Instabilities and nonlinear phenomena

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Most problems in dynamics encountered in physics or in other fields are governed by nonlinear differential equations. In contrast to linear equations, they usually display multiple solutions with different qualitative characteristics, often different symmetries. We study the bifurcations, i.e. the transitions, between these different solutions when a parameter of the system is varied. We show that the dynamics in the vicinity of these bifurcations is governed by universal equations called normal forms that mostly depend on the broken symmetries at the transition. We emphasize the analogy with phase transitions, but also point out differences such as limit cycles or chaotic behaviors which do not occur at equilibrium.

The first part of the course concerns bifurcation theory for maps and ordinary differential equations and an introduction to pattern-forming instabilities and reaction-diffusion equations. Nonlinear waves and solitons as well as instabilities in spatially extended systems are considered in the second part of the lectures, mostly using the concept of amplitude equations which is also applied to problems in condensed-matter physics such as commensurate-incommensurate transitions, magnetic domains and superconductivity.

Through these lectures, our aim is to show that symmetry arguments together with a qualitative analysis of differential equations and the use of perturbation techniques provide tools that can be used to understand many phenomena in various fields of physics and elsewhere.

- **Dynamical systems:** stationary bifurcations, spectra of matrices, Hopf bifurcations, global bifurcations.
- **Convection and Lorenz model:** Rayleigh-Benard convection, linear stability analysis for idealized case, derivation and behavior of Lorenz model, real-world convection behavior.
- **Symmetry:** reflection, rotation, groups
- **Maps and period doubling:** fixed points and iteration, steady bifurcations, period-doubling bifurcations
- **Floquet analysis:** theory, Faraday instability, cylinder wake.
- **Stripes, patterns and instabilities:** Swift-Hohenberg and Newell-Whitehead, squares and hexagons, Eckhaus and zig-zag instabilities.

- **Reaction-diffusion equations:** overview, excitable systems, Turing patterns, spatial analysis.
- **Waves in a nonlinear and dispersive medium:** nonlinear Schrödinger equation, Benjamin Feir instability, solitary waves, bifurcations in conservative systems.
- **Nonlinear waves:** canonical nonlinear wave equations in different fields of physics, shock waves, solitons, methods for exact solutions.
- **Amplitude equations for pattern-forming instabilities:** stationary or Hopf bifurcations in an extended domain, symmetry arguments, potential versus conservative dynamics.
- **The amplitude equation formalism applied to some topics from condensed-matter physics:** commensurate-incommensurate transitions, magnetic domains, superfluids, superconductivity.
- **Broken symmetries and collective modes:** neutral modes related to spontaneously broken symmetries, phase dynamics of patterns, effet of phase modes on secondary instabilities, interaction of phase modes.
- **Topological defects:** kinks and their dynamics, Ising versus Bloch walls and the effect of chirality on their dynamics, vortices or dislocations, spatiotemporal chaos mediated by defect dynamics.

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