Systems out of equilibrium and non-linear dynamics

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The aim of these lectures is to present a unified view on non-equilibrium physics using the framework of statistical and non-linear physics.

Many natural phenomena are far from thermodynamic equilibrium and keep on exchanging matter, energy or information with their surroundings, producing currents that break time-reversal invariance. Such systems lie beyond the realm of traditional thermodynamics: the principles of equilibrium statistical mechanics do not apply to them. At present, there exists no general conceptual framework à la Gibbs-Boltzmann to describe their physics from first principles. The last two decades, however, have witnessed remarkable progress.

In the first part of these lectures we shall explain some recent developments, such as the Work Identities, the Fluctuation Theorem and the Macroscopic Fluctuation Theory which represent the first steps towards a unified approach to non-equilibrium behaviour.

In the second part of this course, we shall study non-equilibrium systems in which the competition between fluctuations and deterministic dynamics leads to original and non-trivial phenomena. For example, we shall investigate the effect of noise in the vicinity of the onset of an instability and in particular probe the impact of stochasticity on the stability and the dynamics of the system.

- **Stochastic evolution of a complex system:** Markov, Langevin and Fokker-Planck dynamics. Time reversibility and its consequences.
- **Thermodynamic fluctuations in the vicinity of equilibrium :** Einstein theory, linear response and Onsager relations. The fluctuation-dissipation relation.
- **Probabilistic approach to rare events:** large deviations. Irreversibility and the Gallavotti-Cohen fluctuation theorem.
- **Work identities of Jarzynski and Crooks:** "violations" of the Second Principle and introduction to stochastic thermodynamics.
- **A field-theoretic description of non-equilibrium fluctuations:** the macroscopic fluctuation theory and its consequences.
- **Interacting particle systems :** determinantal processes and exactly solvable models; the example of the Kardar-Parisi-Zhang growth

process.

- **Effect of noise on an instability :** the additive noise case.
- **Effect of noise on an instability :** the multiplicative noise case. On-off intermittency.
- **Fluctuation driven dynamics:** exit from a potential well. Excitability. Models of Earth magnetic field reversals. Stochastic resonance.
- **Anomalous diffusion and fractional brownian motion:** Wiener-Khintchin theorem. 1/f noise.

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