NEW SCHEMES IN LASER COOLING


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ABSTRACT
We present here several cooling experiments: we first review the usual theory of Doppler cooling and present experimental results on the collimation of an atomic beam (1D cooling) in agreement with this theory. We then show that experimental results for 3D optical molasses with Cesium atoms violate the predictions of the Doppler cooling theory. We propose a possible explanation for these anomalously low temperatures based on an extra cooling force originating from the light polarization gradient. The theoretical limit for this mechanism is the recoil energy. Finally, we present an experiment leading to a cooling below the recoil energy, with a mechanism using coherent population trapping.

Laser cooling consists in producing very low velocity atoms using quasi resonant laser-atom momentum exchanges. During the last few years, a large amount of both theoretical and experimental work has been devoted to this subject [1]. This large interest of the atomic physicist community is due to the variety of applications of laser cooled atoms: high resolution spectroscopy, collision physics, ...

On the other hand, laser cooling remains a very open subject, and new and more efficient cooling schemes can still be found. Some of these new schemes will be presented in the following. We shall first review the usual model of Doppler cooling, initially proposed by Hänisch and Schawlow for free atoms [2]. We will present a 1D experiment which confirms the main theoretical results of Doppler cooling, i.e. a limiting temperature

\[ k_B T > \hbar \Gamma / 2 \]  

where \( \Gamma \) is the natural width of the excited state involved in the laser cooling atomic transition. We will then switch to the 3 dimensional case, where it has recently been demonstrated experimentally, by the NBS Washington group, that the limit (1) could be overcome for 3D laser cooled sodium atoms [3]. We will present a similar experiment that we have performed with Cesium atoms, and which also leads to an anomalously low temperature, violating the limit (1). We will propose a possible explanation based on the multilevel Zeeman structure of the Sodium or Cesium atoms: while these multiple levels could be forgotten in a 1D experiment by a convenient use of optical pumping, they have to be taken account in a 3D experiment and we show that they may be at the origin of the measured very low temperature. In this model, cooling occurs from the light polarization gradient. We will also show that, if this
explanation is valid, the limiting temperature in a 3D cooling is the recoil energy:

$$k_B T > \frac{\hbar^2 k^2}{2m}$$  \hspace{1cm} (2)

which can be much lower than (1). Finally, we will come back to the one dimensional case and we will show that even the limit (2) can be overcome using the so-called “coherent population trapping”. In this last scheme, atoms are optically pumped into a velocity selected non absorbing state, and the final temperature is then only limited by the interaction time [4].

1. ONE DIMENSIONAL DOPPLER COOLING

1.1. Principles and Limits of Doppler Cooling

By Doppler cooling, we refer here to the initial proposal [2] by Hänsch and Schawlow. The idea is to use two laser waves propagating in opposite directions along an Oz axis. The laser frequency is tuned slightly below an atomic transition frequency. Because of the Doppler effect, an atom moving along Oz preferentially absorbs more photons from the counterpropagating wave than from the copropagating one. This creates a net radiation pressure force opposed to the atomic velocity $v_z$, which damps the atomic motion along Oz.

The theory of laser cooling is easily worked out in the low intensity case and for two level atoms. In the low intensity limit, the total force acting on the atom is the sum of the two radiation pressure forces created by the two travelling waves. These two radiation pressure forces are directly obtained from the Lorentzian absorption line shape [5]:

$$f_\pm = \pm \hbar k \vec{u}_z \frac{\Gamma}{4} \frac{\omega_i^2}{(\delta \mp kv_z)^2 + \Gamma^2 / 4}$$  \hspace{1cm} (3)

where $k$ is the laser wave vector, $\vec{u}_z$ is a unit vector along the Oz axis, $\omega_i$ is the Rabi frequency of each laser wave and $\delta = \omega_L - \omega_A$ is the detuning between the laser and the atomic frequencies.

For low velocity atoms:

$$kv \ll \Gamma$$  \hspace{1cm} (4)

the sum of $f_+$ and $f_-$ gives a damping force linear in the atomic velocity:

$$\vec{f} = f_+ + f_- = -\alpha v_z \vec{u}_z$$  \hspace{1cm} (5)

with

$$\alpha = \hbar k^2 \frac{(-\delta) \Gamma \omega_i^2}{(\delta^2 + \Gamma^2 / 4)^2}$$  \hspace{1cm} (6)

We find, as expected, that $\alpha$ is positive (friction force) for negative detunings. One also remarks on eq. (6) that due to the low intensity condition ($\omega_i \ll \Gamma$), $\alpha$ is always much smaller than $\hbar k^2$. In order to get high damping, one could think of increasing the laser power, proportional to $\omega_i^2$. But stimulated emission processes, which have been neglected here, appear for $\omega_i > \Gamma$, and they modify completely the atomic dynamics. We shall not describe here the new phenomena which occur in this high intensity regime, such as cooling with stimulated emission [6].
The steady state temperature of Doppler cooled atoms is found as resulting from the competition between the cooling presented above, and the heating due to the randomness of absorption and emission processes. This heating is described in terms of an atomic momentum diffusion coefficient $D_p$, proportional to the square of each momentum step $\hbar k$ per absorption or emission, and to the rate of these processes [5]:

$$D_p = \frac{\hbar^2 k^2}{2} \frac{\Gamma}{\delta^2 + \Gamma^2/4}$$

(7)

The final temperature is then readily obtained as:

$$k_B T = \frac{D_p}{\alpha} = \frac{\hbar \Gamma}{4} \left( \frac{2 |\delta|}{\Gamma} + \frac{\Gamma}{2 |\delta|} \right)$$

(8)

It is minimal for $\delta = -\Gamma/2$ :

$$k_B T_{\text{min}} = \frac{\hbar \Gamma}{2}$$

(9)

We note that this final temperature is independent of the laser power; it is indeed equal to the ratio between the diffusion and the friction coefficients which both depend linearly on laser power. We also note that the range of velocities that can be Doppler cooled (4) is independent of power, and usually much larger than the final velocity width deduced from (9). Usual atomic resonance lines indeed satisfy the condition:

$$\frac{\hbar^2 k^2}{2m} \ll \hbar \Gamma \Rightarrow \text{final velocity} : \sqrt{\frac{\hbar \Gamma}{2m}} \ll \text{capture range} : \frac{\Gamma}{k}$$

(10)

Note: the coefficient $D_p$ given in (7) represents the momentum diffusion along the $Oz$ axis. The value given here corresponds to the case where all fluorescence photons are emitted along the $Oz$ axis. Actually, taking into account the dipole emission pattern, one can show that $D_p$, given in (7) has to be multiplied by $0.7$ [7] which reduces the achievable temperature for $1D$ cooling:

$$(k_B T)_{1D\text{lim}} = 7\frac{\hbar \Gamma}{20}$$

(9')

On the other hand, $3D$ Doppler cooling of two level atoms leads to eq. (9) because the value (7) for $D_p$ is then correct, since it takes into account the heating of the momentum along $Oz$ due to the presence of laser waves along $Ox$ and $Oy$.

1.2 Transverse Cooling of a Metastable Helium Beam

By irradiating transversally an atomic beam of metastable helium ($^{23}S$ state), we have observed a clear collimation of the beam, corresponding to a cooling of the transverse atomic velocity (Fig. 1.a). The supersonic metastable beam is produced using a colinear electronic excitation of a helium atomic beam. The experiment uses the $|g, J = 1, ^23S > \longrightarrow | e, J = 2, ^23P >$ transition ($\lambda = 1.08\mu m$, $\Gamma^{-1} = 100ns$). A laser standing wave is crossing at right angle this atomic beam. The laser is a home built LNA laser [8] (power $50mW$, linewidth $0.2MHz$). Atomic velocities are damped inside the interaction region and the corresponding collimation is detected $1.4$ meter downstream using an electron multiplier. The two level system is achieved using $\sigma_+$. 
Fig. 1: Transverse cooling of a metastable helium beam

a. A metastable helium beam (2^3 S state) is irradiated at right angle by a red-detuned laser standing wave. The resulting collimation is detected by transverse position profiles with a movable electron multiplier.

b. Evidence for laser induced collimation.

Fig. 2: Experimental set-up for the observation of Cs optical molasses. Laser decelerated Cs atoms are injected into the intersection of 3 orthogonal laser standing waves. The fluorescence of atoms inside the molasses is imaged and detected on a photomultiplier.
polarized light so that the atoms get locked on the \(| J = 1, m = 1 \rightarrow J = 2, m = 2 \rangle \) transition after a few fluorescence cycles.

Experimental results are indicated in Fig. 1.b. We observe a good collimation for a negative detuning, the optimal detuning being between \(-\Gamma/2\) and \(-\Gamma\). The smallest average quadratic transverse velocity deduced from position profiles such as the one of Fig. 1.b is: \(\bar{v}_{exp} = 32 \pm 3\) cm/s. This is slightly larger than the theoretical velocity that one deduces from eq. (9) \((\bar{v} = 23\) cm/s). This discrepancy is due to the finite interaction time \((100\mu s)\) : the equilibrium velocity distribution is not yet reached when the atoms leave the interaction zone. We have done a numerical study of the atomic motion based on the Doppler cooling model, taking into account this finite interaction time; the experimental results appear to be perfectly consistent with the Doppler cooling model.

Let us finally mention the work by de Clercq et al. [9] which has also confirmed the result from eq. (8) by studying the collimation of a Cs atomic beam by a weak laser standing wave.

2. THREE DIMENSIONAL LASER COOLING

Three dimensional laser cooling is achieved by using three mutually orthogonal standing waves, and by loading the interaction volume of these standing waves by atoms slowed by laser deceleration of a thermal atomic beam. The atomic velocity is then strongly damped, and the atoms undergo a random walk inside the three standing waves until they finally leave this region after a fraction of second. These are the so called optical molasses. They have been first observed at Bell Labs in 1985 on sodium atoms, and the temperature of the cold atoms has been found in good agreement with (9) [10]. Similar measurements have also been done in Boulder on Cesium atoms [11]. However, the NBS Washington group recently came to the conclusion that, for sodium atoms, and for detunings larger than \(-\Gamma/2\), experimental temperatures were much lower than the ones predicted by eq. (8) [3]. S. Chu and co-workers have confirmed this result again on sodium atoms [12]. We will present here experimental results, obtained on cesium atoms, which also confirm the anomalously low temperatures for these laser cooled atoms. We will then propose a possible explanation for this result, based on a very efficient damping force originating from light polarization gradient. A similar explanation has been simultaneously proposed by S. Chu at this ELICAP conference [12].

2.1. Temperature Measurements of Laser Cooled Cesium Atoms

The experimental set-up is sketched in Fig. 2. A cesium atomic beam is slowed down by two chirped laser diodes [13-14]. The slow atoms are caught into the molasses region formed by the intersection of three laser standing waves. The cooling laser (ring dye laser coherent 699) is tuned slightly below the \(g, 6s_{1/2} F = 4 \rightarrow e, 6p_{3/2} F = 5 \rangle \) transition \((\lambda = 852\) nm, \(r = 32\) ns). The polarizations of the three standing waves are linear and mutually orthogonal. A repumping laser diode beam, tuned to the \(| g, F = 3 \rightarrow e, F = 3 \rangle \) is mixed to the cooling beam. Note that all laser diodes are free running (no feedback technique as in [11]). The earth magnetic field in the
The detection scheme uses the fluorescence light emitted by the atoms inside the molasses. A typical sequence is shown in Fig. 3 where we have plotted the fluorescence versus time. In the first part of the sequence, the slowing laser diodes are on and fill the molasses. In Fig. 3, the loading consists in 48 chirps of 15 ms duration. The slowing lasers are then blocked by A.O. modulators, and the slow decrease of the fluorescence \(1/e\) time \(0.4\) s corresponds to atoms escaping from the molasses. The longest \(1/e\) time \(t = 0.7\) sec has been obtained with \(\delta = -2\Gamma\) and \(\omega_1 = 0.6\Gamma\) (for the strongest transition \(g, F = m_F = 4 \rightarrow e, F = m_F = 5\)). From the intensity of the fluorescence light, we estimate the atomic density in the molasses to be \(5 \times 10^6\) atoms/cm\(^3\).

To measure the temperature of the cold atoms, we have used the release and recapture method \([10,3]\). This method consists in letting the cold atoms undergo a free fall in the dark by switching off the molasses beam for a given amount of time \(t_{\text{off}}\). When the molasses beams are turned on again, the measure of the fluorescence light gives the fraction of remaining atoms (Fig. 4.a). This fraction, as a function of \(t_{\text{off}}\), is compared to a numerical calculation which is done assuming a gaussian initial velocity distribution. This calculation takes into account the size of the emitting region and the geometry of detection. A least square fit adjustment between experimental and numerical results then leads to a determination of the average quadratic velocity. As an example, Fig. 4.b shows a temperature measurement for \(\delta = -3\Gamma\) and \(\omega_1 = 3.4\Gamma\) for the strongest transition. We find 

\[ v = 7 \text{cm/s} \pm 2 \text{cm/s} \Rightarrow k_B T = 70 \pm 40 \mu K. \]

This is a clear violation of eq. (8) which would give \(T = 380 \mu K\) for this detuning. Finally, various temperature measurements, for different detunings and powers, are presented in Fig. 5. Let us remark that the smallest measured velocities, around \(6 \text{cm/s}\), are in a range where the release and recapture method becomes very imprecise. For such small average quadratic velocities, the departure of the atoms from the molasses region is mainly due to gravity acceleration, and no longer to the initial randomly distributed velocities.

2.2. A Possible Explanation for the Low Temperature 3-D Molasses

2.2.1. Existence of long relaxation times

The model leading to the \(\hbar\Gamma/2\) limit is a two level atom theory. On the opposite, in a 3D experiment, it is not possible to isolate a two level system inside the sodium or cesium level scheme, and one has to deal with all the Zeeman sublevels of the ground state. This considerably modifies the internal dynamics of the atom, and we show in the following that it may lead to lower temperatures.

We start with a very general result giving the order of magnitude of the temperature of laser cooled atoms \([15]\):

\[ k_B T > \hbar / r_{\text{relax}} \]  

(11)
**Fig. 3**: Time evolution of the fluorescence of atoms inside the molasses, when the loading lasers are blocked.

**Fig. 4**: Measurement of the average quadratic velocity of atoms inside the molasses.

a. Typical fluorescence signal in a Release & Recapture sequence.

b. Fit of the fraction of atoms remaining after a given time-off of the molasses laser, with two theoretical results obtained with average quadratic velocities of 7 cm/s and 12 cm/s.

**Fig. 5**: Average quadratic velocity measurements for various powers and detunings.
moves, the internal state does not follow adiabatically the external motion, but there is a time lag which is precisely of the order of $\tau_{\text{relax}}$. This time lag introduces an hysteresis in the atomic motion, which is at the origin of the friction force damping this motion. The longer $\tau_{\text{relax}}$, the more important the friction, and the lower the temperature as indicated in (11).

For a two level atom, we always have $\tau_{\text{relax}} \sim \Gamma^{-1}$. This time does not depend on the laser power, and consequently the final temperature is also independent of the power, at least in the low intensity domain (eq. (9)). On the opposite, for an atom having several ground Zeeman sublevels, relaxation times much longer than $\Gamma^{-1}$ can appear as a result of optical pumping of the atom from one sublevel to another one. These long pumping times, inversely proportional to the laser power, may then lead to very low temperatures proportional to the laser power.

After this very general argument, we now come to the specific case of 3D optical molasses, in which the light polarization gradient, unavoidable due to the 3D nature of the experiment, may be at the origin of low temperatures via such long pumping times. We will first investigate the stationary state of a Cs atom at rest and irradiated by laser light. We will then look for the modification of this state if the atom moves, and we will show that a strong damping force originates from this motion, leading to steady state temperatures smaller than $\hbar \Gamma/2$.

2.2.2. The stationary state of a Cs atom in laser light

We consider a Cs atom at rest irradiated by low intensity laser light resonant or quasi resonant with the $| g, F = 4 \rightarrow e, F = 5 >$ transition. We neglect in the following any other line of the cesium atom. We treat the two cases of circularly and linearly polarized light, and we are concerned with the two following problems.

(i) What are the energy positions of the various atomic ground sublevels, taking into account the level shifts induced by the light irradiation?

(ii) In steady state, what are the most populated ground sublevels due to ”absorption spontaneous emission” processes?

We consider first the case of $\sigma^+$ light (Fig. 6.a). The most light shifted ground sublevel is $| g, F = m_F = 4 >$ since the intensity factor (square of the Clebsh-Gordan coefficient) of the corresponding transition is equal to 1. On the other hand, it is also the only populated level in steady state due to optical pumping.

In the case of linearly polarized light, we choose an axis of quantization parallel to the light polarization ($\pi$ light, Fig. 6.b). From the intensity factors of Fig. 6.b, it is clear that the most light shifted ground sublevel is $| F = 4, m_F = 0 >$, followed by $| F = 4, m_F = \pm 1 >$, and so on. The steady state populations are derived from a rate equation approach, which is valid at low intensity. Equilibrium in the transfers between $| F = 4, m_F = 4 >$ and $| F = 4, m_F = 3 >$ gives for example

$$9 \times 36 \times \text{pop}(m_F = 4) = 1 \times 16 \times \text{pop}(m_F = 3)$$

Solving the whole set of equation, one finds that the most populated level is $| F = 4, m_F = 0 >$, followed by $| F = 4, m_F = \pm 1 >$. 85% of the population is concentrated in these 3 sublevels.

For both circular and linear polarizations, we then reach the conclusion, that we assume to be general, that a cesium atom irradiated on the $| g, F = 4 \rightarrow e, F = 5 >$ transition is pumped by low intensity laser light into the most light shifted ground
Fig. 6: Steady state populations for a Cs atom irradiated by low intensity laser light $\sigma^+$ polarized (a) or $\pi$ polarized (b). The coefficients indicated near each transition are the intensity factors (squares of the Clebsh–Gordan coefficients) multiplied by 45.

Fig. 7: Cooling via motion induced ("non adiabatic") transitions in a light polarization gradient.

a. An atom at rest is pumped in the most light shifted ground level. For red detuned laser light, this is the lowest energy level.

b. Non adiabatic transition towards a less light shifted ground level, which converts atomic kinetic energy into atomic potential energy: the atomic velocity is reduced.

c. Relaxation by optical pumping towards the most light shifted level. The atomic potential energy is converted into fluorescence photons energy.

d–e. Same as b–c with a non adiabatic transition towards an excited level.
state Zeeman sublevel(s). A similar conclusion is reached for sodium atoms irradiated on the \(| g, F = 2 \rightarrow e, F = 3 \rangle \) transition, and is probably a particular case of a much more general property. It will be the starting point of the cooling mechanism explained below.

2.2.3 Laser cooling in a polarization gradient

In the three standing waves forming the optical molasses, the polarization of the light rotates from point to point on the optical wavelength scale. At each point, an atom at rest is pumped in the most light shifted level(s), which, since the light is detuned red from the transition, is the lowest energy state: all other ground sublevels with the same number of laser photons, or excited sublevels with one laser photon less, have an energy larger than the energy of the occupied state (Fig. 7.a).

Due to the polarization gradient, the wave function of the most light shifted level varies from point to point, again on the optical wavelength scale. We take now a moving atom. If the atomic velocity is not infinitely slow, the atom will not follow adiabatically this most light shifted level. Some "non adiabatic transitions" will occur from the lowest energy state to other states (Fig. 7.b). Such a transition increases the atomic potential energy while decreasing the atomic kinetic energy. After this non adiabatic transition, the atom comes back to the most light shifted (lowest energy) level by optical pumping : atomic potential energy is then converted into field energy (Fig. 7.c). Globally, such a cycle "non adiabatic transition-optical pumping relaxation" leads to a transfer of atomic kinetic energy into field energy.

This mechanism therefore gives rise to a dissipation process which does not exist for a two-level system. Since analytical calculations are difficult for sodium or cesium atoms where many levels play a role, we will present here a very simplified model whose purpose is to give an order of magnitude of the temperature achievable in such a system.

2.2.4 A simplified model of "multi-level atom cooling"

We consider here an atomic system with two ground states. This atom is irradiated by laser light with a polarization gradient, and we assume that, at any point \( \vec{r} \), an atom at rest is pumped in a state \(| g_1(\vec{r}) \rangle \), the other state \(| g_2(\vec{r}) \rangle \) having a zero steady state population. Due to the polarization gradient, \( g_1 \) and \( g_2 \) rotate on the optical wavelength scale. In the following, we will not take into account explicitly any excited state, and we will treat the effect of the laser irradiation in a phenomenological way:

(i) relaxation to the steady state \( \rho_{11} = 1, \rho_{22} = 0 \) with a time constant \( \tau_r \).

(ii) different light shifts \( E_1 \) and \( E_2 \) for the two levels \( g_1 \) and \( g_2 \), \( g_1 \) being the lowest energy state \( (E_1 < E_2) \).

The phenomenological atom-laser hamiltonian can thus be written:

\[
H = E_1 | g_1 \rangle \langle g_1 | + E_2 | g_2 \rangle \langle g_2 |
\]

and the evolution of the density matrix of an atom at rest is:

\[
\dot{\rho}_{11} = \rho_{22} / \tau_r \\
\dot{\rho}_{22} = -\rho_{22} / \tau_r \\
\dot{\rho}_{12} = (i\omega_{21} - 1/2\tau_r)\rho_{12} \\
\dot{\rho}_{21} = -(i\omega_{21} + 1/2\tau_r)\rho_{21}
\]

(13)
with $\hbar \omega_{21} = E_2 - E_1 > 0$. The force operator along a given $z$ direction is obtained from the spatial gradient of the Hamiltonian:

$$F_z = - \sum_i \left\{ \left( \frac{d}{dz} E_i \right) | g_i > < g_i | + E_i \left( \frac{d}{dz} | g_i > \right) < g_i | g_i > \left( \frac{d}{dz} < g_i | \right) \right\}$$  \hspace{1cm} (14)

From the orthonormality of the basis $g_1, g_2$, we can always take

$$\begin{align*}
\frac{d}{dz} | g_1 > &= \mu | g_2 > \\
\frac{d}{dz} | g_2 > &= - \mu^* | g_1 >
\end{align*}$$  \hspace{1cm} (15)

where $\mu$ is a complex number of modulus of the order of $k = 2\pi/\lambda$. The average of $F_z$, which gives the average force acting on the atom, is then

$$f_z = \sum_i - \left( \frac{d E_i}{dz} \right) \rho_{ii} + (E_2 - E_1)(\mu \rho_{12} + \mu^* \rho_{21})$$  \hspace{1cm} (16)

We now calculate the values of the $\rho_{ij}$'s coefficients for a moving atom. For $\rho_{11}$, we get for example:

$$(\rho_{11})_{\text{moving}} = < g_1 | \rho | g_1 > = < g_1 | \dot{\rho} | g_1 > + < \dot{g}_1 | \rho | g_1 > + < g_1 | \rho | \dot{g}_1 >$$

$$= \rho_{22}/\tau_r + v_z (\mu^* \rho_{21} + \mu \rho_{12})$$  \hspace{1cm} (17)

Similar equations can be derived for $\rho_{22}, \rho_{12}$ and $\rho_{21}$. The system giving the steady state is readily solved to first order in the velocity:

$$\begin{align*}
\rho_{11} &= 1 + O(v^2) \\
\rho_{12} &= v_z \mu^* (i \omega_{21} - 1/2\tau_r) \\
\rho_{22} &= 0 + O(v^2) \\
\rho_{21} &= -v_z \mu (i \omega_{21} + 1/2\tau_r)
\end{align*}$$  \hspace{1cm} (18)

This steady state solution has a clear interpretation: it corresponds to the density matrix of the two level system $g_1, g_2$, with a weak motion-induced coupling $\mu^* v_z$ between the stable state $g_1$ and the unstable state $g_2$.

From eq. (5), we get the friction coefficient $\alpha$:

$$\alpha = - \frac{df_z}{dv_z} = \hbar |\mu|^2 \frac{\omega_{21}/\tau_r}{\omega_{21}^2 + 1/4\tau_r^2}$$  \hspace{1cm} (19)

This coefficient is positive provided $\omega_{21} > 0$, which means that the atom at rest has to be optically pumped into the lowest energy state (negative detuning). Such an expression of $\alpha$ can also be found starting from the non adiabatic transition probability per unit time between the stable state $g_1$ and the unstable state $g_2$ (Bethe formula):

$$\gamma_{1 \rightarrow 2}^{n.a.} = |\mu v_z|^2 \frac{1/\tau_r}{\omega_{21}^2 + 1/\tau_r^2}$$

Multiplying this rate by the potential energy gain $\hbar \omega_{21}$ for such a transition and equating the result to the rate of loss of kinetic energy $-m v_z \frac{dv_z}{dt}$, one gets $m \frac{dv_z}{dt} = -\alpha v_z$ with $\alpha$ given by (19).
In order to comment this expression, we need to come back to the physical problem. The coefficient $\mu$ is related to the gradient of the states $g_1$ and $g_2$. Its order of magnitude is $k = 2\pi/\lambda$. The parameters $\omega_{21}$ and $\tau_r$ represent respectively light shifts and pumping times for an atom irradiated by low intensity laser light. For $|\delta| \gg \Gamma$, they can be estimated by:

$$\begin{cases} 
\omega_{21} & \approx \omega_1^2/|\delta| \\
1/\tau_r & \approx \Gamma\omega_1^2/\delta^2 
\end{cases}$$

We finally get a friction coefficient of the order of

$$\alpha \approx \hbar k^2 \Gamma/\delta$$

The remarkable point about this result is that $\alpha$ is independent of the laser power. This has to be contrasted with the Doppler cooling result (eq. (6)) where $\alpha$ was proportional to the laser power. This means that, in the low power regime that we are investigating, the friction due to the polarization gradient is much stronger than the friction due to the Doppler effect. The counterpart of this advantage lies in the smallness of the range on which the force is linear in velocity. Eq. (18) is indeed valid only for

$$\mu v_x \ll 1/\tau_r$$

which can also be written, using (20) and $\mu \approx k$:

$$kv_x \ll \Gamma\omega_1^2/\delta^2$$

For low laser power, this range is much smaller than the range found for Doppler cooling (eq. (4)). These results can be summarized as:

**Doppler cooling**

- friction $\propto$ power
- range independent of power

**Polarization gradient cooling**

- friction independent of power
- range $\propto$ power

Of course, in a real experiment, one takes advantage of both coolings: atoms with velocity up to $\Gamma/k$ are first Doppler cooled, and then reach lower temperatures by polarization gradient cooling.

It remains now to calculate the momentum diffusion coefficient and to deduce then the final equilibrium temperature. The diffusion coefficient $D_p$ can be estimated as the sum of the diffusion coefficient for a two level system, $D_{TLS}$ given in eq. (7), and of an extra diffusion coefficient $D_{pol}$ describing the fluctuations of the polarization gradient force $F_z$. This extra coefficient can be calculated from [15]:

$$D_{pol} = \int_0^\infty dt \left\{ -\frac{1}{2} (F_z(\tau)F_z(0) + F_z(0)F_z(\tau)) > -f_z^2 \right\}$$
where $F_z(r)$ represents the operator $F_z$ taken in Heisenberg representation at time $r$. We get after some algebra:

$$D_{pot} = \hbar^2 \left| \mu \right|^2 \frac{1}{2r} \frac{\omega_{21}^2}{\omega_{21}^2 + 1/4r^2} \tag{25}$$

Using now the estimates (20), we find that $D_{pot}$ is of the order of $D_{TLS}$ (eq. (7)), so that the total diffusion coefficient is, for $|\delta| > \Gamma$:

$$D_p = D_{TLS} + D_{pot} \simeq \hbar^2 \hbar^2 \Gamma \omega_1^2 / \delta^2 \tag{26}$$

We finally get the temperature

$$k_B T = D_p / \alpha \simeq \hbar \omega_1^2 / \delta \tag{27}$$

This very simple result has to be compared with the one obtained for a two level system (eq. (8)). It may lead to much lower temperature and it is consistent with two experimental observations:

(i) at a given power ($\omega_1$ fixed), the temperature decreases when the detuning increases.

(ii) at a given detuning, reducing the power decreases the temperature.

One may ask about the lowest achievable temperatures in this model. Eq. (7) suggests that infinitely low temperatures could be reached, by reducing the laser power or increasing the detuning. Actually, this is not possible. One must indeed check that the validity condition for a linear expansion of the force (eq. (23)) is satisfied at least for atoms having reached the steady state temperature (27). This implies

$$\hbar v_z \ll \Gamma \omega_1^2 / \delta^2 \quad \text{with} \quad v_z = \sqrt{k_B T / m} = \sqrt{\hbar \omega_1^2 / m \delta} \tag{28}$$

which can be written:

$$v_z \gg \frac{\hbar k |\delta|}{m} \frac{1}{\Gamma} \tag{29}$$

Consequently, the lowest achievable temperature in this model remains larger than the recoil energy. We can notice that it would be anyway meaningless to look for velocities smaller than $\hbar k / m$ within this model. This is indeed a semi classical treatment, whose validity is restricted to atomic de Broglie wavelengths smaller than the optical wavelength. This in turns imposes a velocity uncertainty larger than $\hbar k / m$ because of Heisenberg inequality.

To conclude this section, let us recall that the results given by the model described above should just be considered as orders of magnitude for the force or the temperature. Several crude approximations have been made as for example neglecting the contribution of excited states to the non adiabatic transition processes. We are presently working on a numerical solution of a $W$ system (two ground states and three excited states) in a $\sigma_+ - \sigma_-$ configuration (two counterpropagating plane waves with respectively $\sigma_+$ and $\sigma_-$ polarizations). Preliminary results seem to confirm the variations found here for the force with respect to detuning and laser power.
3. COOLING BELOW THE RECOIL LIMIT

We now come to the last part of this paper, which concerns the recent observation of 1D laser cooling below the recoil limit [4]. In order to reach temperature lower than the Doppler limit \( k_B T = \hbar \Gamma /2 \), mechanisms based on Raman two photon processes in a three-level atom have been proposed [16–18], but for free or weakly bound atoms, the predicted temperatures remain higher than the recoil limit. It has also been suggested that optical pumping in translation space might be used to cool the translational degrees of freedom below the recoil limit [19]. Here, we present a cooling scheme which is based on velocity selective optical pumping, using coherent population trapping.

Coherent population trapping [20] occurs in a 3-level \( \Lambda \)-shaped system, irradiated by two coherent light waves (Fig. 8.a). If the Raman resonance condition is satisfied, the atoms may be pumped into the non absorbing coherent superposition:

\[
|\psi_{NC} > = (| g_+ > - | g_- >)/\sqrt{2}
\]  

(30)

where they remain trapped. For two counterpropagating laser waves, because of Doppler effect, the Raman resonance condition is fulfilled only for an atom at rest, so that coherent population trapping is now velocity selective.

A rigorous analysis requires the introduction of translational quantum numbers. For example, \( |e,p> \) represents an atom in the \( e \) state with a momentum \( p \) along the laser beam (Oz). By interaction with both lasers, this state is coupled only to \( |g_-,p-\hbar k> \) and \( |g_+,p+\hbar k> \). The state generalizing (30)

\[
|\psi_{NC}(p) > = (| g_-,p-\hbar k > - | g_+,p+\hbar k >)/\sqrt{2}
\]  

(31)

is not coupled to \( |e,p> \). However, for \( p \neq 0 \), it is not a perfect trapping state because it is not stationary (the energies of \( |g_+,p+\hbar k> \) and \( |g_-,p-\hbar k> \) differ by \( 2\hbar kp/M \)) so that an atom optically pumped into \( |\psi_{NC}(p)> \) will eventually go into absorption-fluorescence cycles that will modify its momentum.

The only perfectly trapping state is therefore

\[
|\psi_{NC}(0)> = (| g_-,\hbar k > - | g_+,\hbar k >)/\sqrt{2}
\]  

(32)

Note that this state is a double momentum state: a measurement of atomic momentum (along \( z \)) will give either \( +\hbar k \) or \( -\hbar k \) as a result. Suppose now that we have an atom in a state different from \( |\psi_{NC}(0)> \). The succession of “absorption spontaneous emission” processes then causes a random walk in momentum space. Eventually, this random walk stops if the atom falls into the state \( |\psi_{NC}(0)> \) given in (32). This therefore provides a mechanism for pumping and accumulating atoms into the non-absorbing superposition of states \( |\psi_{NC}(0)> \) (or \( |\psi_{NC}(p)> \) with \( p \) very small). This qualitative treatment, has been confirmed by a numerical solution of generalized optical Bloch equation [4] (quantum treatment of both internal and external atomic degrees of freedom). It predicts that, for sufficiently long interaction times, the final atomic momentum distribution along Oz exhibits two peaks emerging at \( \pm \hbar k \) above the initial distribution.

This cooling process has been demonstrated with an experimental setup similar to the one described in § 1.2. The \( \sigma_+ - \sigma_- \) configuration is obtained using two
quarterwave plates and a retroreflection. Figure 8.b shows the transverse velocity profiles with and without laser. The two peaks at about \( \pm \frac{\hbar k}{M} \) (\( \pm 9, 2, \text{cm/s}^{-1} \)) clearly appear well above the initial distribution. A measurement of the standard half width at \( \exp(-1/2) \) gives \( 6 \text{cm/s} \), which corresponds to a temperature about \( 2 \mu \text{K} \). This experimental curve is in reasonable agreement with the theoretical prediction.

We have thus demonstrated that this velocity selective optical pumping into a non-absorbing state is a very efficient process to accumulate atoms in an extremely narrow velocity class. By increasing the coherent interaction time, still narrower velocity distributions could be produced, allowing one to reach temperatures in the nanoKelvin range. Several developments of this work can be considered: extensions to other level schemes; direct observation of the coherence between the two components of \( |\psi_{NC}(0)\rangle \) propagating along different directions; generalization to more than one dimension.

Finally, let us emphasize that this cooling mechanism is quite different from the previously demonstrated ones, since it is not due to a friction force but to diffusion into the cooled velocity class. Another important feature is that the cooled atoms no longer interact with the laser field which then causes no perturbation, neither on external degrees of freedom (no diffusion) nor on internal degrees of freedom (no light shifts). This particularity may be essential for future applications.

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