Is Field Quantization Essential for Discussing Atoms in Laser Beams?

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1- Introduction

Laser-atom interactions are usually described by "optical Bloch equations" very similar to the ordinary Bloch equations used in nuclear magnetic resonance.

In these equations, we have driving terms describing the Rabi precession of the atomic system induced by a c-number incident laser field and phenomenological damping terms associated with various processes such as spontaneous emission or collisions. But we don't see any photon or field operator. Is field quantization essential for the physical problems described by such equations? This is the question we would like to discuss in this lecture.

2- Incident Field and Vacuum Field

We first review a few simple results concerning the radiation field and its expansion in "normal modes" of vibration [1].

Each mode of the free radiation field is dynamically equivalent to a fictitious one-dimensional harmonic oscillator. Field quantization is achieved by quantizing each of these oscillators. The energy levels of each mode are therefore labelled by an integer quantum number n, and have an energy which can be analyzed in terms of elementary excitations $\hbar \omega$, which are nothing but photons associated with this mode.

In the ground state of a quantum oscillator, we have $\langle X \rangle = \langle P \rangle = 0$, but $\langle X^2 \rangle \neq 0$ and $\langle P^2 \rangle \neq 0$ (where X and P are the position and momentum operators). This pure quantum result is a consequence of the commutation relation $[X, P] = i \hbar$, which prevents a simultaneous cancellation of the kinetic ($P^2$) and potential ($X^2$) energies [2]. A similar result holds for the ground state of the quantized radiation field (all modes i being in their ground states $|0_i\rangle$). We have $\langle E \rangle = \langle B \rangle = 0$, but $\langle E^2 \rangle \neq 0$ and $\langle B^2 \rangle \neq 0$ (where E and B are the noncommuting electric and magnetic field operators). These so-called "vacuum fluctuations" have a spectral power density equal to $\hbar \omega / 2$ per mode $\omega$, and, consequently, a very short correlation time.

In laser-atom experiments, the atom interacts, not only with the laser mode, which is excited and contains photons, but with all other modes i, which are initially in their ground state $|0_i\rangle$:

\[ \text{Laser mode } \quad \longleftrightarrow \quad \text{Atom } \quad \longleftrightarrow \quad \text{All other modes } \quad \text{i in } |0_i\rangle \]

As a consequence of these interactions, photons are transferred from the laser mode to the initially empty modes, i.e. incident laser photons are scattered by the atom in all directions.
When the laser is in a coherent state, it has been shown by Mollow that the problem can be, after a unitary transformation, formulated in a different but completely equivalent way [3]:

\[
\begin{array}{c}
\text{c-number} \\
\text{Atom} \quad \text{Laser field}
\end{array} \quad \text{Quantum vacuum field}
\]

(All modes \( i \) in \( |0_i\rangle \))

The atom interacts now with a c-number time dependent laser field (corresponding to the coherent state of the laser mode) and with the quantum vacuum field. A quantum description of the incident laser field is therefore not essential (although it may be useful and can give some physical insight, as in the dressed-atom approach to resonance fluorescence [4]), and the question asked in the introduction should be reformulated: Is the quantum nature of the vacuum field essential for the atomic evolution?

3- Vacuum Field - Atom Coupling. Atomic Langevin Equation

The vacuum field appears as a "large reservoir" introducing damping and fluctuations in the atomic evolution. If one starts from the coupled Heisenberg equations for atomic and field operators, one can derive an atomic equation of motion very similar to the Langevin equation in the theory of brownian motion [5,6,7].

Three types of forces appear in this Langevin equation, the driving force due to the laser field, a vacuum "friction force", introducing damping and shift in the atomic evolution, and a vacuum "Langevin force", introducing fluctuations. The important point is that one cannot have the friction force without the Langevin force. Fluctuations are always associated with dissipation.

Optical Bloch equations are obtained by taking the average of the Langevin equation in the vacuum state of the field. The Langevin force has a zero average value and disappears. It therefore appears that the Langevin equation has a richer physical content since it deals with operators and fluctuations rather than with average values.

4- Quantum Nature of the Langevin and Friction Forces

The Langevin force is of first order in the coupling constant (electric charge \( e \)), and is proportional to the quantum vacuum field. Even if it has a zero vacuum average value, the Langevin force is essential. Without such a force, atomic commutation relations would not be conserved in the time evolution [8,9] (atoms would collapse!), and one would get wrong results for atomic correlation functions, in contradiction with experiment [10]. Even if it does not appear explicitly in optical Bloch equations, the Langevin force is essential in the derivation of the quantum regression theorem [11] allowing to compute atomic correlation functions from optical Bloch equations.

The quantum nature of the friction force (introducing damping and shift) is less obvious. Such a force, which is of second order in \( e \), has a nonzero vacuum average value. Two extreme physical points of view are usually taken for interpreting the friction force. In the first one, the vibration of the electric charge induced by vacuum fluctuations is considered as the basic physical mechanism. In the second point of view, one instead invokes the interaction of the atomic dipole moment with its self-field (classical concept of radiation reaction). It seems now generally admitted [12,13] that these two points of view can be mixed in arbitrary proportions. The splitting of the total friction force into a vacuum-fluctuations part and a radiation-reaction part seems to depend on the order which is chosen between two commuting
atomic and field operators appearing in the initial atomic Heisenberg equa-
tion.

Actually, we have recently removed such an indetermination by physical ar-

guments [14]. By requiring the vacuum-fluctuations and radiation-reaction
forces to be separately hermitian (which is a necessary condition if we want
them to have a physical meaning), we have selected one particular order among
several mathematically equivalent ones (the completely symmetrical order),
and we have obtained a well-defined separation between the effects of vacuum
fluctuations and those of radiation reaction. We find, for example, that all
radiation reaction effects are independent of $\hbar$ and strictly identical to those
derived from classical radiation theory. They introduce a correction to the
kinetic energy associated with the electromagnetic inertia of the electron.
They produce a rate of emission proportional to the square of the acceler-
ation of the radiating charge. On the other hand, all vacuum fluctuation effects,
which are proportional to $\hbar$, can be interpreted by considering the vibration
of the electron, induced by a random field having a spectral power density
equal to $\hbar\omega/2$ per mode. In particular, they introduce a correction to the po-
tential energy due to the averaging of the Coulomb potential seen by an elec-
tron vibrating in vacuum fluctuations (WELTON's picture [15]). They also sta-
bilize the ground state by introducing a rate of energy gain which compensates
the rate of energy loss due to radiation reaction [16]. On the other hand,
the two spontaneous emission rates are equal for an atomic excited state. The
spin anomaly can also be simply interpreted as being due to radiation reaction
which slows down the cyclotron motion of the electric charge in a uniform ma-
gnetic field, more efficiently than the Larmor precession of the spin [17]
(in the nonrelativistic domain, electric effects predominate over magnetic
ones). A complete relativistic calculation confirms the validity of this in-
terpretation [18].

5- Quantum Effects Observable in Laser Experiments

The laser field itself does not need a quantum description. But it allows one to
bring atoms in nonequilibrium situations where the interaction with the va-
cuum field can give rise to observable quantum effects. We review now a few
examples of such situations.

5.1- Non Classical Fluorescence Light

It has been recently observed [19] that the photoelectrons detected in the
fluorescence light emitted by a single 2-level atom irradiated by a resonant
laser beam are "antibunched". If $P(T)$ is the distribution of time intervals
$T$ between two successive photodetections, one observes that $P(T)$ is an in-
creasing function of $T$ around $T = 0$. This is a pure quantum effect, since
$P(T)$ would always be a decreasing function of $T$ for a classical fluctuating
field [HANBURY-BROWN and TWISS effect]. The quantum interpretation of the
antibunching is straightforward [20]. The first spontaneous emission process
projects the atom into the ground state, and the atom must be reexcited by the
laser before being able to emit a second photon.

Because of the strong correlations which exist between two successi-
vely emitted photons, the photons can be emitted more regularly than in a
sequence of random events. The variance $(\Delta N)^2$ of the number of photons emit-
ted during a time $T$ can be, in certain conditions, smaller than the average
value $\bar{N}$ [21]. A classical field would always give $(\Delta N)^2 \geq \bar{N}$. Subpoissonian
photon statistics in resonance fluorescence have been recently observed [22].

Finally, we can mention quantum effects which could be observed in photon-
counting experiments performed on frequency-filtered fluorescence photons. The
fluorescence spectrum emitted by a strongly driven 2-level atom is the well known Mollow's triplet [23], consisting of a central component (c), at the laser frequency, with two high (h) and low (l) frequency sidebands. Suppose that, with frequency filters, one detects only the photons emitted in the l or h sidebands. It can be shown [24] that photons l and h are emitted in alternance: l h l h l h ... In other words, if N_l and N_h are the numbers of l and h photons emitted during a time Tt, one predicts that \( \Delta(N_l - N_h)^2 = 0, +1 \), whereas a classical field would give \( \Delta(N_l - N_h)^2 \approx N_l + N_h \). Such an experiment has not yet been done, although time correlations between l and h photons have been observed [25].

5.2 - Fluctuations of Radiative Forces [26]

Consider an atom in a travelling resonant laser wave. Let \( \Delta N \) be the number of fluorescence cycles (absorption-spontaneous emission) occurring during a time \( \Delta T \). During the absorption process, the atom gets the momentum \( \hbar k \) of the absorbed photon. Since spontaneous emission can occur with equal probabilities in two opposite directions, the momentum taken away by the fluorescence photon is zero on the average. It follows that the atom experiences a mean "radiation pressure force" given by \( \hbar k \langle \Delta N \rangle / \Delta T \), which has useful applications, for example for radiative cooling [27]. Actually, the previous argument gives only the mean force. Spontaneous emission introduces two types of fluctuations. First, \( \Delta N \) fluctuates around its average value \( \langle \Delta N \rangle \). Secondly, the fluorescence photons are emitted in random directions, so that the recoil due to spontaneous emission fluctuates around zero. These quantum fluctuations are responsible for a diffusion of the atomic momentum, both in the direction of the laser beam and in the transverse direction, and introduce a quantum limitation to radiative cooling.

In a laser beam, the atom also experiences radiative dipole forces, proportional to the gradient of the light intensity, and which can be interpreted in the following way. For a 2-level atom in an inhomogeneous laser wave, there are two types of dressed states 1 and 2, with opposite energy gradients \( V_{E_1} = -V_{E_2} \). The mean dipole force is the average of the two forces \( -V_{E_1} \) and \( -V_{E_2} \) weighted by the probabilities of occupation \( \pi_1 \) and \( \pi_2 \) of states 1 and 2. Spontaneous emission introduces random jumps between the two types of states, changing in a random way the sign of the force. The corresponding quantum fluctuations of the dipole force introduce a diffusion of atomic momentum which limits the stability of optical traps for neutral atoms [26].

5.3 - Fluctuations in Superfluorescence

Consider an ensemble of 2N 2-level atoms, all prepared at time \( t = 0 \) in the upper state by a laser pulse. In the DICKE's model of superradiance, the subsequent evolution of the system is analogous to the spontaneous emission of a large angular momentum \( J = N \) starting from the state \( |J = N, M = N> \). One can also describe the process in terms of a pendulum starting from its metastable (upwards) equilibrium position. Without fluctuations, the pendulum would remain indefinitely in this position. Actually, the quantum fluctuations of the atomic dipole moments, and those of the quantum vacuum field play an essential role in the initial phase of the process by removing the pendulum from its metastable position. They introduce a small "tipping angle". The large fluctuations which are observed in the delay of the superfluorescence pulse are essentially due to this quantum initial phase [28]. For multilevel atoms, fluctuations also appear on the polarization of the pulse [29].
In conclusion, field quantization is essential, not for laser-atom interactions, but for vacuum field-atom interactions. Laser-atom interactions are important for achieving situations where the interaction with the quantum vacuum field leads to observable effects. They provide a great stimulation for new physical insights (interpretation of radiative corrections and spontaneous emission rates), and new investigations (for example, reduction of the shot noise by the use of "squeezed states" [30]).

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   Kimble H.J. and Mandel L.: reference 10
   Dalibard J. and Reynaud S.: in Les Houches 1982, same reference as 18