Superconducting Electrometer for Measuring the Single Cooper Pair Box

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Abstract

We discuss for the single Cooper pair box the contributions to relaxation and to decoherence of the electromagnetic environment, of the offset charge noise, and of a measuring Single Electron Transistor. We show that a single Cooper pair transistor can also be used for that purpose. Experimentally, we have operated such a device by measuring the variations of its critical supercurrent with the gate voltage using a SQUID series array amplifier. We describe the characteristics of this new electrometer and compare different schemes for measuring the critical current.

0.1 Introduction

The giant leap that quantum mechanics could bring to computing science\cite{1} motivates an intense research of systems suitable for implementing quantum bits (qubits) and quantum algorithms. The requirements are formidable: the quantum states of the elementary qubits should be manipulable at will without significant loss of coherence over times much longer than the duration
of elementary transformations, the couplings between qubits should be fully controllable, and the state of a qubit should be readable reliably. Furthermore, the implementation of the error correcting codes necessary to fight the unavoidable residual decoherence would require to perform measurements on some qubits during the computation process in order to perform adequate correction manipulations[2]. At the present time, quantum entanglement up to four qubits[3] and operation of elementary quantum gates[4] have already been demonstrated in quantum-optics based systems. Although less developed, microfabricated solid state systems are more appealing because they could be integrated on a large scale far more easily. The most advanced results reported so far with solid state systems have been obtained on qubits based on flux-states of small superconducting loops[5], and on charge states of small superconducting islands[6, 7]. In particular, Rabi precession between the two states of a charge qubit has been demonstrated in the single Cooper pair box over a few tens of oscillations[7], and longer coherence times are expected. The understanding of all decoherence sources which limit the duration of coherent oscillations in this system is thus an important issue. In this work, we discuss the influence on the single Cooper pair box of the residual dissipation in the box circuit, of the offset charge noise, and of the measuring system. In order to reduce the back-action of the measuring apparatus, we consider a new type of electrometer based on the superconducting version of the single electron transistor[8]. Finally, we report the first electrometry measurements performed with such an electrometer, and we discuss the different possible measuring set-ups.

0.2 Description of the single Cooper pair box

The single Cooper pair box[6], described in Fig. 1, consists of a single superconducting island connected to a voltage source $U$ through a small capacitor $C_g$ on one side and through a small Josephson junction[9], with capacitance $C_J$ and Josephson energy $E_J$, on the other side. When the superconducting gap $\Delta$ in the junction electrodes is larger than the charging energy $E_c = e^2/2C_\Sigma$ (with $C_\Sigma = C_J + C_g$), two charge states $|n\rangle$ and $|n+1\rangle$ differing by one Cooper pair in the island form, close to their electrostatic energy degeneracy point, a two-level-system well decoupled from other degrees of
Figure 1: Top: schematic circuit of the single Cooper pair box. The dashed line encloses the box island. Middle: realistic circuit with a residual series impedance. Bottom: representation of the electromagnetic modes coupled to the box.
freedom. The effective spin 1/2 hamiltonian of this two-level system writes:

\[ H_0 = -\frac{1}{2} \vec{B} \cdot \vec{\sigma} \]  

(1)

where \( \vec{\sigma} \) is a vector of Pauli operators, and \( \vec{B} \) a fictitious field with components \( \{E_J, 0, 4E_c(1 - n_g)\} \), with \( n_g = C_g U/e \) (see Fig. 2). The ground and excited states of the above hamiltonian, \( |\sigma_B = 1\rangle \) and \( |\sigma_B = -1\rangle \), are the two states of the qubit. Their energy difference is \( \hbar \Omega = E_J / \sin \theta \), where \( \theta \) is the angle between \( \vec{B} \) and the \( z \) axis. This description is however oversimplified, and the qubit is coupled to other degrees of freedom, both at thermal equilibrium and out of thermal equilibrium. We will consider in this work the effect of residual electromagnetic dissipation in the Cooper pair box circuit, the effect of moving charges in the neighborhood of the box island, and the effect of an electrometer measuring the state of the qubit from the electrostatic potential of the box island.

Figure 2: Fictitious spin 1/2 representation of the Cooper pair box. The vector \( \vec{B} \) is the effective field acting on the spin.

0.3 Relaxation and decoherence induced by the electromagnetic environment

The electromagnetic degrees of freedom of the box circuit can be modeled by inserting a small series impedance \( z(\omega) \), as shown in Fig. 1. This impedance
incorporates the effect of the voltage source and wiring impedances. Its effect is to couple the box to a set of bosonic electromagnetic modes with frequencies \( \{ \omega_j \} \) through the hamiltonian:

\[
h = -\sqrt{\pi} \sum_j \hbar \omega_j \sqrt{\frac{Z_j}{R_K}} (b_j - b_j^\dagger) \sigma_z ,
\]

where \( Z_j, b_j \) and \( b_j^\dagger \) are the impedance, the annihilation and creation operators of mode \( j \), respectively, and \( R_K = \hbar/e^2 \) is the resistance quantum. These modes, which form the electromagnetic environment of the charge qubit, are assumed at thermal equilibrium. Any summation \( \sum_j f(\omega_j) Z_j \) over the set of modes is performed in the following way[10]:

\[
\sum_j f(\omega_j) Z_j = \frac{2}{\pi} \int_0^\infty f(\omega) \text{Re } Z(\omega) \frac{d\omega}{\omega} ,
\]

with, in the weak coupling regime \( \kappa = C_q/(C_j + C_g) \ll 1 \) relevant for the experiments, \( \text{Re } Z(\omega) = \kappa^2 \text{Re } z(\omega) \). The coupling hamiltonian (2) induces transitions between the box and the modes of the environment. The standard second order perturbation theory yields for the downward and upward transition rates \( \Gamma_\downarrow \) and \( \Gamma_\uparrow \) between the excited state and the ground state of the box:

\[
\Gamma_\downarrow(\uparrow) = \frac{(2\pi)^2 \sin^2 \theta \ S(\pm(\omega))}{\hbar R_K} ,
\]

where \( S(\omega) \) is the spectrum of the voltage fluctuations across the effective impedance \( Z(\omega) \):

\[
S(\omega) = \frac{\hbar |\omega|}{2\pi} \left[ \coth(\frac{\hbar \omega}{2k_B T}) + 1 \right] \text{Re } Z(\omega) .
\]

The total relaxation rate \( \Gamma_\downarrow^{ENV} = \Gamma_\downarrow + \Gamma_\uparrow \) is the decay rate of the diagonal part of qubit matrix density. In the low temperature regime \( T \ll \hbar \Omega/k_B \), one has \( \Gamma_\uparrow \simeq 0 \), and the total relaxation rate is simply:

\[
\Gamma_\downarrow^{ENV} \simeq 4\pi \Omega \sin^2 \theta \ \kappa^2 \text{Re } z(\Omega)/R_K .
\]

Numerically, one gets \( \Gamma_\downarrow^{ENV} = 50 \text{ kHz} \) for the box parameters \( \Omega/2\pi = 10 \text{ GHz} \) \( (\hbar \Omega/k_B \simeq 0.5 \text{ K}) \), \( \kappa = 2.5\% \), \( \text{Re } z(\Omega) = r = 5 \Omega \), and \( \theta = \pi/4 \).
We now discuss the decay of the coherence amplitude \( \langle X' \mid X'(t) \rangle \) of a state prepared at \( t = 0 \) as \( \mid X' \rangle = 1/\sqrt{2} (\mid \sigma_B = 1 \rangle + \mid \sigma_B = -1 \rangle) \). Although on-resonance modes exchanging energy with the box contribute to the decay of coherence, out-of-resonance oscillators also contribute because they get entangled with the box. The coherence amplitude \( \langle X' \mid X'(t) \rangle \) picks an extra decay factor\(^\text{[11]} \):
\[
A(t) = \exp \left( 4 \cos^2 \theta, \text{Re}[J(t)] \right)
\]
where \( J(t) \) is the phase correlation function which appears in the theory of Coulomb blockade\(^\text{[12]} \):
\[
J(t) = 2 \int_0^\infty \frac{d\omega \ \text{Re} Z(\omega)}{\omega} \frac{\exp(-i\omega t) - 1}{1 - \exp(-\hbar \omega/k_B T)}
\]
(8)

In the simple case when \( z(\omega) = r \), the function \( J(t) \) can be calculated exactly. The long time behavior of \( \text{Re} J(t) \) is:
- At zero temperature: \( \text{Re} J(t) \sim -2\frac{2\pi e^2}{\hbar R_K}(\gamma + \ln(t/\kappa r C_J)) \)
- At finite temperature: \( \text{Re} J(t) \sim -2\frac{\pi e^2 r}{R_K}(k_B T t/\hbar) \)

At zero temperature, \( A(t) \) follows a power-law with a small exponent, and decoherence is weak. At temperatures \( T > 10 \text{ mK} \), the classical regime is almost reached, and \( A(t) \) decays exponentially at a rate \( \Gamma_2^{\text{ENV}} \):
\[
\Gamma_2^{\text{ENV}} = 8\pi \left( \frac{k_B T}{\hbar} \right) \frac{k^2 r}{R_K} \cos^2 \theta.
\]
(9)

This result can be easily retrieved by performing the following semiclassical average:
\[
A(t) = \left\langle \exp i \int_0^t (\Omega(t') - \bar{\Omega}) \ dt' \right\rangle
\]
where \( \Omega(t) \) is now the time dependent transition frequency modulated by the thermal fluctuations of the voltage across the impedance \( z(\omega) \). In the classical regime, this entanglement is dominated by a random phase factor between the two states of the coherent superposition. It is worth noticing that the entanglement between the qubit and its environment is not an irreversible process by itself, and that one could in principle recover to some extent the loss of coherence due to low frequency modes using echo or even more sophisticated pulse techniques, analogous to those developed in nuclear magnetic
resonance (NMR). If the impedance \( z(\omega) \) is frequency independent, the ratio between the relaxation rate and the decoherence rate due to thermally excited oscillators reduces to:

\[
\frac{\Gamma_{1}^{ENV}}{\Gamma_{2}^{ENV}} = \frac{\hbar \Omega}{2k_B T} \tan^2 \theta .
\]

Since the experiments are performed in the low temperature regime \( k_B T \ll \hbar \Omega \), one has \( \Gamma_{1}^{ENV} / \Gamma_{2}^{ENV} \gg 1 \) in practice. In this case, the electromagnetic environment of the qubit relaxes the whole qubit density matrix at the rate \( \Gamma_{1}^{ENV} \).

0.4 Relaxation and decoherence induced by the offset charge noise

It is well known that the island of a Single Electron Transistor (SET) [13] is subject to an offset charge noise with a \( 1/f \) spectrum[14]. This noise is attributed to a set of charges randomly fluctuating between two positions in the junction barriers and/or in the insulators close to the SET island. Occasionally, slow two-level fluctuators (TLF) have been directly observed over long times. The \( 1/f \) character of this TLF noise has been probed up to about 10 MHz. Its typical intensity is \( S_q(f) = e^2 B / f \), with \( B \approx 10^{-7} \). Like the electromagnetic environment, the charge noise induces relaxation and decoherence on the box quantum states. Although the spectral density of the charge noise has only been measured at frequencies much smaller than \( \Omega \), recent experiments on the single electron pump[15] have provided experimental evidence that the charge noise extends up to 100 GHz, i.e. well above \( \Omega \). Amazingly, the spectral density estimated from the measured transition rate of otherwise forbidden transitions falls rather close to the extrapolated value of the \( 1/f \) spectrum, with \( B' \approx 10^{-8} \). This noise should result in upward as well as downward transitions at a rate \( \Gamma_1^{TLF} \):

\[
\Gamma_1^{TLF} = \frac{2\pi B'}{\Omega} \left( \frac{E_c}{\hbar} \right)^2 \sin^2 \theta .
\]

The estimated value is about \( (1 \times \sin^2 \theta) \) MHz, significantly larger than the estimated contribution of the box circuit electromagnetic environment.

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The decay rate of the coherence amplitude picks an extra contribution from the low-frequency offset charge fluctuations. Because of divergences inherent to the $1/f$ spectrum, the experimental protocol has to be precisely defined. When a measurement, performed during a short time $\tau$, is averaged over a time $t_{av} \gg \tau$, the transition frequency has drifted away from its initial value and one finds for the decay function:

$$A'(\tau) \simeq \exp \left[ -8B \cos^2 \theta \left( \frac{E_c \tau}{\hbar} \right)^2 \ln(t_{av}/\tau) \right].$$

(13)

The contributions of low frequency TLF to the decay of a coherence signal can be suppressed using an echo pulse sequence similar to those used in NMR to compensate for inhomogeneous line-broadening. In this case, the decay function does not depend on the averaging time and writes:

$$A'(\tau) \simeq \exp \left[ -8B \cos^2 \theta \left( \frac{E_c \tau}{\hbar} \right)^2 \frac{\log(2)}{2} \right].$$

(14)

The decay is still fast, and coherence is lost after a time $\sim (20/\cos \theta)$ ns for $B = 10^{-7}$, $E_c = 0.5 \text{kB} \text{K}$. The $1/f$ noise is thus a serious limitation, and its reduction to a level significantly smaller than commonly achieved is an important issue.

0.5 Relaxation and decoherence induced by a measuring SET electrometer

The state of the box can be measured either by measuring the electrostatic potential of the island[6], or by connecting it to an extra small probing tunnel junction which directly exchanges electrons with it[7]. Whereas this latter method results in a destructive measurement of the qubit state, the first one discussed here could in principle allow quantum non demolition (QND) measurements. For that purpose, the measuring electrometer should be able to distinguish both states of the qubit without inducing transitions between them. Repeated measurements of the qubit state should then give the same answer. An important characteristic of any measuring electrometer is thus the amount of information it can provide before irreversible transitions induced by the measuring system or by other relaxation mechanisms occur. Figure 3 shows a measuring set-up with a SET electrometer capacitively coupled through $C_c$ to the box island, a set-up which has been used
to measure the island potential in the box ground state[6]. The measuring process has been theoretically investigated in great detail for this set-up[16]. The back-action of the SET results from the fluctuations of the electrostatic potential of its own island while the current is flowing. When one electron enters or exits the island, the voltage varies by \( \delta V = e/C_{\text{SET}} \), where \( C_{\text{SET}} \) is the capacitance of the SET island. Typically, \( \delta V \) is a few hundreds of microvolts. At a practical working point, the correlation time of these voltage fluctuations is \( \tau_c \approx 1/R_{\text{SET}}C_{\text{SET}} \approx e/I \), where \( R_{\text{SET}} \) is the tunnel resistance of the SET junctions, and \( I \) is the average current. The spectrum is thus lorentzian with a cut-off frequency \( \tau_c^{-1} \). Its detailed form depends on the biasing point, and is precisely known for SETs with junction resistances larger than \( R_K \) [17]. Low frequency fluctuations result in decoherence, and a SET at the threshold can reach the quantum limit in the sense that the qubit state is dephased just on the time scale needed to measure it[16]. However, fluctuations at the qubit transition frequency \( \Omega \) induce transitions at a significant rate when \( \Omega \tau_c < 1 \). In this regime, the induced relaxation rate follows from Eq. (4) using the spectral density of the SET island voltage and the relevant

![Diagram](image_url)
coupling factor $\kappa_{SET} = C_e/C_J \ll 1$ between the box and the SET:

$$\Gamma^\text{SET}_\downarrow \approx \frac{(2\pi)^2 \sin^2 \theta \ \kappa^2_{SET}}{\hbar R_K} \frac{(\delta V/2)^2}{\tau_c}. \quad (15)$$

More precise estimates can be obtained to take into account the precise working point of the SET and the effect of the rf drive in the case of an rf-SET[18]. The question thus arises if a SET is able to measure the box state before the induced relaxation has destroyed the qubit. A figure of merit can be defined as $f^{SET} = 1/ \left( \Gamma^\text{SET}_\downarrow \tau_m \right)$, $\tau_m$ being the minimum time necessary to perform a measurement of the box state. Assuming that the measurement accuracy is solely limited by the SET intrinsic noise and not by the electronic amplifiers measuring the SET current, one finds $f^{SET} \approx \cot^2 \theta$. A usual SET can thus perform a single measurement of the qubit[16], but not by a large margin. Note that a SET operated in the strong tunneling regime could possibly achieve a better performance. From the experimental point of view, the sensitivity of the best rf-SET is still presently limited by the noise of the microwave amplifier at the carrier frequency but the intrinsic limit is not beyond reach. In the following, we examine another type of electrometer based on the superconducting version of the SET, the SSET.

### 0.6 The superconducting SET electrometer

The SSET is almost equivalent to a single small Josephson junction whose critical current $I_c(n_g)$ is periodically modulated by the gate charge[8]. The only difference with a single junction is that the current-phase is not strictly sinusoidal, and that higher energy states can be excited. In the case when the gate charge is modulated at frequencies smaller than the band gap, the adiabatic approximation holds, and the SSET behaves as a tunable junction. The modulation pattern is determined by the ratio $E_c/E'_J$, where $E'_J = I_0 \hbar/2e$ is the Josephson energy of each junction with critical current $I_0$. For practical values $E_c/E'_J \sim 1$, the maximum critical current of a SSET, obtained for a reduced gate charge $n_g = 1 \ [\text{mod} 2]$, is of the order of $I_0/2$, and the slope $dI_c/dn_g$ is of the order of $I_0$ at practical working points. The predicted critical current for a SSET is compared in Fig. 4 with the average maximum supercurrent measured in a current-biased set-up[8]. In this case, the current-voltage $I - V$ characteristic is hysteretic and, upon ramping the bias current, the junction switches out of the zero-voltage-state at a switching
current $I_S$. The values of $I_S$ are distributed with an histogram whose average value and width depend on the electromagnetic impedance as seen from the SSET[19]. The average value is smaller than the critical current but almost scales with the predicted variations for $I_c(n_g)$. Depending on the biasing circuitry impedance, the critical current of a SSET can be measured in two different ways, which yields to two very different types of electrometers.

Straight line: predicted variations for the critical current of a SSET with $E_c = 0.66 \, k_B K$ and $E_j = 0.50 \, k_B K$. Dots: Measured average switching current $I_S$ for this SSET in a moderate damping circuit at $T = 20 \, \text{mK}$. Dashed line: theoretical predictions.

![Graph showing critical current and switching current variations](image.png)

Figure 4: Straight line: predicted variations for the critical current of a SSET with $E_c = 0.66 \, k_B K$ and $E_j = 0.50 \, k_B K$. Dots: Measured average switching current $I_S$ for this SSET in a moderate damping circuit at $T = 20 \, \text{mK}$. Dashed line: theoretical predictions.

### 0.6.1 electrometry based on the switching of an ac-shunted SSET

The variations of the average switching current with the gate charge can be used for electrometry. The obtention of narrow switching histograms suitable for electrometry requires to damp the dynamics of the phase across the junction. Switching histograms with an average close to the critical current and
a relative width smaller than $10^{-3}$ have been obtained in the case of a single junction using an $RC$ ac-shunting circuit[19]. However, the bias current ramp-rate required for measuring a box is larger than used in the previous experiment[19], and we have found that histograms get wider when the ramp rate increases. Experimentally, we have measured switching histograms of an $RC$-shunted SSET using ramping speeds up to $0.2 \times I_S/\mu s$. Preliminary results show switching histograms narrow enough to discriminate the two box states within a measuring time of $1 \mu s$.

0.6.2 electrometry based on the dc-shunted SSET

When a Josephson junction is shunted by a small enough resistor, its $I - V$ characteristic is no longer hysteretic. However, its measurement is far more difficult than in the unshunted case because the voltage across the junction is too small to be measured using room temperature amplifiers. Recently, our group has used the SQUID series array amplifiers developed at NIST-Boulder[20] to measure the full $I - V$ characteristic of a small Josephson junction. The array we have used consists of 100 dc-SQUIDs in series, and delivers a signal large enough to be amplified by room temperature amplifiers without degradation[21]. Closed loop operation of the arrays is possible within a few MHz bandwidth. The experimental results on single junctions[22], in excellent agreement with the calculated $I - V$ characteristics[26], show that the classical regime for the phase dynamics was indeed reached. This result made possible the design of an electrometer based on the measurement of a shunted SSET[23]. For that purpose, we have implemented the set-up schematically described in Fig. 5, in order to measure the current $I_R$ through the shunting resistance $R_s$ of a SSET. More precisely, a fraction of this current flows in the input coil of a SQUID array amplifier. In order to avoid any high frequency resonance, an extra ac-shunt has been placed on-chip across the SSET, but careful mounting of discrete components should however be sufficient to obtain an adequate environment.

**sensitivity and bandwidth** The sensitivity of the electrometer is set by the intrinsic SSET sensitivity $dI_c/dn_q \approx I_0$, by the ratio $dI_R/dI_c \lesssim 1$ at the bias point of the device, and by the noise of the SQUID array referred to its input $S_I \approx 3 \text{ pA}/\sqrt{\text{Hz}}$. The intrinsic noise floor which results from the thermal fluctuations of $I_R$ is much smaller and will be considered later.
Figure 5: Schematic circuit of a SSET electrometer. The SSET is displayed as a tunable junction. A fraction $\sim 1/3$ of the dc-current in the shunt resistance passes in the input coil of a 100 SQUID series array. $R_d$ and $C_d$ form an on-chip damping circuit, $R_c$ is a contact resistance, and $R_1$, $R_2$ and $R_3$ are surface-mounted components. The low-pass filters F prevent the Josephson oscillations in the SQUIDs from disturbing the SSET.
on. The sensitivity, expressed in $e/\sqrt{\text{Hz}}$, is thus $s \approx S_I/I_0$. Although a larger critical current $I_0$ should result in better sensitivity, too large values of $I_0$ result in a strong renormalisation of the island charging energy due to virtual quasiparticle tunneling and correspondingly to a strong reduction of the modulation depth of the critical current. In practice, we have found that the optimal value of $I_0$ is in the range 20 – 40 nA, which would result in a sensitivity $s \approx 10^{-4}e/\sqrt{\text{Hz}}$ for a maximal coupling between the SSET and the array. This figure is significantly worse than the sensitivity $s \approx 7 \times 10^{-5}e/\sqrt{\text{Hz}}$ already achieved with the rf-SET, but two-stage SQUID amplifiers might allow to improve the present sensitivity by about one order of magnitude. In this design, the SSET is connected to the input coil of a single dc-SQUID, which is itself in series with the input coil of a SQUID array. The bandwidth is limited by the input circuit and by the electronics backing the array. As seen from the array input coil, the SSET behaves as a source with a resistive impedance which is of the order of the series resistance in the input coil circuitry, 10 $\Omega$ in our case. For an input coil inductance of 200 nH, the resulting bandwidth is about 10 MHz, but can be made larger if needed.

**experimental results** We have fabricated SSETs using 3 angle deposition through a shadow mask[24]. The two first aluminum layers form the junction electrodes, and the third gold layer forms normal wires which help eliminating spurious quasiparticles in the superconducting electrodes, and connect the SSET to an on-chip $RC$ damping circuit fabricated by optical lithography. The SSET junction have an area of 80×130 nm$^2$ and a tunnel resistance of 7.5 k$\Omega$. The samples were mounted in a shielded box fitted with coaxial connections to the SQUID series array box, and microfabricated RC filters[25] were installed on the bias and gate lines of the electrometer. As shown in Fig. 6 for a series of bias current values, the current through the shunt resistor of the SSET is modulated by the gate voltage, with a measured period corresponding to $2e$. At larger bias currents, the modulation depth decreases progressively. We have also determined the IV characteristic of the SSET for different gate voltages. The extremal IVs, obtained for $n_g = 0$ and $n_g = 1$, are shown in Fig. 7. The overall agreement between the experimental results and the theoretical predictions[26] for a tunable Josephson junction is satisfactory. The parameters are $E_c = 0.55 \ k_B K$, and $I_0 = 40$ nA which are in good agreement with the values estimated from the area and tunnel resistance
of the junctions, and from the superconducting gap $\Delta = 180 \, \mu eV$. This corresponds to a ratio $E_c/E'_f = 0.55$. The damping resistance as seen from the SSET is $R_{\text{eff}} = 38 \, \Omega$, larger than the estimated value 25 $\Omega$ taking into account all components in the circuit. The effective temperature of 200 mK needed to fit the data is larger than the fridge temperature 40 mK because filtering between the SQUID array and the SSET has been greatly reduced compared to previous experiments on a single junction. This excess noise temperature is however tolerable for electrometry applications, and can be reduced if needed. We have also applied to the SSET gate a $0.2e$ step and

![Graph](image)

Figure 6: Open symbols: variations of the current through the shunt resistor $I_R$ with the gate charge $n_g$ at $T = 40$ mK and for bias currents $I = 11, 21, 31, 40$ nA, from bottom to top. Full lines: theoretical predictions at $T = 200$ mK based on the effective junction model.

recorded the response of the electrometer. In order to be sure that the measured signal originates from the SSET and not from a direct coupling to the SQUID electronics, we have subtracted the traces with the SSET

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Figure 7: Open symbols: extremal $I-V$ characteristics of the SSET obtained for $n_g = 0$ and $n_g = 1$, at $T = 40$ mK. Full lines: theoretical predictions at $T = 200$ mK.

on and off. The applied step is compared in Fig. 8 with the electrometer response for two different output bandwidths, averaged over 4000 traces. The results demonstrate that the overall system has a bandwidth of about 1 MHz, probably limited by the extra filtering installed on the SQUID array lines. The noise level corresponds to the estimated one of $3.10^{-4} e/\sqrt{\text{Hz}}$, taking into account that only a fraction of the modulated current goes through the array input coil, and numerical factors. Although a faster response would be convenient, it would be of little use for measuring the state of a Cooper pair box if the sensitivity is not improved. Indeed, with the achieved noise level, one would need a typical time of a few $\mu$s to perform a measurement of the box state, assuming a coupling factor $\kappa_{\text{SSET}} = 2.5\%$. Improving the sensitivity is thus mandatory, and two stage SQUID amplifiers will be tested in the future for that purpose.

**back-action noise** The fluctuations of the island voltage $V$ are very different from those in the SET because the SSET island is not sequentially charged and discharged. Since the island voltage varies periodically with the superconducting phase across the SSET, the island voltage fluctuations fol-
Figure 8: From top to bottom: applied charge step at the gate of the SSET
electrometer; electrometer response averaged over 4000 steps and measured
within 10 MHz and 1 MHz bandwidths, respectively. Curves have been
offsetted for clarity.
low the phase fluctuations. Assuming that the box transition frequency is lower than the SSET band gap, the dynamics of the SSET can be calculated using an adiabatic approximation, and we find for the relaxation rate $\Gamma_1^{\mathrm{SSET}}$ due to the measuring SSET:

$$\Gamma_1^{\mathrm{SSET}} \sim \frac{R_s I_c^2}{\hbar \Omega} \kappa_{\mathrm{SSET}}^2 \sin^2 \theta.$$  \hspace{1cm} (16)

For the electrometer we have operated, this rate would be of the order of 1 MHz, assuming again $\kappa_{\mathrm{SSET}} = 2.5\%$. The charge sensitivity of the SSET as an electrometer is limited by the thermal fluctuations of the voltage at the working point[27, 23]. The intrinsic figure of merit of the shunted SSET for the measurement of the box is:

$$f^{\mathrm{SSET}} = \frac{1}{\Gamma_1^{\mathrm{SSET} \tau_m}} \sim \frac{\hbar \Omega}{k_B T} \cot^2 \theta,$$  \hspace{1cm} (17)

where $\tau_m$ is again the minimum time needed to discriminate the two states of the box. Although the factor $\hbar \Omega/k_B T$ can be large, the practical figure of merit of the present electrometer would be smaller than one because the measuring time would be limited by the SQUID array and not by the intrinsic noise of the shunted SSET.

The decoherence results from the low frequency fluctuations of the island voltage. Although the spectrum of the phase fluctuations is known[27], the complicated relation between the phase across the SSET and the voltage in the island does not allow to deduce the island voltage fluctuation spectrum except in some limits. In the non-running state, the fluctuations of the phase are small, and a perturbative calculation of around the average value leads to:

$$\Gamma_2^{\mathrm{SSET}} = \frac{4 k_B T^2}{\hbar} \frac{\kappa_{\mathrm{SSET}}^2}{R_K} \frac{R_s}{r} \left( \frac{E_C}{E_J} \right)^2 g^2 \cos^2 \theta,$$  \hspace{1cm} (18)

where

$$g = \frac{I_c}{\sqrt{I_c^2 - I_{\mathrm{SSET}}^2}} \frac{\partial (C_{\mathrm{SSET} V/e})}{\partial (\arcsin (I_{\mathrm{SSET}}/I_c))}$$  \hspace{1cm} (19)

is an increasing function of the dc current $I_{\mathrm{SSET}} < I_c$ with $g(0) = 0$. For our electrometer biased at $I_{\mathrm{SSET}} = I_c/2$, the decoherence rate of a box coupled to it with $\kappa_{\mathrm{SSET}} = 2.5\%$ would be of the order of a few MHz. This implies that, for a SSET used in the switching mode, the bias current has to be
reduced close to zero during box manipulations. When the ratio $E_c/E_j$ is larger than one, the variations of the island voltage are almost proportional to $\cos(\delta)$, and the spectrum can be calculated numerically along the lines of refs.[27] in all regimes. We find that the spectrum of the island voltage fluctuations has a peak at the Josephson frequency, and that the induced decoherence rate is the largest when the current through the electrometer is close to its maximum value. Quantitatively, the SSET we have operated would result in a decoherence rate $\Gamma_{2}^{\text{SSET}} \approx 0.3 \text{ MHz}$ at the working point used for electrometry.

**alternate high bandwidth set-up using a SQUID array** In the shunted SSET set-up, the bandwidth is ultimately limited by the time constant of the $RL$ circuit at the input of the array. This limitation is not mandatory, and an alternate high bandwidth set-up is possible. For that purpose, a SSET, directly connected in parallel with the input coil of a SQUID, is current biased well below its critical current. Due to the residual resistance in the coil wiring, the dc bias current flows through the SSET only, and the phase difference across the SSET adjusts to accommodate it. When the critical current of the SSET is ac-modulated, its effective inductance is varied accordingly, and the distribution of the bias current between the SSET and the SQUID input coil is modulated. In this set-up, the SSET behaves as a charge to current transducer, and the sensitivity, bandwidth and back-action are entirely determined by the current measuring stage.

### 0.7 Conclusions

We have evaluated in the single Cooper pair box the relaxation and decoherence rates due to electromagnetic dissipation in the box circuit itself and to external charges moving in the neighborhood of the box. The offset charge noise is likely the dominant source of relaxation and of decoherence, which rises an important problem. We have also evaluated the relaxation and decoherence rates due to a measuring electrometer coupled to the box island, and discussed in particular set-ups based on unshunted or shunted SSETs. Experimentally, we have operated an electrometer based on the continuous measurement of a shunted SSET with a SQUID series array. The sensitivity achieved by this system was about $3.10^{-4}e/\sqrt{\text{Hz}}$, limited by the SQUID array noise, within a few MHz bandwidth. Such an electrometer would be
useful in Cooper pair box experiments only if its sensitivity is significantly improved. Other measuring strategies can however be used for that purpose. In particular, the switching of an unshunted SSET is a simple one because it does not require a cold amplifier and is well suited for pulsed operation.

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References


