Entanglement creation and characterization in a trapped-ion quantum simulator

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- Highly entangled state or noisy mess?
- How to characterize entangled states with >8 ions
- How coherent is the engineered spin-spin interaction?
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Outline:
• Trapped-ion experiments: time scales and tools
• Making trapped ions interact with each other
• Experimental characterization of entangled states
Quantum physics with linear ion strings

- Ion loading ~ minutes
- Ion storage ~ day(s)
- Individual experiments ~ 20 ms
  (initialization coherent interaction, + measurement)
  → ~ $10^5 - 10^6$ quantum measurements per day
- Ion creation
- Laser cooling → quantized ion motion
- Coherent excitation → spin-spin interactions
- Ion detection → quantum measurements
Quantum physics with linear ion strings

Trap frequencies:
\[ \nu_z \propto 1 \text{ MHz} \]
\[ \nu_{x,y} \propto 5 \text{ MHz} \]

Length scales

<table>
<thead>
<tr>
<th>ion distance ( d )</th>
<th>laser wavelength ( \lambda )</th>
<th>ion localisation ( z_0 )</th>
<th>Bohr radius ( a_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ( \mu \text{m} )</td>
<td>700 ( \text{nm} )</td>
<td>10 ( \text{nm} )</td>
<td>50 ( \text{pm} )</td>
</tr>
</tbody>
</table>

- Spatially resolved fluorescence
- Individual addressing
- No direct state-dependent interactions between ions
How to make ions interact

State-dependent interactions via

• Coulomb interaction (collective motional modes) + lasers / \( \mu \)-waves
• Rydberg interactions
• coupling to other quantum systems:
  – photons (cavity-QED experiments)
  – atomic quantum gases
  – transmission lines

\{ Entangling quantum gates

Many-body Hamiltonian
Realization of a transverse-field Ising model

\[ H = \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x + B \sum_i \sigma_i^z \]

\[ J_{ij} > 0, \quad B \gg J_{ij} \]
Encoding a (pseudo-)spin in a trapped ion

$P_{1/2}$

$\tau \approx 7 \text{ ns}$

$S_{1/2}$

$\langle \sigma_z \rangle$

$Qubit$/pseudo-spin

$\tau \approx 1.1 \text{ s}$

$40^{\text{Ca}^+}$

Quantum state detection

Quantum state manipulation

729 nm

Quantum state manipulation

$\equiv |\uparrow\rangle$

$\equiv |\downarrow\rangle$
Experimental setup

Linear trap with anisotropic harmonic potential:
\[ \omega_\perp / \omega_{ax} \approx 15 - 20 \quad \Rightarrow \quad \text{linear strings of up to 20 ions} \]

Spatially resolved fluorescence: detection of individual spin states

Focused steerable laser beam: coherent single-spin manipulation

Beam switching time \( \sim 10 \mu s \)
Measuring spins and spin correlations

\[ \equiv |\downarrow\downarrow\downarrow\downarrow\longrightarrow\downarrow\downarrow\rangle \]
\[ \equiv |\downarrow\downarrow\downarrow\downarrow\longleftrightarrow\downarrow\downarrow\rangle \]
\[ \equiv |\downarrow\downarrow\downarrow\downarrow\longrightarrow\uparrow\downarrow\rangle \]
\[ \equiv |\downarrow\downarrow\downarrow\downarrow\longleftrightarrow\uparrow\uparrow\rangle \]
\[ \equiv |\uparrow\uparrow\downarrow\downarrow\longleftrightarrow\uparrow\uparrow\rangle \]
\[ \equiv |\downarrow\downarrow\downarrow\downarrow\longrightarrow\downarrow\downarrow\rangle \]

\[ \rightarrow \text{any correlation } \langle \sigma^{(1)}_{\alpha_1} \sigma^{(2)}_{\alpha_2} \cdots \sigma^{(N)}_{\alpha_N} \rangle \]

Measurement of \( \sigma_x \)

(single)-spin rotation + fluorescence measurement \( |\Psi\rangle \rightarrow \begin{array}{c} U \end{array} \rightarrow \begin{array}{c} \text{π/2-pulse} \end{array} \rightarrow \begin{array}{c} |\uparrow\rangle \end{array} \rightarrow \begin{array}{c} |\downarrow\rangle \end{array} \rightarrow \begin{array}{c} |\rightarrow\rangle_x \end{array} \rightarrow \begin{array}{c} |+\rangle_x \end{array} \)
Quantum tomography: density matrix reconstruction

\[ \equiv |\downarrow\downarrow\downarrow\downarrow\rightarrow\downarrow\downarrow\rangle \quad \implies \text{any correlation } \langle \sigma^{(1)}_{\alpha_1} \sigma^{(2)}_{\alpha_2} \ldots \sigma^{(N)}_{\alpha_N} \rangle \text{ can be measured.} \]

Quantum tomography:

**1 qubit:**
\[
\rho = \frac{1}{2} \left( \langle I \rangle I + \langle \sigma_x \rangle \sigma_x + \langle \sigma_y \rangle \sigma_y + \langle \sigma_z \rangle \sigma_z \right)
\]

**N qubits:**
\[
\rho = \sum_{i=1}^{4^N} \langle A_i \rangle A_i \quad A_i \sim \sigma^{(1)}_{\alpha_1} \sigma^{(2)}_{\alpha_2} \ldots \sigma^{(N)}_{\alpha_N} \quad \sigma_\alpha \in \{I, \sigma_x, \sigma_y, \sigma_z\}
\]
How to make spins interact with each other

Coloumb interaction

collective modes of ion motion

$U \propto \frac{1}{r}$

Coloumb interaction + laser light

variable-range effective spin-spin interaction

$U \propto \frac{1}{r^\alpha}$

$0 < \alpha < 3$
Coupling to transverse vibrational modes

\[
\begin{align*}
|\downarrow\rangle|n\rangle & \leftrightarrow |\uparrow\rangle|n-1\rangle \\
|\downarrow\rangle & \leftrightarrow |\uparrow\rangle \\
|\downarrow\rangle|n\rangle & \leftrightarrow |\uparrow\rangle|n+1\rangle
\end{align*}
\]

18-ion spectra
all modes laser-cooled to ground state

Spin-spin interaction by off-resonant laser coupling to vibrational modes

\[
H \propto \sigma_i^x \sigma_j^x
\]
Variable-range interactions by coupling to transverse modes

Example: 11 ions

\[ H = \sum_{i<j} J_{ij} \sigma^x_i \sigma^x_j \]

\[ J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m} \sum_m \frac{b_{i,m} b_{j,m}}{\Delta^2 - \nu_m^2} \]

Spin-spin coupling \( J_{ij} \) (Hz)

- 'COM'
- 'Tilt'

strength decreases with distance
Measurement of the coupling matrix

Protocol:

1. Initialize ions in state $|\uparrow\rangle_i |\downarrow\rangle_j$

2. Switch on Ising Hamiltonian

$$|\uparrow\rangle_i |\downarrow\rangle_j \leftrightarrow |\downarrow\rangle_i |\uparrow\rangle_j$$

3. Measure coherent hopping rate
Spread of correlations after local quenches

\[ H \approx \sum_{i<j} J_{ij} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + B \sum_i \sigma_i^z \quad B \gg J_{ij} \]

Ground state: all spins aligned with transverse field

1. Local quench: flip one spin

2. Spread of entanglement

3. Measure magnetization or spin-spin correlations
Spread of correlations after a local quench

P. Jurcevic et al., Nature 511, 202 (2014)
P. Richerme et al., Nature 511, 198 (2014)

\[ J_{ij} \approx J_0 \frac{1}{|i-j|^{\alpha}} \]
Spread of entanglement after a local quench

z-Magnetization

7 ions $\alpha \approx 1.75$

density matrix reconstruction of spins 3 + 5
9 ms after the quench

P. Jurcevic et al., Nature 511, 202 (2014)
Creation of complex N-particle quantum states

\[ H = \sum_{i < j}^{N} J_{ij} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + B \sum_{i=1}^{N} \sigma_i^z \]

- **N/2 excitations**
- **complex quantum states**
- **subspace dimension exponentially with N**
- **quasi-particles:** spin waves (delocalized excitation)
- **ground state:** all spins aligned with external field

\[ B \gg J_{ij} \]
Characterization of large complex entangled states

1. Quantum state tomography of subsystems

N. Friis et al., PRX 8, 021012 (2018)

2. Matrix-product state tomography

B. Lanyon et al., Nat. Phys. 13, 1158 (2017)

3. Entropy measurements by random unitaries

unpublished data

T. Brydges et al., manuscript in preparation

4. Complex quantum states as a resource for variational eigensolvers

unpublished data
Neighbouring spins get entangled ...

... and disentangled with correlations spreading further out

B. Lanyon, C. Maier et al., Nat. Phys. 13, 1158 (2017)

with 20 ions: N. Friis et al., PRX 8, 021012 (2018)
Entanglement creation across the 8-spin chain

z-Magnetization dynamics

Propagation of spin correlation

Entanglement between the ends of the chain!
Quantum tomography of large multiparticle states with little entanglement

At early times: n-qubit entangled state, with finite correlation length

Strategy: find compact matrix product state representation of the global state by measuring local spin correlations

T. Baumgratz et al,
PRL 111, 020401 (2013)
M. Cramer et al,
Nat. Comm. 1, 149 (2010)
Step 1: Search for MPS state that optimally reproduces the experimentally observed local spin correlations

But what does the MPS state tell us about the state in the lab?

Step 2: Find a certificate: determine minimum fidelity of the lab state with the MPS state reconstruction
MPS tomography results for 8-spin quench

MPS tomography:
- Resource-efficient in the number of particles
- but: restricted to states with little entanglement

Dashed lines: MPS reconstruction for model state

Data points: Certified minimum fidelity of lab state with MPS state

B. Lanyon, C. Maier et al., Nat. Phys. 13, 1158 (2017)
State reconstruction:
Direct fidelity estimation vs. lower fidelity bounds

Direct fidelity estimation

$f(\ket{\psi_{\text{rec}}(t)}, \hat{q}_{\text{lab}})$

$|\langle \psi_{\text{rec}}(t) | \psi_{\text{theory}}(t) \rangle|^2$

14 ions

$R = 3$

B. Lanyon, C. Maier et al., Nat. Phys. 13, 1158 (2017)
Characterization of complex entangled states

- How mixed is the quantum state?
  
  measure the purity \[ P = \text{Tr}(\rho^2) \]

  (How close to unitary is the quantum dynamics generating the state?)

- How much entanglement is generated by non-equilibrium quantum dynamics?

  measure entanglement entropy

  von-Neumann entropy \[ S = -\text{Tr}(\rho_A \log \rho_A) \]

  Renyi entropy \[ S^{(2)} = -\log_2 \text{Tr}(\rho_A^2) \]

  measurement of nonlinear functionals of the density matrix
Measuring the purity / second Renyi entropy $S^{(2)}$

Measurement options:

- quantum state tomography
- joint measurement on two copies of a system

\[ P = \text{Tr}(\rho \otimes \rho \mathcal{O}) \]

resources scale exponentially with system size
difficult with ions


• random measurement on two virtual copies of a system

\[ P = \text{Tr}(U_\alpha \rho U_\alpha^\dagger \mathcal{O})^2 \]

average over:

van Enk, Beenacker, PRL 108, 110503 (2012)
A. Elben et al., PRL 120, 050406 (2018)

random gate circuits
global or local quenches
Random unitaries: single qubit

Single qubit

\[ P(\uparrow) = \frac{1}{2} (1 + |\vec{a}| \cos \theta) \]

length of Bloch vector

\[ \text{Tr}(\rho^2) = \frac{1}{2} (1 + 3|\vec{a}|\cos^2 \theta) = \frac{6}{3} P(\uparrow)^2 - 1 \]

\[ = \frac{1}{3} \quad \text{uniformly distributed} \]

Pure state:

\[ |\vec{a}| = 1 \rightarrow \text{Tr}(\rho^2) = 1 \]

Clearly, the purity of the state is maximal in the pure case.

Fully mixed state:

\[ |\vec{a}| = 0 \rightarrow \text{Tr}(\rho^2) = \frac{1}{2} \]
Local random unitaries on multiple qubits

\[ U_A = \bigotimes u_i \]
\[ u_i \in \text{CUE}(2) \]

Average over random measurements

\[ \text{Tr} \left[ \rho_A^2 \right] = \sum_{s_A, s'_A} A_{s_A s'_A} \frac{P(s_A) P(s'_A)}{P(s_A)} \]

analytically known

\[ P(s_A) = \text{Tr} \left[ U_A \rho_A U_A^\dagger |s_A \rangle \langle s_A| \right] \]
Local random unitaries: experimental realization

Single-qubit random unitaries are realized by

\[ U(\alpha, \beta, \gamma) = Z(\alpha)X(\beta)Z(\gamma) = Z(\alpha)Y(\pi/2)Z(\beta)Y(-\pi/2)Z(\gamma) \]

\( (\alpha, \beta, \gamma \text{ drawn from suitable distributions}) \)

\[ U(\alpha, \beta_2, \gamma_2)U(\alpha_1, \beta_1, \gamma_1) : \text{Concatenation of two such random unitaries to make the experiment robust against calibration errors.} \]

Resulting pulse sequence:

\[ \rho \]

\[ : \text{global } \pi/2 \text{ pulses} \]

\[ : \text{single-qubit } z\text{-rotations} \]
Measurement scheme

$H_{XY}$
Purity and Renyi entropy measurement

Entanglement growth

Total purity nearly invariant

Number of projective measurements $N_U N_M$

unpublished data
Entropy measurements

Subsystems acquire high entropies over time (hard to measure)

unpublished data

T. Brydges et al, manuscript in preparation
Interplay of interactions and disorder

\[ H_{XY} = \hbar \sum_{i<j} J_{ij} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + \hbar \sum_j (B + b_j) \sigma_j^z \]

long range interaction  
local disorder potentials

T. Brydges et al, manuscript in preparation

unpublished data

Disordered system

Disorder strength \( \Delta/J = 3 \)  \( \Delta/J = 0 \)

Mutual information (correlations) decaying with distance
Complex quantum states as a resource for variational quantum eigensolvers (VQE)

**Goal:** find the ground state energy of the Hamiltonian \( H = \sum_l a_l H_l \)

„Quantum-classical hybrid approach“:

- use quantum co-processor for calculating \( \langle H \rangle \) for a variational state
- classical computer for updating parameters of the variational state

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<tr>
<th>B</th>
<th>VQE implementation</th>
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<tbody>
<tr>
<td>classical computer</td>
<td>variational optimization</td>
</tr>
<tr>
<td>4</td>
<td>( \langle H \rangle = \sum_\ell a_\ell \langle H_\ell \rangle ) iteration ( i=0,1,... ) for each ( R )</td>
</tr>
<tr>
<td>quantum processor</td>
<td>parametrized state preparation</td>
</tr>
<tr>
<td>1</td>
<td>(</td>
</tr>
<tr>
<td>2</td>
<td>Measurement of observables</td>
</tr>
<tr>
<td>3</td>
<td>[ \langle \varphi(\theta_i)</td>
</tr>
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</table>
VQEs for spin lattice Hamiltonians

Mapping a 1d lattice Schwinger model to a spin lattice Hamiltonian:

\[ H^T = w \sum_{n=1}^{N-1} \left[ \sigma_n^+ \sigma_{n+1}^- + \text{H.c.} \right] + \frac{m}{2} \sum_{n=1}^{N} (-1)^n \sigma_n^z + J \sum_{n=1}^{N-1} L_n^2, \]

with

\[ L_n = \varepsilon_0 - \frac{1}{2} \sum_{\ell=1}^{n} \left( \sigma_{\ell}^z + (-1)^{\ell} \right) \]

complicated spin-spin interaction
Preparing the Schwinger ground state in a trapped-ion experiment and measuring its energy

State preparation:

$$|\psi(\delta t, \Theta)\rangle = \prod_{j=1}^{Q} R_j^j(\Theta_1^Z) \cdots R_{N-1}^j(\Theta_2^Z) R_N^j(\Theta_1^Z) \times e^{-i\delta t \tau_j^{H_{\text{eff}}}} |\psi_i\rangle.$$ 

Energy measurement:

Measurement of all 1- and 2-body correlations for determining $\langle H_T \rangle$

$H_T$: Schwinger target Hamiltonian

$H_{\text{eff}}$: trapped-ion spin-spin Hamiltonian
Experimental results

unpublished data

Variational optimization

work in progress...
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MPS tomography + entanglement witnesses:
Summary and outlook

Trapped-ion quantum simulations

- Realization of long range Ising models in trapped ions
- Fully addressable for up to 20 ions
- Single-shot measurements of arbitrary spin correlations
- Entanglement characterization in small subsystems by tomography and random measurements

P. Jurcevic et al., Nature 511, 202 (2014)  
B. Lanyon, C. Maier et al., Nat. Phys. 13, 1158 (2017)  
N. Friis et al., PRX 8, 021012(2018)  
T. Brydges et al., in preparation

Outlook:

- Exploration of non-equilibrium quantum dynamics in larger systems
- Scaling the system up: longer 1d strings, experiments with planar ion crystals