low velocities ($kv \ll \Gamma$) (see also the insert) and for intermediate velocities ($\Gamma \ll kv \ll kv_{cr}$). The structures appearing in the exact solution in the high-velocity domain are a signature of the breakdown of the adiabatic approximation ("Doppleron" resonances).

We see that the agreement between the two theories is very good for these two velocity ranges. Note that it is possible to connect the two results for the dressed atom [expressions (5.24a) and (5.24b)] by calculating f from the general expression (5.12) for the populations. Actually, we have done such a numerical calculation, and the predictions of the two theories (dressed atom and continued fractions) then coincide perfectly, provided that the adiabatic approximation holds. When the velocity becomes too large, resonances appear on the solid curve of Fig. 7 (continued-fractions result), which are a signature of the breakdown of the adiabatic approximation. These are related to the so-called "Doppleron resonances."[7]

The force has been expressed in Fig. 7 in units of $\hbar k\Gamma/2$, which is the saturation radiation pressure. We see that the averaged dipole force can exceed this radiation pressure by 2 orders of magnitude for realistic Rabi frequencies ($\tilde{\omega}_1 = 1000\Gamma$ is achieved for sodium atoms with 1-W laser power focused on 100 μm). This indicates the very rich potentialities of this system, for example, for slowing down an atomic beam by sweeping a standing wave over the Doppler profile. The idea would be to change the relative frequencies of the two counterpropagating waves so that the nodes and the antinodes of the standing wave would move with a velocity v_{SW}. The velocity appearing in the expressions (5.24) of the dipole force would now be the relative velocity $v - v_{SW}$ between the moving atom and the standing wave. An appropriate sweeping of v_{SW} would then permit the decelerating force to be kept close to the optimal value of Fig. 7. Kazantsev has also suggested[23] that a swept standing wave be used to accelerate beams of neutral atoms.

6. CONCLUSION

In conclusion, we have derived the following results in this paper.

The dipole force experienced by an atom in a gradient of light intensity is closely related to the spatial variation of the energy levels of the combined system: atom + laser photons (dressed states) and to the redistribution of populations between these levels induced by the atomic motion (NA effects) or by spontaneous emission of fluorescence photons (radiative relaxation). We have shown that the work done by the dipole force during a small atomic displacement $d\mathbf{r}$ corresponds to the sum of the changes of the atomic potential energy and of the field energy, the latter change being related to an asymmetry of the fluorescence spectrum emitted by the atom during the displacement.

As in the Stern–Gerlach effect, two different forces with opposite signs are associated with the two types of position-dependent dressed states. An atom initially at rest in $\mathbf{r}$ undergoes spontaneous transitions between these two types of states. The corresponding picture of a two-valued instantaneous force fluctuating around a mean value leads to the correct values for the mean dipole force and for the atomic-momentum diffusion coefficient associated with intensity gradients.

For a very slowly moving atom ($kv \ll \Gamma$), we have interpreted the velocity-dependent dipole force as being due to the finite time constant of radiative relaxation, which introduces a time lag in the variation of the dressed-state populations of the moving atom. We have explained in this way why the velocity-dependent dipole force damps the atomic

Mechanical Effects of Light

motion when the laser is detuned above the atomic frequency and why, in a high-intensity standing wave, the sign of the force is opposite the one obtained by adding the radiation pressures of the two counterpropagating waves.

We have also considered the case of an atom moving in a high-intensity standing wave with intermediate velocities ($\Gamma \ll kv \ll kv_{cr}$). We have shown in this case that the dipole force averaged over λ is inversely proportional to v, and we have interpreted this result as being due to the fact that one sideband of the fluorescence specrum is emitted preferentially in the antinodes of the standing wave, whereas the other one is emitted in the nodes. Potentialities of such a dipole force for efficient slowing down of atoms have been pointed out.

It thus appears that the dressed atom provides useful physical insights in the dipole force in the high-intensity domain ($\omega_1 \gg \Gamma$) where perturbative approaches are no longer valid. We have also shown that such an approach leads to tractable mathematical expressions that are in good quantitative agreement with the prediction of other exact solutions (analytical or numerical) when they exist.

ACKNOWLEDGMENTS

REFERENCES

1. J. P. Gordon and A. Ashkin, Phys. Rev. A **21**, 1606 (1980).
2. R. J. Cook, Comments At. Mol. Phys. **10**, 267 (1981).
3. V. S. Letokhov and V. G. Minogin, Phys. Rep. **73**, 1 (1981).
4. A. P. Kazantsev, G. I. Surdutovich, and V. P. Yakovlev, J. Phys. (Paris) **42**, 1231 (1981).
5. D. J. Wineland and W. M. Itano, Phys. Rev. A **20**, 1521 (1979).
6. S. Stenholm, Phys. Rev. A **27**, 2513 (1983).
7. E. Kyrölä and S. Stenholm, Opt. Commun. **22**, 123 (1977).
8. A. F. Bernhardt and B. W. Shore, Phys. Rev. A **23**, 1290 (1981).
9. R. J. Cook, Phys. Rev. A **20**, 224 (1979).
10. V. G. Minogin and O. T. Serimaa, Opt. Commun. **30**, 373 (1979).
11. V. G. Minogin, Opt. Commun. **37**, 442 (1981).
12. J. Dalibard and C. Cohen-Tannoudji, J. Phys. B **18**, 1661 (1985).
13. R. J. Cook, Phys. Rev. A **22**, 1078 (1980).
14. C. Cohen Tannoudji and S. Reynaud, in *Multiphoton Processes*, J. H. Eberly and P. Lambropoulos, eds. (Wiley, New York, 1978), p. 103.
15. C. Cohen-Tannoudji and S. Reynaud, J. Phys. B **10**, 345 (1977).
16. S. Reynaud, Ann. Phys. (Paris) **8**, 315 (1983).
17. B. R. Mollow, Phys. Rev. **188**, 1969 (1969).
18. A. Messiah, *Mécanique Quantique II* (Dunod, Paris, 1964), p. 642.
19. The phase of the standing wave is constant so that radiation pressure does not play any role here.
20. The counterpart of this advantage is that the diffusion coefficient is bigger in a strong standing wave, so that the equilibrium velocities after cooling are larger than for usual radiation-pressure cooling (See Ref. 5).
21. E. Fiordilino and M. H. Mittleman, Phys. Rev. A **30**, 177 (1984).
22. I. S. Gradshteyn and L. M. Ryzhik, *Tables of Integrals, Series, and Products* (Academic, New York, 1980).
23. A. P. Kazantsev, Sov. Phys. Usp. **21**, 58 (1978).

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