Calcul de $G^{(2)}(\vec{r},\vec{r}\,')$

$$\hat{\Psi}^+(\vec{r}) = \sum_\alpha \psi_\alpha^*(\vec{r})\,\hat{a}_\alpha^+ \qquad \hat{\Psi}(\vec{r}) = \sum_\beta \psi_\beta(\vec{r})\,\hat{a}_\beta$$

$$\langle \hat{\Psi}^+(\vec{r})\,\hat{\Psi}^+(\vec{r}\,')\,\hat{\Psi}(\vec{r}\,')\,\hat{\Psi}(\vec{r})\rangle =$$

$$\sum_\alpha\sum_\beta\sum_\gamma\sum_\delta \psi_\alpha^*(\vec{r})\psi_\beta^*(\vec{r}\,')\psi_\gamma(\vec{r}\,')\psi_\delta(\vec{r})\,\langle \hat{a}_\alpha^+\hat{a}_\beta^+\hat{a}_\gamma\hat{a}_\delta\rangle$$

En utilisant le théorème de Wick, et de nouveau l'expression de $\hat{\Psi}$ et $\hat{\Psi}^+$ en fonction des $\hat{a}$ et $\hat{a}^+$, on obtient

$$G^{(2)}(\vec{r},\vec{r}\,') = \langle \hat{\Psi}^+(\vec{r})\hat{\Psi}^+(\vec{r}\,')\,\hat{\Psi}(\vec{r}\,')\,\hat{\Psi}(\vec{r})\rangle =$$

$$\langle\hat{\Psi}^+(\vec{r})\hat{\Psi}(\vec{r}\,')\rangle\langle\hat{\Psi}^+(\vec{r}\,')\hat{\Psi}(\vec{r})\rangle + \langle\hat{\Psi}^+(\vec{r})\hat{\Psi}(\vec{r})\rangle\langle\hat{\Psi}^+(\vec{r}\,')\hat{\Psi}(\vec{r}\,')\rangle$$

$$= |G^{(1)}(\vec{r},\vec{r}\,')|^2 + G^{(1)}(\vec{r},\vec{r})\,G^{(1)}(\vec{r}\,',\vec{r}\,')$$

On en déduit

$$g^{(2)}(\vec{r},\vec{r}\,') = \frac{G^{(2)}(\vec{r},\vec{r}\,')}{G^{(1)}(\vec{r},\vec{r})\,G^{(1)}(\vec{r}\,',\vec{r}\,')} =$$

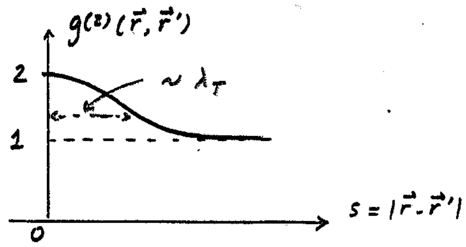
$$= 1 + \frac{|G^{(1)}(\vec{r},\vec{r}\,')|^2}{G^{(1)}(\vec{r},\vec{r})\,G^{(1)}(\vec{r}\,',\vec{r}\,')}$$

c'est à dire encore

$$g^{(2)}(\vec{r},\vec{r}\,') = 1 + |g^{(1)}(\vec{r},\vec{r}\,')|^2$$

$$g^{(2)}(\vec{r},\vec{r}) = 1 + 1 = 2$$

Variations de $g^{(2)}$



Calcul de $G^{(3)}(\vec{r},\vec{r}\,',\vec{r}\,'')$

$$G^{(3)}(\vec{r},\vec{r}\,',\vec{r}\,'') =$$

$$\langle \hat{\Psi}^+(\vec{r})\,\hat{\Psi}^+(\vec{r}\,')\,\hat{\Psi}^+(\vec{r}\,'')\,\hat{\Psi}(\vec{r}\,'')\,\hat{\Psi}(\vec{r}\,')\,\hat{\Psi}(\vec{r})\rangle =$$

$$\begin{aligned}
= \; & G^{(1)}(\vec{r},\vec{r})\,G^{(1)}(\vec{r}\,',\vec{r}\,')\,G^{(1)}(\vec{r}\,'',\vec{r}\,'') \\
+ \; & G^{(1)}(\vec{r},\vec{r})\,G^{(1)}(\vec{r}\,',\vec{r}\,'')\,G^{(1)}(\vec{r}\,'',\vec{r}\,') \\
+ \; & G^{(1)}(\vec{r}\,',\vec{r}\,')\,G^{(1)}(\vec{r},\vec{r}\,'')\,G^{(1)}(\vec{r}\,'',\vec{r}) \\
+ \; & G^{(1)}(\vec{r}\,'',\vec{r}\,'')\,G^{(1)}(\vec{r},\vec{r}\,')\,G^{(1)}(\vec{r}\,',\vec{r}) \\
+ \; & G^{(1)}(\vec{r},\vec{r}\,')\,G^{(1)}(\vec{r}\,',\vec{r}\,'')\,G^{(1)}(\vec{r}\,'',\vec{r}) \\
+ \; & G^{(1)}(\vec{r},\vec{r}\,'')\,G^{(1)}(\vec{r}\,'',\vec{r}\,')\,G^{(1)}(\vec{r}\,',\vec{r})
\end{aligned}$$

$3! = 6$ termes

Expression de $g^{(3)}(\vec{r},\vec{r}\,',\vec{r}\,'')$

$$g^{(3)}(\vec{r},\vec{r}\,',\vec{r}\,'') = \frac{G^{(3)}(\vec{r},\vec{r}\,',\vec{r}\,'')}{G^{(1)}(\vec{r},\vec{r})\,G^{(1)}(\vec{r}\,',\vec{r}\,')\,G^{(1)}(\vec{r}\,'',\vec{r}\,'')}$$

$$= 1 + |g^{(1)}(\vec{r},\vec{r}\,')|^2 + |g^{(1)}(\vec{r}\,',\vec{r}\,'')|^2 + |g^{(1)}(\vec{r}\,'',\vec{r})|^2$$
$$+ 2\,\mathrm{Re}\left[g^{(1)}(\vec{r},\vec{r}\,')\,g^{(1)}(\vec{r}\,',\vec{r}\,'')\,g^{(1)}(\vec{r}\,'',\vec{r})\right]$$

Quand $\vec{r} = \vec{r}\,' = \vec{r}\,''$, tous les $g^{(1)}$ valent 1, et l'on a donc

$$g^{(3)}(\vec{r},\vec{r},\vec{r}) = 6 = 3!$$

Quand $|\vec{r}-\vec{r}\,''| \gg \lambda_T$ et $|\vec{r}\,'-\vec{r}\,''| \gg \lambda_T$, on peut negliger $g^{(1)}(\vec{r},\vec{r}\,'')$ et $g^{(1)}(\vec{r}\,',\vec{r}\,'')$ de sorte que

$$g^{(3)}(\vec{r},\vec{r}\,',\vec{r}\,'') \underset{\vec{r}\,''\to\infty}{\simeq} g^{(2)}(\vec{r},\vec{r}\,') = 1 + |g^{(1)}(\vec{r},\vec{r}\,')|^2$$

Quelques références

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