Overview of the course

Collective Phenomena:

From Phase Transitions to Statistical Field Theory

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From thermodynamics to statistical mechanics

A 'complex system' in the 1800s: steam engines

Second law of thermodynamics, notion of entropy (Clausius)

Microscopic foundations? (Boltzmann, Gibbs, ...)

Predict the macroscopic behavior of a system from the knowledge of its microscopic components and of their interactions.
Phase Transitions:

Van der Waals (Nobel Prize 1910)

$V < V_1 < V < V_2 < V$

$F_1$ (liquid) $F_2$ (gas)

$P$ $V$

$T = \text{const} (< T_c)$

$V_1$ $V_2$

$0 \text{ K}$ $0$ Pressure Temperature

$V < V_1$ $V_1 < V < V_2$ $V_2 < V$
'Modern' Statistical Physics: universality

Critical point
Liquid - Vapor

Phase Transition
Paramagnetic-Ferromagnetic

\[ C_{\text{Vol}} = \text{cte} \times |T-T_c|^{-\alpha} \]
\[ \rho_L - \rho_V = \text{cte} \times |T-T_c|^{\beta} \]
\[ \frac{dV}{dP} = \text{cte} \times |T-T_c|^{-\gamma} \]
\[ <\rho_0 \rho_q> = \text{cte} / q^{2-\eta} \]

\[ \gamma = 1.36 \quad (\text{Ni, Fe, ...}) \]
\[ = 1.22 \quad (\text{CrBr}_3, \text{fluids}) \]

\[ \alpha = -0.1 \quad (\text{Ni, Fe, ...}) \]
\[ = +0.1 \quad (\text{alloys e.g. CrBr}_3, \text{fluids}) \]
Phase Transitions and Universality

Existence of a common mathematical description (« field theory »)

⇒ Models need not be exact to be correct!
Critical exponents and field theory

Exponents depend on dimensions $d$ (space) and $n$ (order parameter)

Various applications: liquid crystals, superconductivity, polymers, ...
Relationship with polymers ... 

Random Walk

Polymer \approx \text{Self-Avoiding Random Walk}

Number of S.A.W. with N steps

\[ c \ N^{\nu-1} \ z^N \]

etc ...

Exponents for polymers in dim. \( d \) = exponents for field theory in dim. \( d \) and for \( n \rightarrow 0 \) !
Dynamics close to equilibrium ...

\[ P(\text{configuration}) = \exp\left(-\frac{E(\text{configuration})}{k_B T}\right) / Z \]

Time = 0 : Configuration \rightarrow equilibrium

\textit{thermalization time?}

time = cte \times |T-T_c|^{-(\text{dynamical exponent})}
... and out-of-equilibrium

Nucleation of supercooled water (video)

Nucleation time, shape of the critical bubble?

2nd order phase transitions and kinetic of domain growth (scaling laws, universal dynamical critical exponents ...)

Free energy

Space of « configurations »
Organisation of lectures and tutorials

Documents related to the lectures:

http://enseignement.phys.ens.fr/master/?lang=en

Take handwritten notes!

Two exams:
- after lecture 7(±1): check what you know
- after lecture 15: final exam (written, 3h)

Books:
- Théorie statistique des champs by C. Itzykson et J-M. Drouffe (volume 1)
- Introduction to Statistical Field Theory by E. Brézin
- Cours de 3e année de l'Ecole Polytechnique by A. Georges, M. Mézard, R.M.
- Quantum field theory and critical phenomena, J. Zinn-Justin (difficult!)