Spin-injection Spectroscopy of a Spin-orbit coupled Fermi Gas

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Why spin-orbit coupling?

A little bit of History


- In 1980: Quantum Hall state. First topological state characterized by a topological invariant [von Klitzing et al. 1980]

- 2008-2010: a new topological class predicted and discovered, where time-reversal symmetry is preserved. Spin-Orbit coupling plays a crucial role [M.Z. Hasan and C.L. Kane, RMP, 2010]

Also: Modified interactions, unconventional pairing, Majorana fermions

Cold atoms: (very often) constitute optimal system thanks to purity and control
Spin-orbit Hamiltonian

- Electron moving in an electric field creates a momentum-dependent magnetic fields in the moving frame

- In 2D semiconductor electric field can arises from structure

\[ \mathcal{H} = \frac{\hbar^2 k^2}{2m} \mathbb{I} - \mu \cdot [\mathbf{B} + \mathbf{B}_{SO}(k)] \]

\[ -\mu \cdot \mathbf{B}_{SO}(k) \propto \begin{cases} 
\sigma_x k_y - \sigma_y k_x & \text{Rashba} \\
-\sigma_x k_y - \sigma_y k_x & \text{Dresselhaus}
\end{cases} \]

Provides a good description of 2D SOC in solids
How does the Hamiltonian look like?

Reminder

2-level system + electric field

\[ \vec{E} = E_0 \bar{e} \cos(\omega t + \phi) \]

RWA approx.:

\[ -\vec{d} \cdot \vec{E} = \frac{\hbar \Omega}{2} \left( \sigma_x \cos \phi - \sigma_y \sin \phi \right) \]

By adiabatic elimination of the excited state, the Raman process can be described as the interaction of a 2-level system with a field

\[ E_0 \bar{e} \cos(\Delta \omega t + Qx) \]

\[ -\vec{d} \cdot \vec{E} = \frac{\hbar \Omega_R}{2} \left( \sigma_x \cos Qx - \sigma_y \sin Qx \right) \]
Engineering SO coupling

\[ \mathcal{H} = \frac{\hbar^2 k^2}{2m} \mathbb{I} + \frac{\hbar \Omega_R}{2} (\sigma_x \cos Qx - \sigma_y \sin Qx) + \frac{\delta}{2} \sigma_z \]

Local pseudo-spin rotation of angle \( Qx \) around the z-axis

\[ \mathcal{H} = \frac{\hbar^2 k^2}{2m} \mathbb{I} + \frac{\hbar^2 Q}{2m} \sigma_z k_x + \frac{\hbar \Omega_R}{2} \sigma_x + \frac{\delta}{2} \sigma_z + \frac{E_R}{4} \mathbb{I} \]

Global rotation \( \sigma_z \rightarrow \sigma_y, \sigma_y \rightarrow \sigma_x, \sigma_x \rightarrow \sigma_z \)

\[ \mathcal{H} = \frac{\hbar^2 k^2}{2m} \mathbb{I} + \frac{\hbar^2 Q}{2m} \sigma_y k_x + \frac{\hbar \Omega_R}{2} \sigma_z + \frac{\delta}{2} \sigma_y + \frac{E_R}{4} \mathbb{I} \]

\[ -\mu \cdot \mathbf{B}_{SO}(k) \quad -\mu \cdot \mathbf{B} \]

Momentum dependent Zeeman field

“equal Rashba and Dresselhaus contributions”

Engineering SO coupling

Define quasi-momentum $q$

\[
\begin{align*}
\left| \downarrow, \vec{k} \right\rangle & \quad q = k + Q/2 \\
\left| \uparrow, \vec{k} + \hbar \vec{Q} \right\rangle & \quad q = k - Q/2 \quad + Q/2
\end{align*}
\]

quasi-momentum space

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Engineering SO coupling

Define quasi-momentum $q$

\[ q = k + Q/2 \]

\[ q = k - Q/2 \]

\[ + \frac{Q}{2} \]

\[ - \frac{Q}{2} \]

**quasi-momentum space**

\[ \mathcal{H}_{SO} = \left( \frac{\hbar^2 (q - Q/2)^2}{2m} + \frac{\hbar^2 (q + Q/2)^2}{2m} \right) \]

\[ \delta = 0 \]
• Fermionic $^6\text{Li}$ atoms sympathetically cooled by $^{23}\text{Na}$

• Relevant states are 2$^{nd}$ and 3$^{rd}$ lowest states at 11G

• Interactions are negligible ($20a_0$)
Coupling spin and momentum via Raman

Laser 1  Fermion  Laser 2

\[ \nu_2 - \frac{\nu}{\lambda} \quad \nu_1 + \frac{\nu}{\lambda} \]

Vary detuning
Short pulse

State-selective imaging after TOF provides spin and momentum information
Start with a mixture of $|\downarrow\rangle$ and $|\uparrow\rangle$, and apply a Raman pulse for a given $\delta$.

Check for the linear dependence of the transfer with momentum $q$ (Doppler shift $\propto k_x Q$)
Start with state $|\downarrow\rangle$
Pulsing on Raman Beams

- Atomic system is coherent over many cycles
- Momentum-dependent Rabi oscillations

Probability of transfer:

\[ P_\uparrow = \frac{\Omega^2}{\Omega^2 + \Delta^2} \sin^2 \left( \frac{1}{2} \sqrt{\Omega^2 + \Delta^2} t \right) \]

with \( \Delta = \Delta(q) \)

Start with state |↓⟩
Adiabatic Sweep

- Start with state $\downarrow$
- Set large initial detuning ($|\delta| \gg E_R$) and then sweep

$\delta(t = 0) < 0$
Adiabatic Sweep

- Start with state $|\downarrow\rangle$
- Set large initial detuning ($|\delta| \gg E_R$) and then sweep

$$\delta(t = 0) < 0$$

$$\delta(t = 0) > 0$$
Spin-injection spectroscopy

• How to characterize Hamiltonian?
  – Can topology be measured?
• Condensed matter: transport, (spin-)ARPES, STM ...
• Cold atom analog:
  momentum resolved RF (Jin, Koehl)
    (=photoemission spectroscopy)
• Photoemission Spectroscopy probes dispersion $E(k)$
What has been done so far

Ian Spielman’s group

P. Wang et al *arXiv:1204.1887*
(Jing Zhang’s group)

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Can Topology be measured?

Spin-injection spectroscopy:
Measures spin, energy, momentum

1. Inject atoms from “reservoir”
2. Project into free space
3. Spin-selective imaging

→ Reconstruct $E(k)$ along with “color” of band
Experimental Setup

- 1\textsuperscript{st} and 4\textsuperscript{th} states used as reservoir states
Spin-injection spectroscopy
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Increasing Raman Intensity
Spin-injection spectroscopy

Increasing Raman Intensity

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Creating a Spinful Lattice

- Add RF coupling -> lattice system with full bandgaps and spinful bands

Creating a Spinful Lattice

• Add RF coupling -> lattice system with full bandgaps and spinful bands

The Spin-Orbit band structure is periodically repeated

• In repeated scheme
Bandstructure of Raman + RF lattice

- Degenerate point inside spin orbit gap
Bandstructure of Raman + RF lattice

- Bandgap opens between $2^{nd}$ and $3^{rd}$ band
Bandstructure of Raman + RF lattice

- Larger RF, gap between lowest bands
Spin-injection Spectra

Increasing Raman Intensity

Increasing RF Intensity
Spin-injection Spectra

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Increasing RF Intensity

Increasing Raman Intensity

\begin{align*}
\Delta v/E_R &:\quad 0.00 & 0.40 & 0.93 \\
\hbar \Omega_{RF}/E_R &:\quad 0.00 & 0.11 & 0.28
\end{align*}
Spin-injection Spectra

Increasing Raman Intensity

Increasing RF Intensity

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Spin-injection Spectra

Increasing Raman Intensity

Increasing RF Intensity

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Spin-injection Spectra

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Spin-injection Spectra

$E/E_R$

$h\Delta V/E_R$

$E/E_R$

$q/Q$

$k_x/Q$

$h\Delta V/E_R$

$k_x/Q$

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Spin-injection Spectra

Δ₁ → Raman gap
Δ₂ → RF gap
Δ₃ → Raman + RF gap
Reconstructing the Bandstructure

- In addition to dispersion, can reconstruct eigenstates
- TOF gives eigenstate in the basis of free space spin/momentum states
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Summary and Outlook

• Summary:
  – SO-coupled Fermi gas
  – Spinful lattice
  – Spin-injection spectroscopy
  – Band and eigenstate reconstruction

• Future:
  – Interactions : p-wave
  – Pairing in 1D tubes : Majorana edge mode ?

For details see:
Collaborators

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Experimental Setup

- Fermionic \(^6\text{Li}\) atoms sympathetically cooled by \(^{23}\text{Na}\)

- Relevant states are 2\(^{\text{nd}}\) and 3\(^{\text{rd}}\) lowest states at 11G

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Spin-injection spectroscopy

Experiment

Theory

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SO-coupling in a Fermi gas

• Direct demonstration of SO-coupling through Rabi oscillations

• Controlled adiabatic loading of SO-coupled bands.

• Reversibility of loading shows adiabaticity.
The spin-orbit Hamiltonian

- The SO Hamiltonian

\[ \mathcal{H} = \frac{\hbar^2 k^2}{2m} - \frac{g \mu_B}{\hbar} \mathbf{S} \cdot (\mathbf{B}^{(D)} + \mathbf{B}^{(R)} + \mathbf{B}^{(Z)}) \]

\[ \mathbf{B}^{(R)} = \alpha(-k_y, k_x, 0) \quad \mathbf{B}^{(D)} = \beta(k_y, k_x, 0) \]

- Raman Coupling Hamiltonian

\[ \mathcal{H}_{SO} = \left( \frac{\hbar^2 k^2}{2m} + \frac{\delta \hbar \Omega_R}{2} - \frac{\hbar^2 (k+Q)^2}{2m} - \frac{\delta}{2} \right) \]

maps to 1D spin-orbit Hamiltonian with

\[ \alpha = \beta = \frac{\hbar^2 Q}{2mg \mu_B} \quad B_z^{(Z)} = \frac{\hbar \Omega_R}{g \mu_B} \quad B_y^{(Z)} = \frac{\hbar \delta}{g \mu_B} \]

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Spin-injection spectroscopy

- Spin-injection spectroscopy on a spinful lattice
The spin-orbit Hamiltonian

- Raman coupled atomic system maps to SO Hamiltonian.
- Rotating-Frame approximation:
  \[
  \mathcal{H}_{SO} = \left( \frac{\hbar^2 k^2}{2m} + \frac{\delta}{2} - \frac{\hbar \Omega_R}{2} \right)
  \]
  \[
  \frac{\hbar^2 (k+Q)^2}{2m} - \frac{\delta}{2}
  \]
- Write in terms of COM momentum $q$ (spin-dependent transformation):
  \[
  \uparrow \quad q = k + \frac{Q}{2}
  \]
  \[
  \downarrow \quad q = k - \frac{Q}{2}
  \]

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The spin-orbit Hamiltonian

- Raman coupled atomic system maps to SO Hamiltonian.
- Rotating-Frame approximation:

\[ \mathcal{H}_{SO} = \left( \frac{\hbar^2 k^2}{2m} + \frac{\delta}{2} + \frac{\hbar \Omega_R}{2} \right) - \frac{\hbar^2 (k+Q)^2}{2m} - \frac{\delta}{2} \]
• Raman coupled atomic system maps to SO Hamiltonian.

• Rotating-Frame approximation:

\[ \mathcal{H}_{SO} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} + \frac{\delta}{2} & \frac{\hbar \Omega_R}{2} \\ \frac{\hbar \Omega_R}{2} & \frac{\hbar^2 (k+Q)^2}{2m} - \frac{\delta}{2} \end{pmatrix} \]

• Write in terms of COM momentum \( q \) (spin-dependent transformation):

\[ \uparrow \quad q = k + Q/2 \]
\[ \downarrow \quad q = k - Q/2 \]

• Amplitude of Raman beams give splitting
• Detuning imbalances the two wells
Raman coupled atomic system maps to SO Hamiltonian.

Rotating-Frame approximation:

\[ \mathcal{H}_{SO} = \left( \frac{\hbar^2 k^2}{2m} + \frac{\hbar}{2} + \frac{\hbar \Omega_R}{2} - \frac{\hbar \delta}{2} \right) \]

Write in terms of COM momentum \( q \) (spin-dependent transformation):

\[ \uparrow \quad q = k + Q/2 \]
\[ \downarrow \quad q = k - Q/2 \]

Amplitude of Raman beams give splitting
Detuning imbalances the two wells
SO-coupling in a Fermi gas

• When SO coupling is ramped slowly:
  – Spin composition follows effective magnetic field
  – Process is reversible
  – By changing detuning, either upper band or lower band
Spin-injection spectroscopy

• How to characterize Hamiltonian?
  – Can we measure topology?
• Condensed matter: transport, ARPES, STM ...
• Cold atom analog: photoemission spectroscopy (PES) has been
• PES probes $E(k)$
  – Transfer to hyperfine states outside system with RF
  – Measure momentum in TOF
  – Use RF frequency, free particle dispersion and momentum to reconstruct $E(k)$
Creating a Spinful Lattice

• Add RF coupling -> lattice system with full bandgaps and spinful bands

Detecting Spin Texture

- Image Sequence: TOF + state-selective imaging
Detecting Spin Texture

- Image Sequence: TOF + state-selective imaging
Detecting Spin Texture

- Image Sequence: TOF + state-selective imaging
• Raman Beams couple two hyperfine states

• SO coupling along one direction

• Recoil momentum: \( Q \)

• Recoil energy: \( E_R = \frac{\hbar^2 Q^2}{2m} \)
SO-coupling in Ultracold Atoms

• Realized in bosons:
  – Modified dispersion
  – Synthetic higher-order partial waves
  – Synthetic magnetic field


• Recently realized in fermions

  P. Wang et al *arXiv:1204.1887*
  L. W. Cheuk at el *arXiv:1205.3483*
Bandstructure of Raman + RF lattice

\[ \frac{E}{E_R} \]

\[ q/Q \]

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Bandstructure of Raman + RF lattice

\[ \frac{E}{E_R} \]

\[ q/Q \]

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Bandstructure of Raman + RF lattice

$E/E_R$ vs $q/Q$
A Spin Diode

- Spin diode when the Fermi level is inside the spin gap
Experiment vs Simulation

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Why spin-orbit coupling?

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices


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