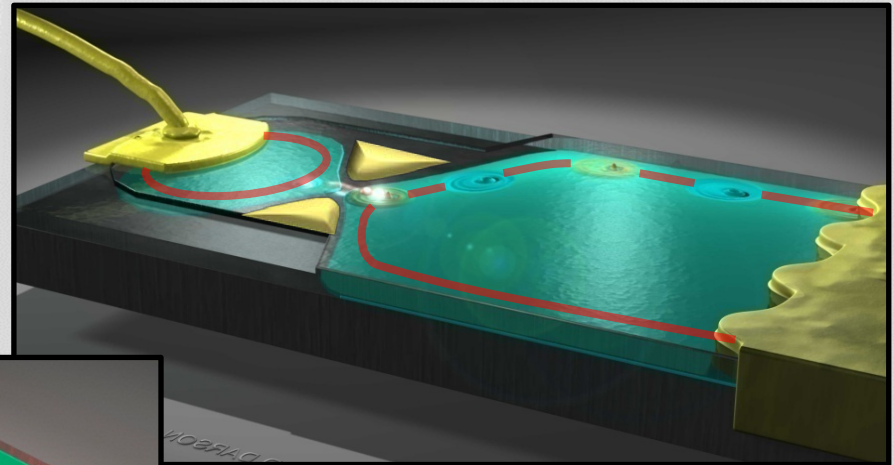
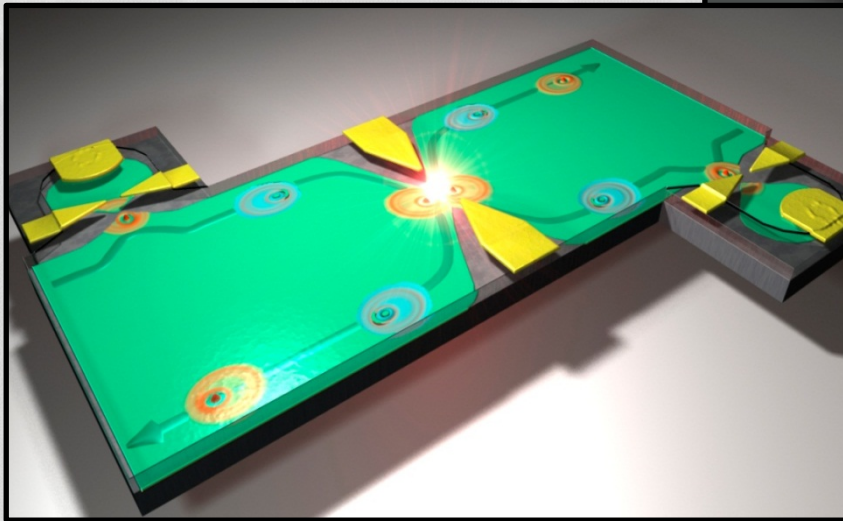




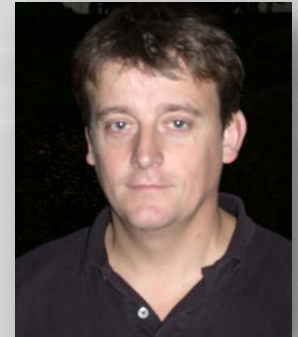
# Quantum optics with edge states

Bernard Plaçais

[placais@lpa.ens.fr](mailto:placais@lpa.ens.fr)

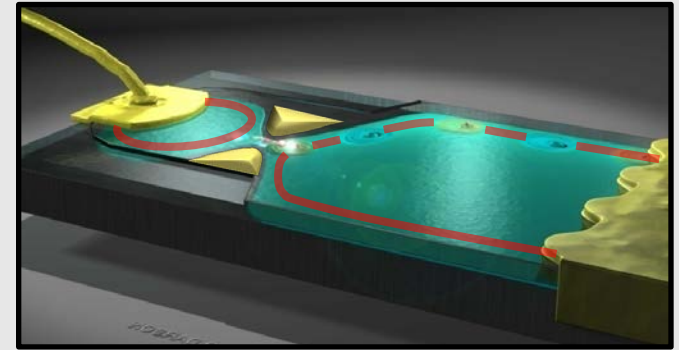


Gwendal Fève



Jean-Marc Berroir

Theory : P. Degiovanni et al. (ENS-Lyon), T. Martin et al. (CPT-Marseille)  
M. Buttiker et al. (Uni Genève)



### Part 3 : Introduction to electrons optics

- Electron optics principles
- Single electron sources
- Mesoscopic capacitor : average current and noise

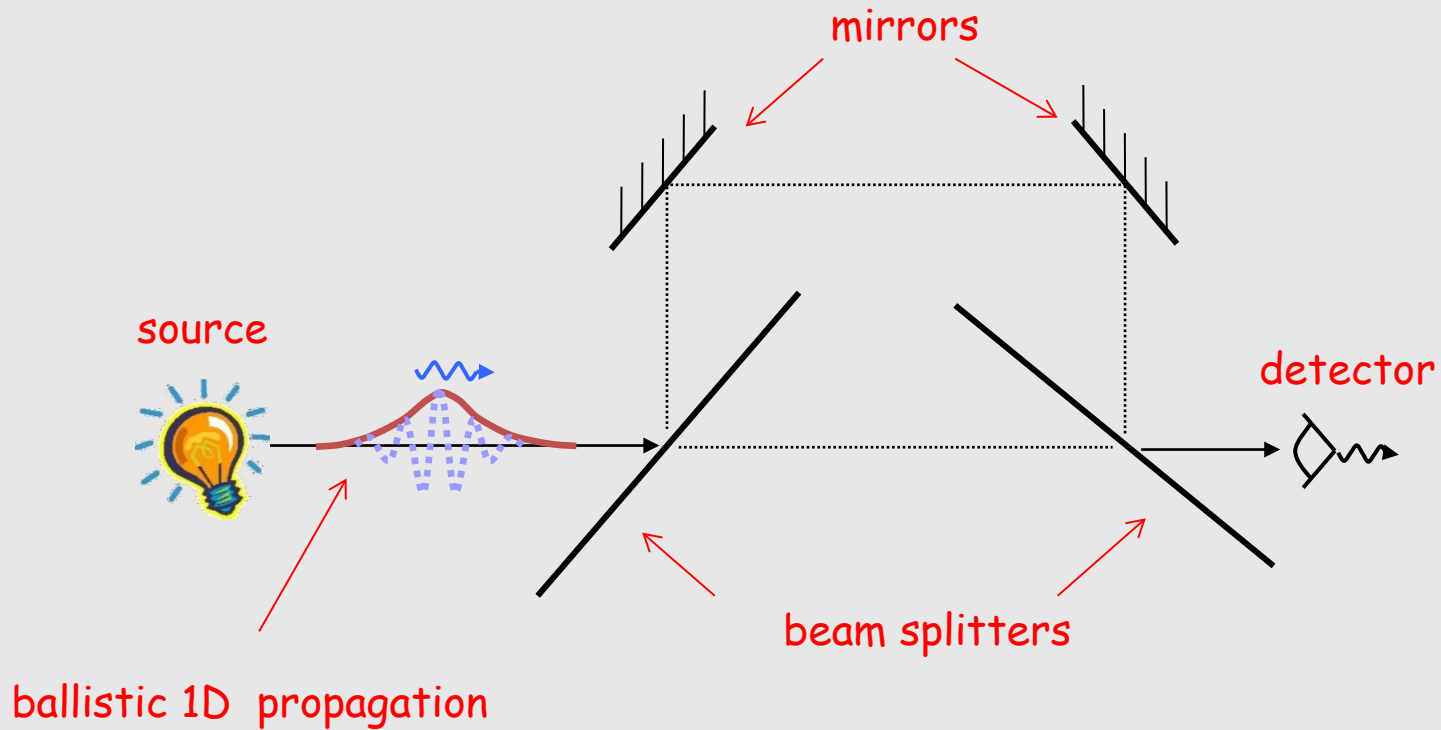
### Part 4 : Quantum optics experiments

- Partitioning single electrons
- Colliding indistinguishable electrons
- Decoherence and charge fractionalization

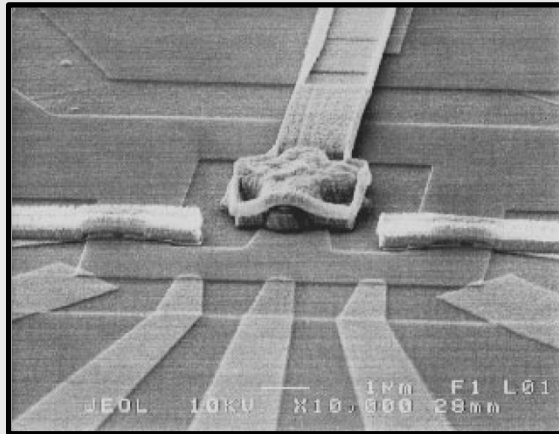
*Electron quantum optics in ballistic chiral conductors,*

*Bocquillon et al., Annalen der Physik 526, 1 (2014)*

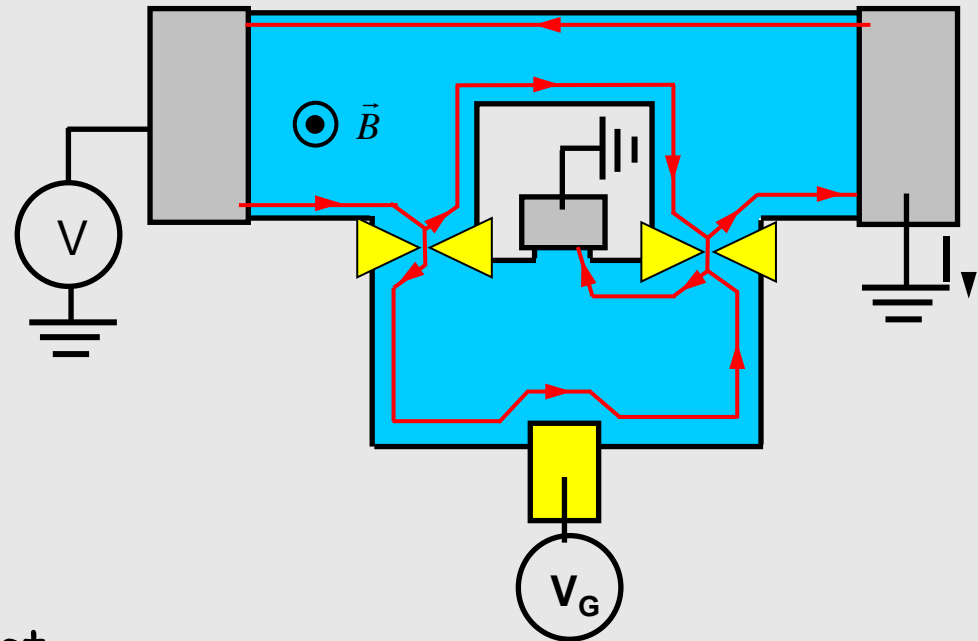
What do we need to realize an interferometer ?



# Mach-Zehnder interferometry with a biased contact (two-path interferometer)



Y. Ji et al., *Nature* 422, 415 (2003)



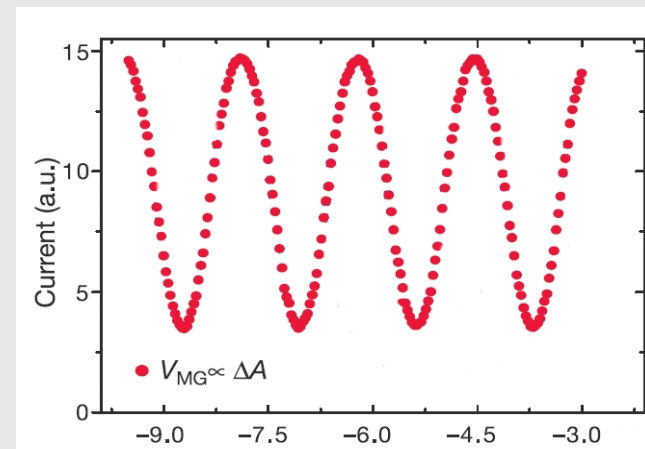
Coherent source = biased contact

« Natural source » not noisy

Coherence length = contrast

$$L_\phi = 20 \mu\text{m} \text{ at } 20 \text{ mK}$$

$$\tau_\phi \approx 200 \text{ ps for } v_D \approx 10^5 \text{ ms}^{-1}$$

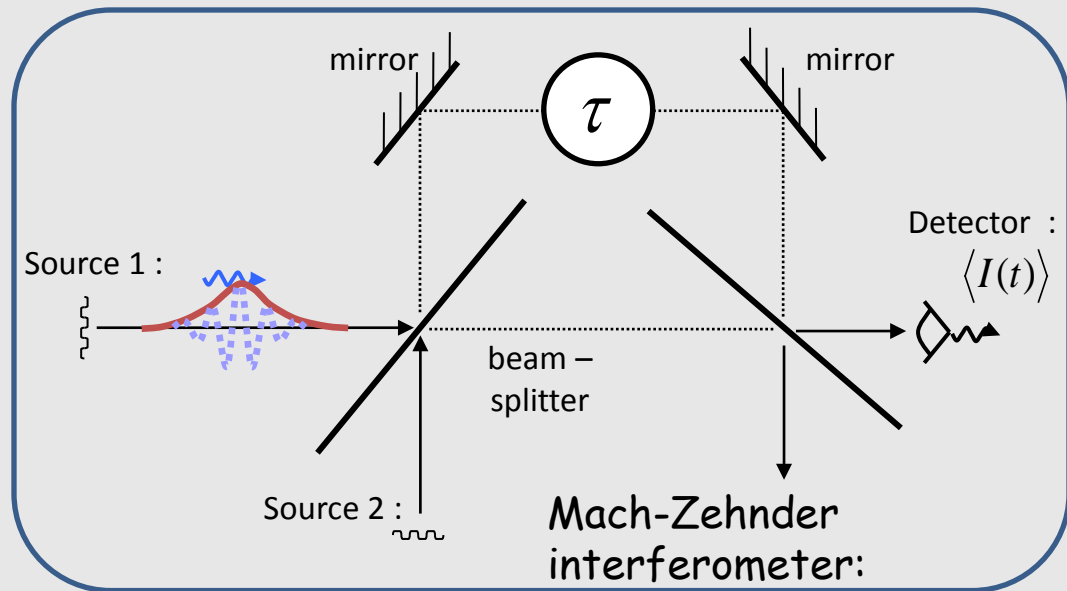


## Coherence properties of light

Wavelike description encoded in the first order coherence of the electromagnetic field

$$G^{(1)}(t, t + \tau) \propto \langle E(t)E(t + \tau) \rangle$$

Measured by e.g. Mach-Zehnder interferometry

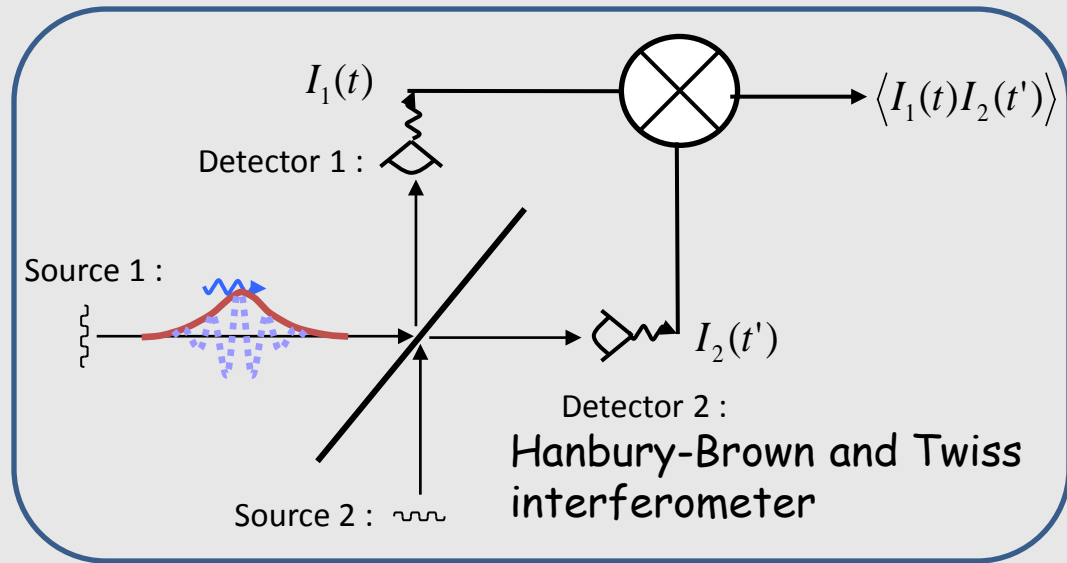


## Statistical properties of light

Corpuscular description encoded in intensity correlations

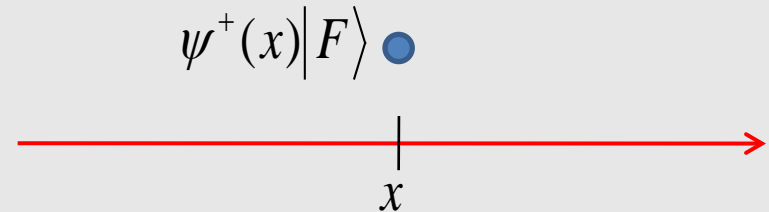
$$G^{(2)}(t, t + \tau) \propto \langle I_{ph}(t)I_{ph}(t + \tau) \rangle$$

Measured by Hanbury-Brown and Twiss (HBT) interferometry



## Electron field operator

$$\psi(x) = \frac{1}{\sqrt{\hbar v_F}} \int d\epsilon a(\epsilon) e^{i\epsilon x / (\hbar v_F)}$$



## Chiral, 1D, propagation

$$\psi(x, t) = \psi(x - v_F t) \quad \psi(x) \leftrightarrow E^+(x) \quad \psi^\dagger(x) \leftrightarrow E^-(x)$$

## Charge density and electrical current

$$I_{el}(x, t) = e v_F \psi^\dagger(x, t) \psi(x, t) \quad I_{ph}(x, t) \propto E^+(x, t) E^-(x, t)$$

$$G^{(1)}(\epsilon, \epsilon') = \langle a^\dagger(\epsilon) a(\epsilon') \rangle$$

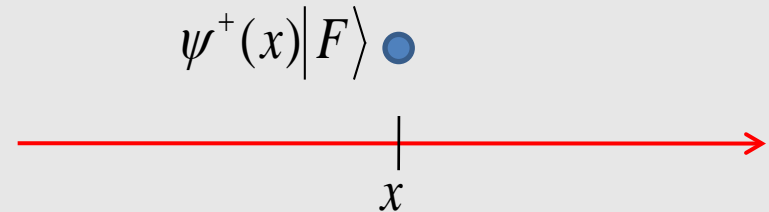
## Vacuum is the Fermi sea ! transport subtracts the Fermi sea

$$G^{(1)}(x, x') = G_F^{(1)}(x, x') + \Delta G^{(1)}(x, x') \quad G_F^{(1)}(\epsilon, \epsilon') = \theta(\epsilon - \epsilon_F) \delta(\epsilon - \epsilon')$$

$$I(x) = e v_F \Delta G^{(1)}(x, x)$$

## Electron field operator

$$\psi(x) = \frac{1}{\sqrt{\hbar v_F}} \int d\epsilon a(\epsilon) e^{i\epsilon x / (\hbar v_F)}$$



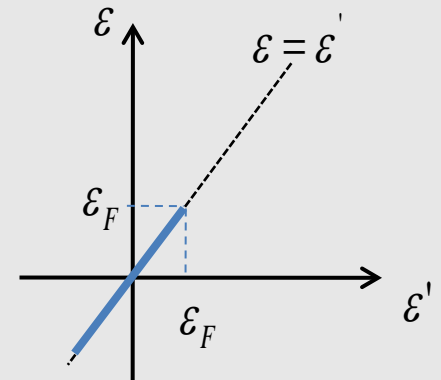
## Chiral, 1D, propagation

$$\psi(x, t) = \psi(x - v_F t) \quad \psi(x) \leftrightarrow E^+(x) \quad \psi^\dagger(x) \leftrightarrow E^-(x)$$

## Charge density and electrical current

$$I_{el}(x, t) = e v_F \psi^\dagger(x, t) \psi(x, t) \quad I_{ph}(x, t) \propto E^+(x, t) E^-(x, t)$$

$$G^{(1)}(\epsilon, \epsilon') = \langle a^\dagger(\epsilon) a(\epsilon') \rangle$$



## Vacuum is the Fermi sea ! transport subtracts the Fermi sea

$$G^{(1)}(x, x') = G_F^{(1)}(x, x') + \Delta G^{(1)}(x, x')$$

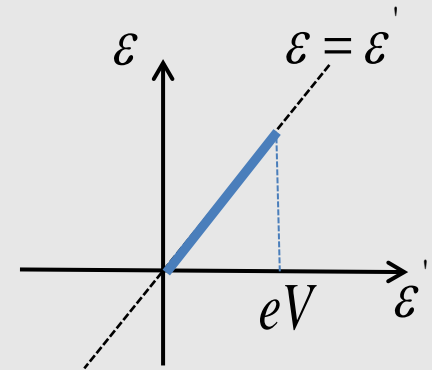
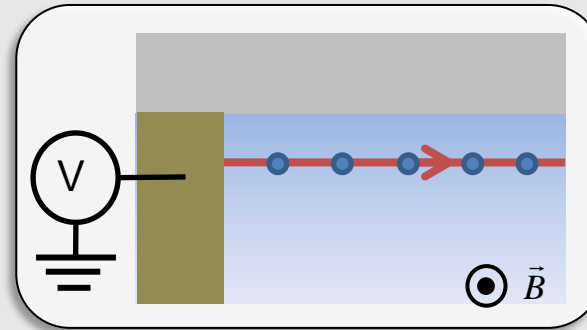
$$G_F^{(1)}(\epsilon, \epsilon') = \theta(\epsilon - \epsilon_F) \delta(\epsilon - \epsilon')$$

$$I(x) = e v_F \Delta G^{(1)}(x, x)$$

## Stationary emitters :

$$\Delta G^{(1)}(x, x') = \Delta G^{(1)}(x - x')$$

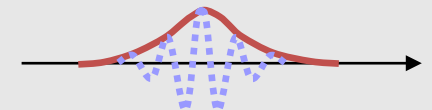
$$\Delta G^{(1)}(\epsilon, \epsilon') \propto \delta(\epsilon - \epsilon')$$



*control of the populations (no off – diagonal element in  $\Delta G$  !)*

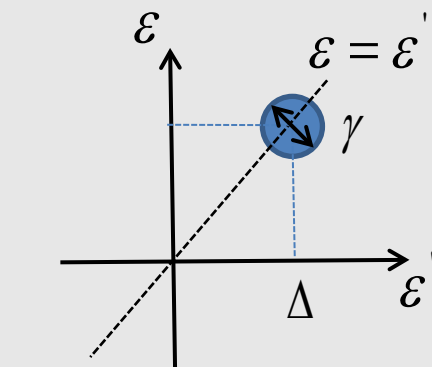
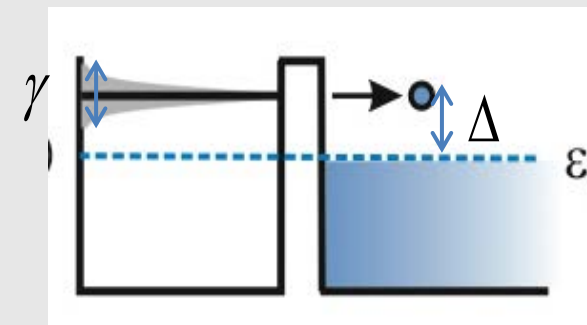
## Single electron state, wave packet $\varphi(x)$ , « a Landau QP »

$$\psi^\dagger[\varphi]|F\rangle = \frac{1}{\sqrt{h\nu_F}} \int dx \varphi(x) \psi^\dagger(x) |F\rangle$$



$$\Delta G^{(1)}(x, x') = \varphi^*(x) \varphi(x')$$

$$\Delta G^{(1)}(\epsilon, \epsilon') = \varphi^*(\epsilon) \varphi(\epsilon')$$



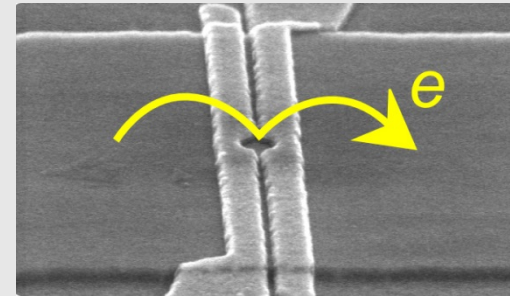
*electrons ( $\epsilon > 0$ ) holes ( $\epsilon < 0$ )*

*access to the coherence (fragile off – diagonal terms in  $\Delta G$ )*



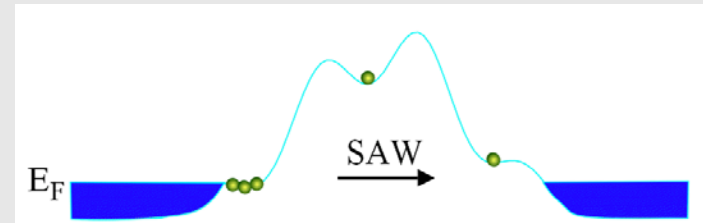
## Electron pumps

- M.D. Blumenthal et al., *Nature Physics* 3, 343 (2007)  
 F. Hohls et al., *Phys. Rev. Lett.* 109, 056802 (2012)  
 J. Fletcher et al., *Phys. Rev. Lett.* **111**, 216807 (2013)



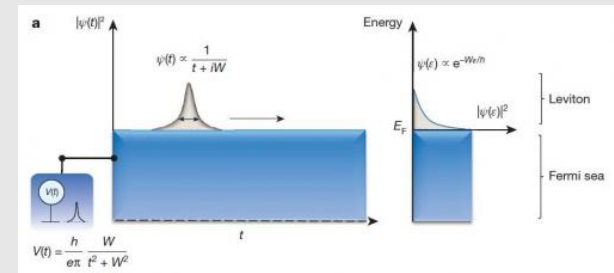
## Electrons flying on SAW (flying Q-dot)

- R. McNeil et al., *Nature* 477 (7365), 439 (2011)  
 S. Hermelin et al., *Nature* 477 (7365), 435 (2011)



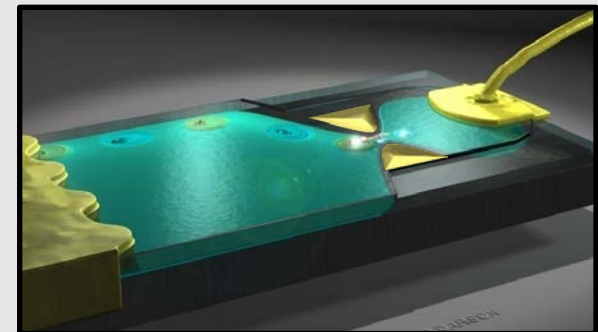
## Lorentzian voltage pulse (Levitons)

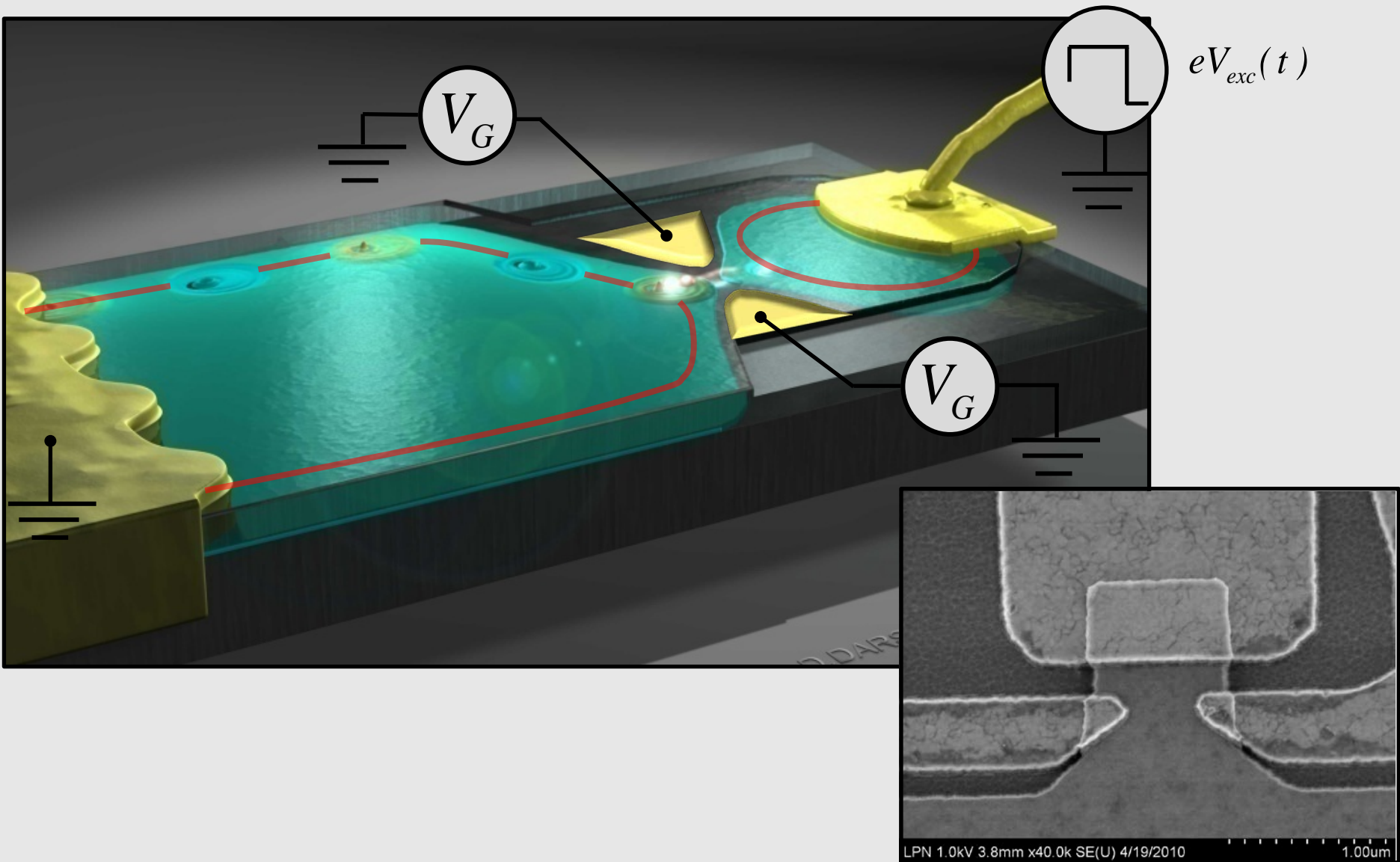
- D. A. Ivanov, et al., *Phys. Rev B* 56, 6839 (1997)  
 J. Dubois et al., *Nature* 502, 659 (2013)

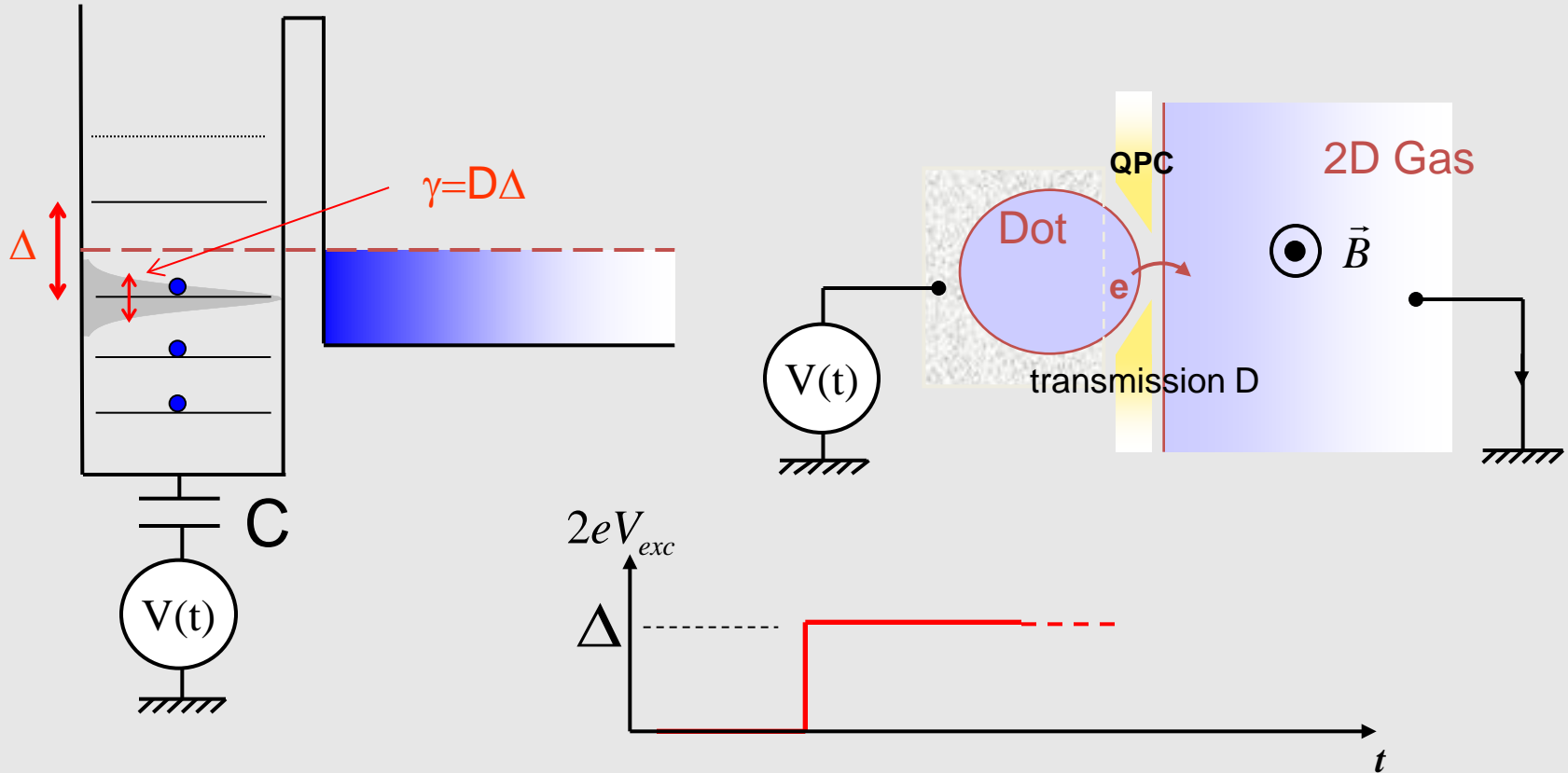


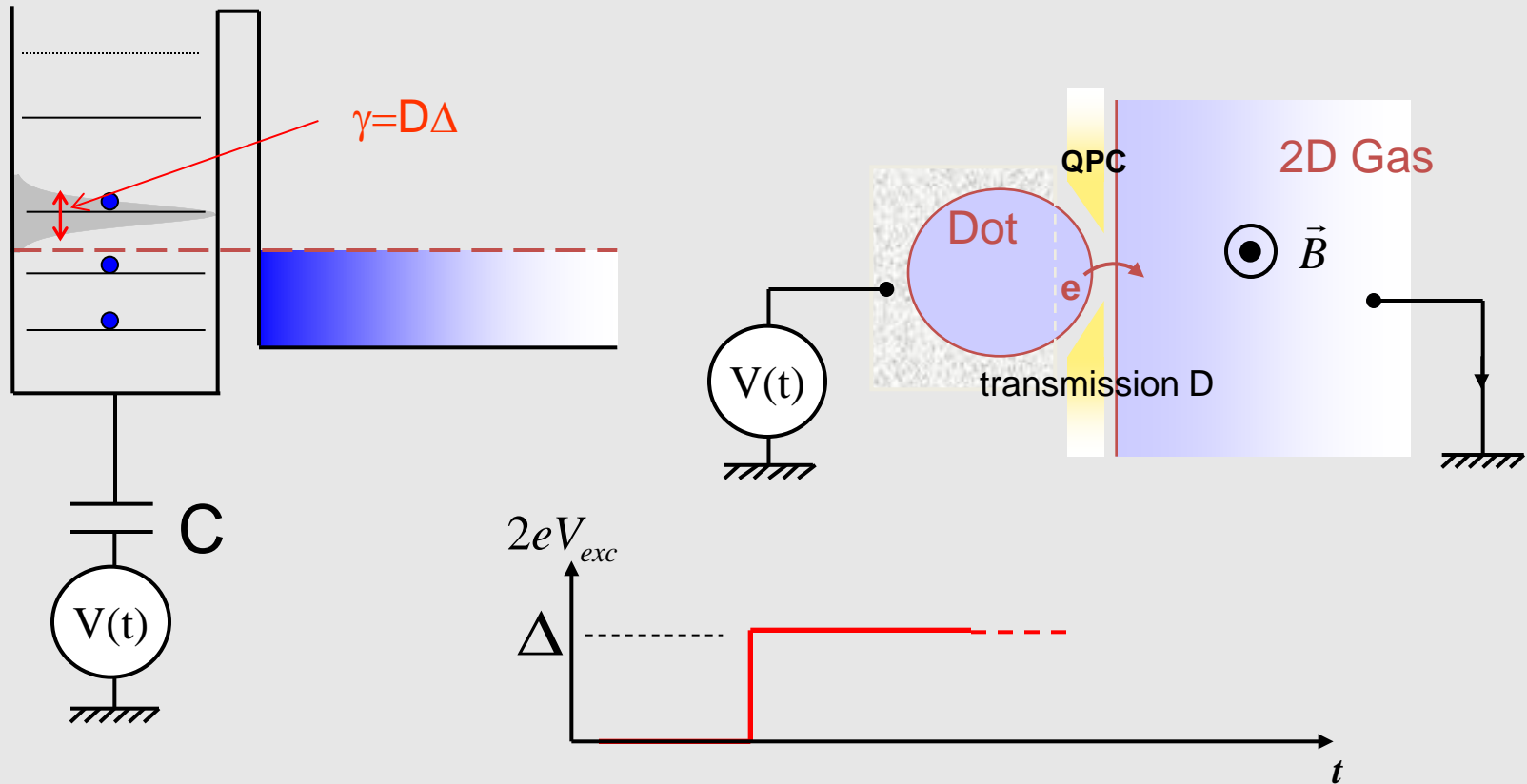
## Mesoscopic capacitor (electron/hole, energy resolved)

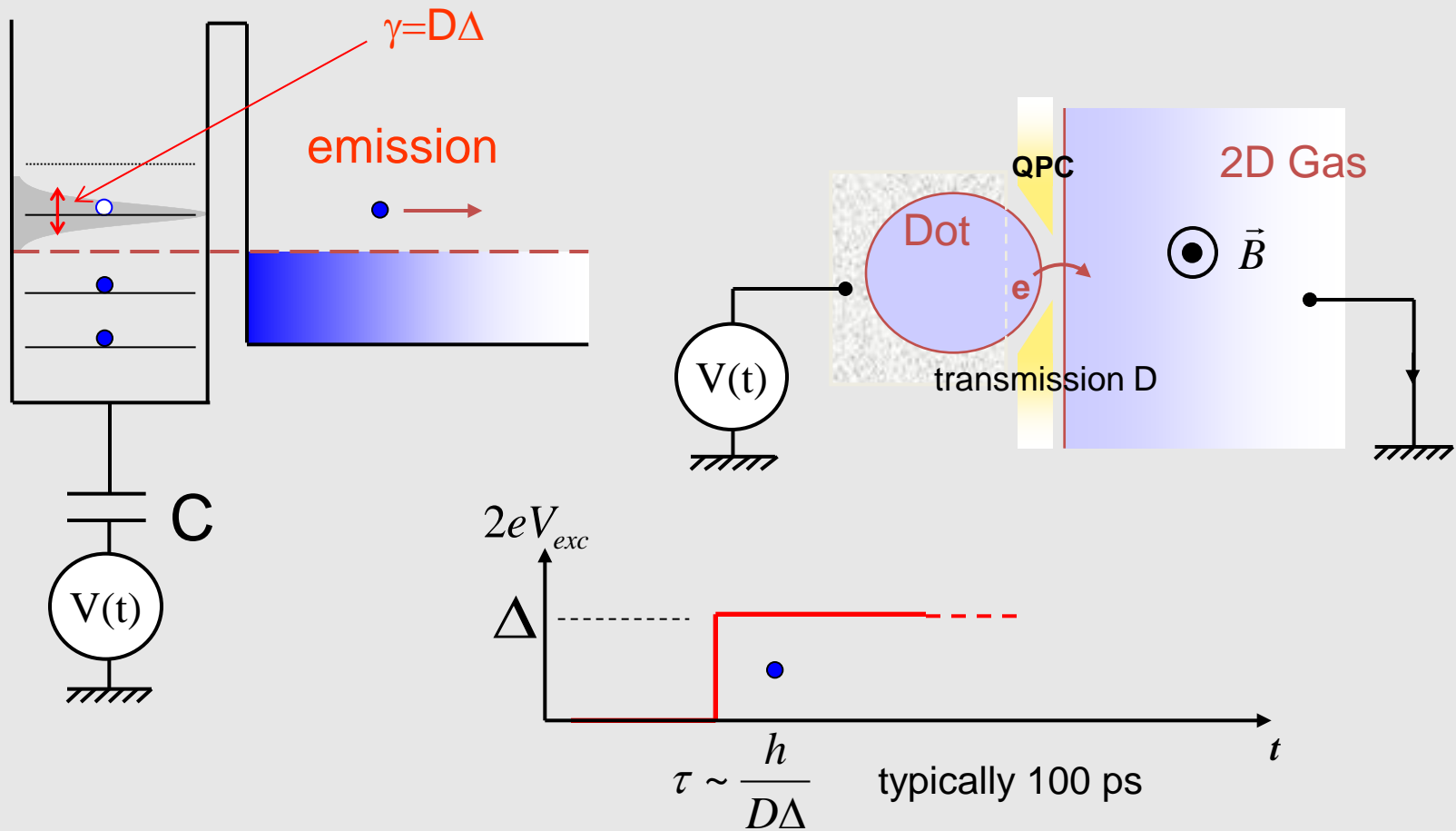
- G. Fève et al., *Science* 316, 1169 (2007)

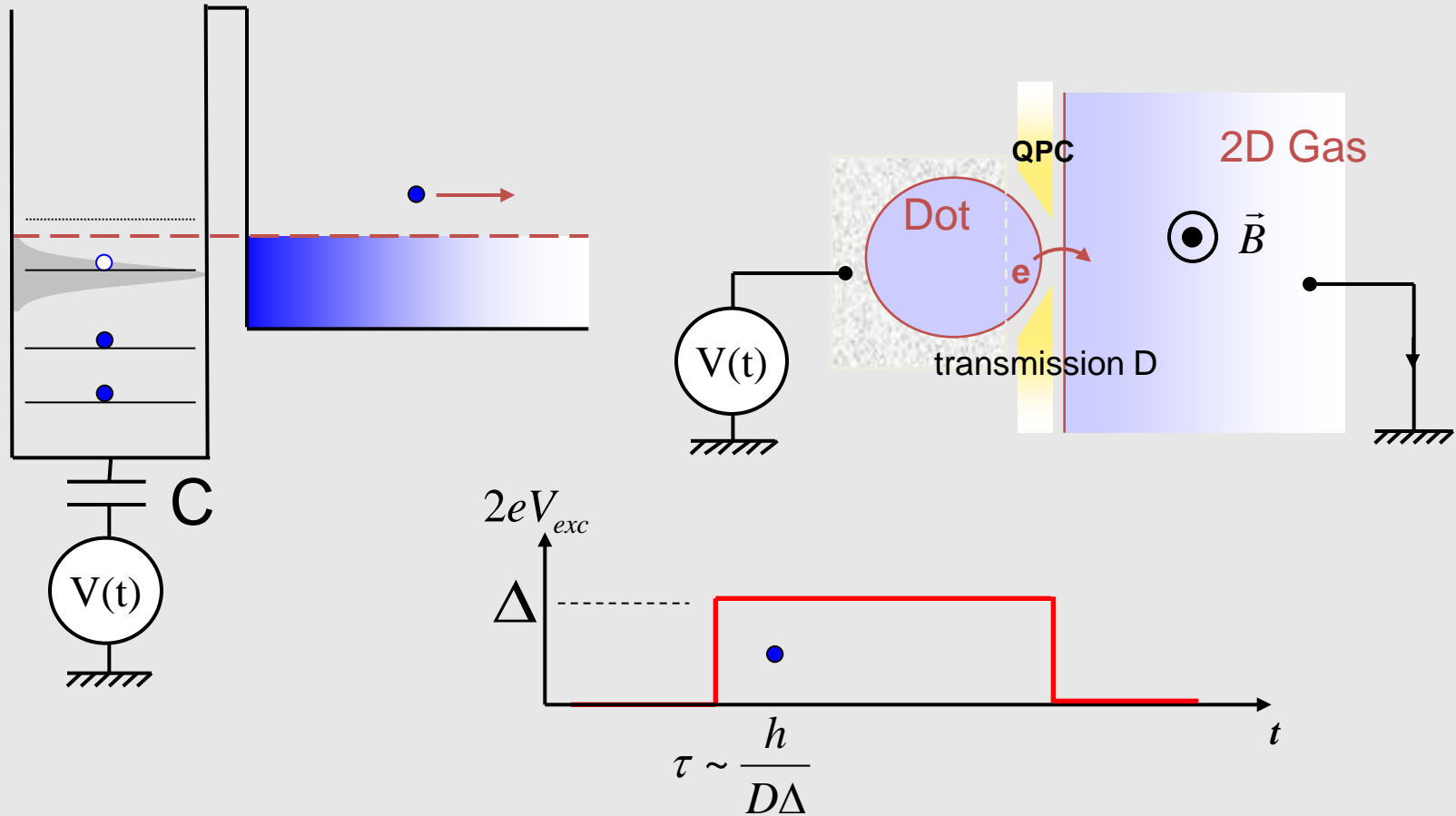


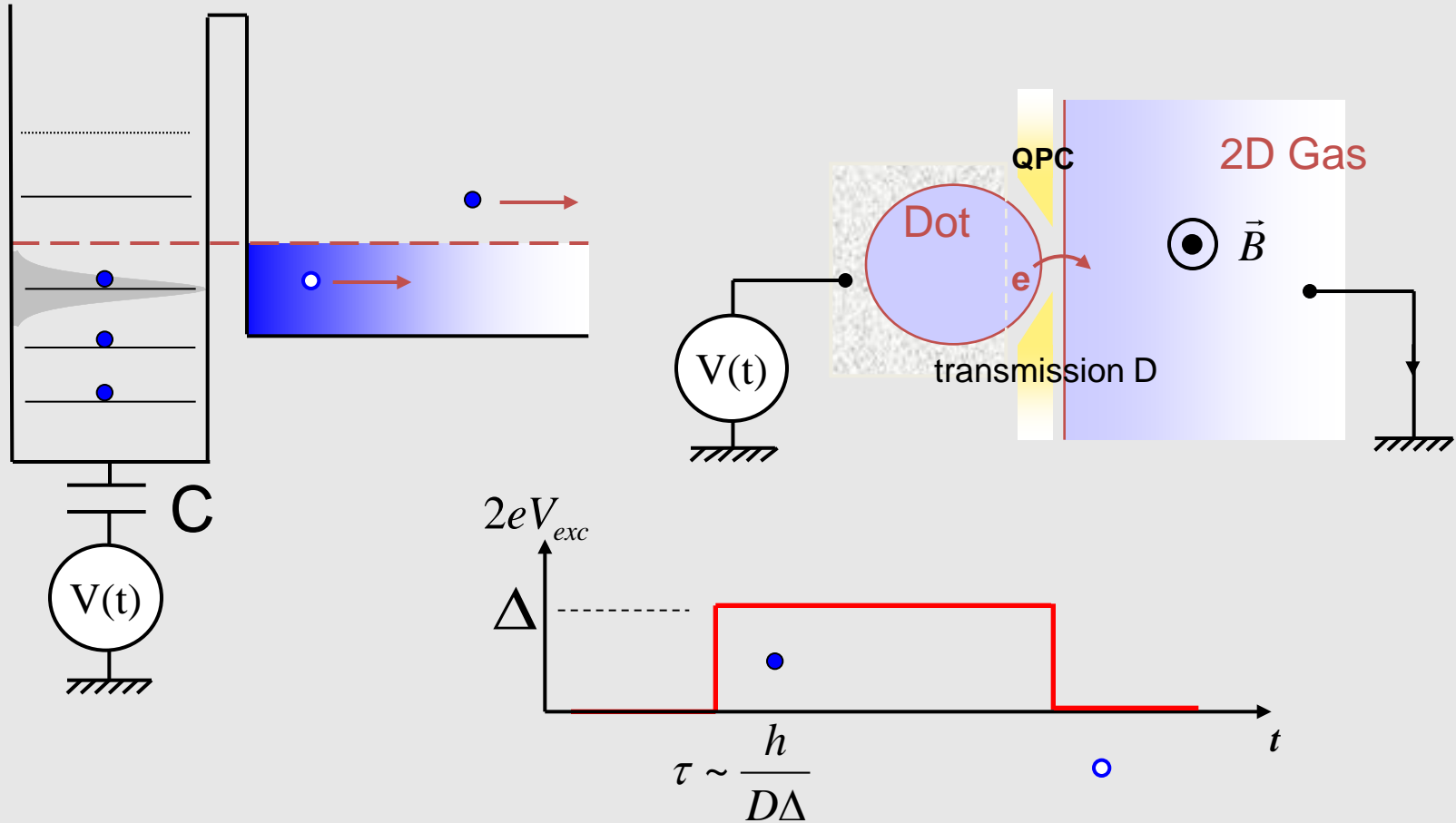






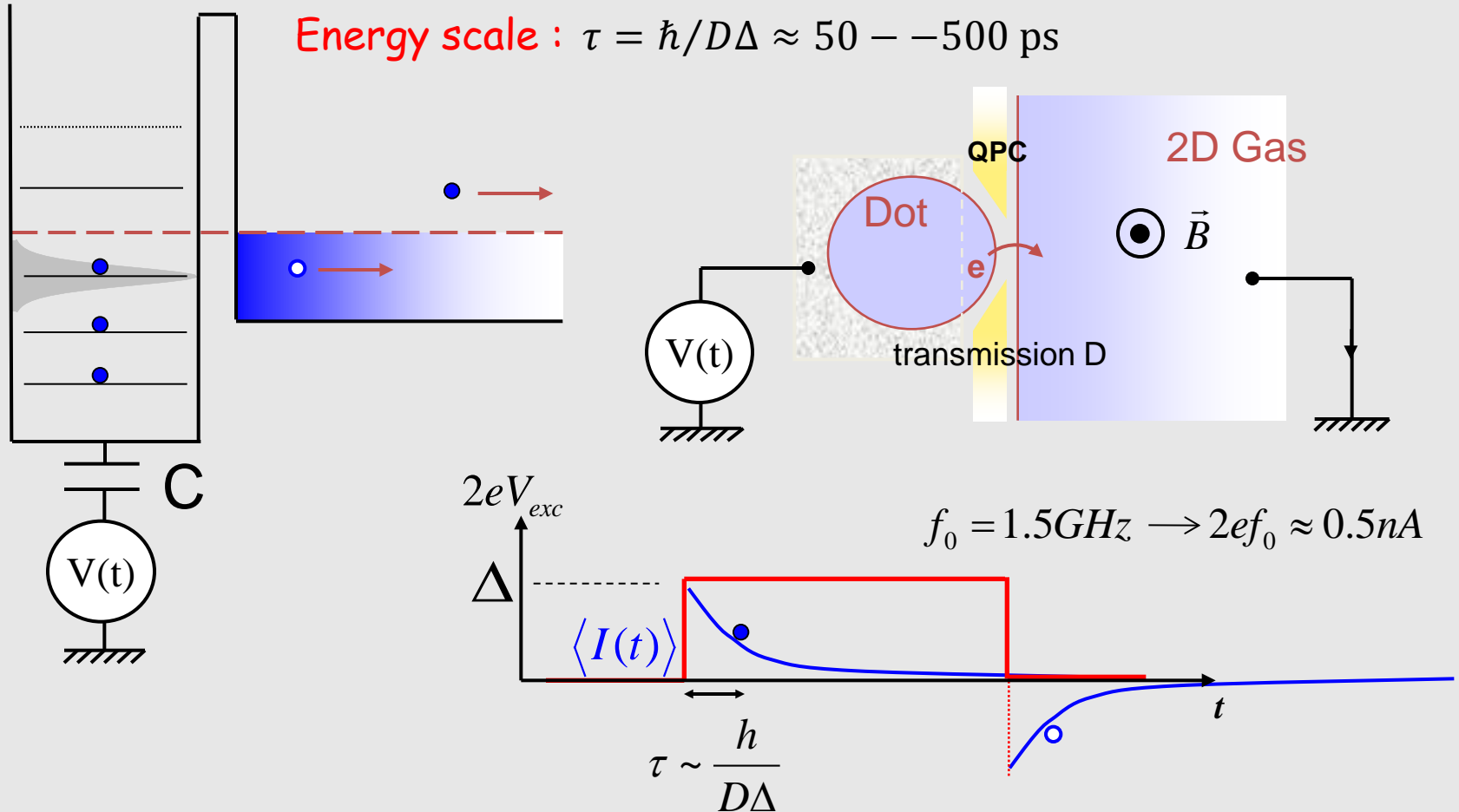






Energy scale (1 Kelvin  $\equiv 86 \mu\text{eV} \equiv 20.8 \text{ GHz}$ ) :  $\Delta \approx 2 \text{ Kelvin}$

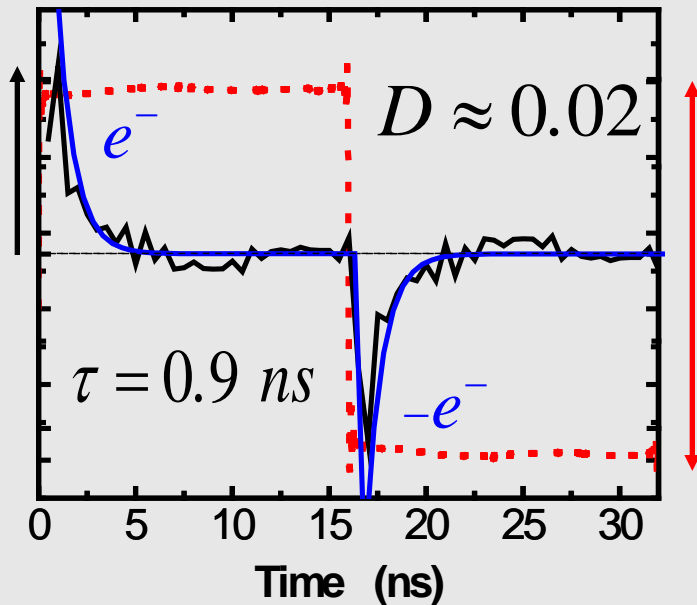
Energy scale :  $\tau = \hbar/D\Delta \approx 50 - 500 \text{ ps}$



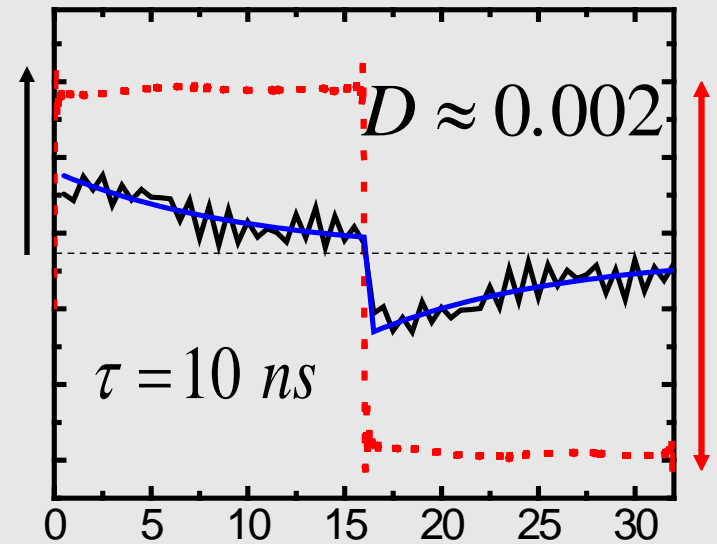


## Measuring single electron current pulses ?

Use a slow 32 MHz clock and a 1GHz fast averaging card (Acquiris)

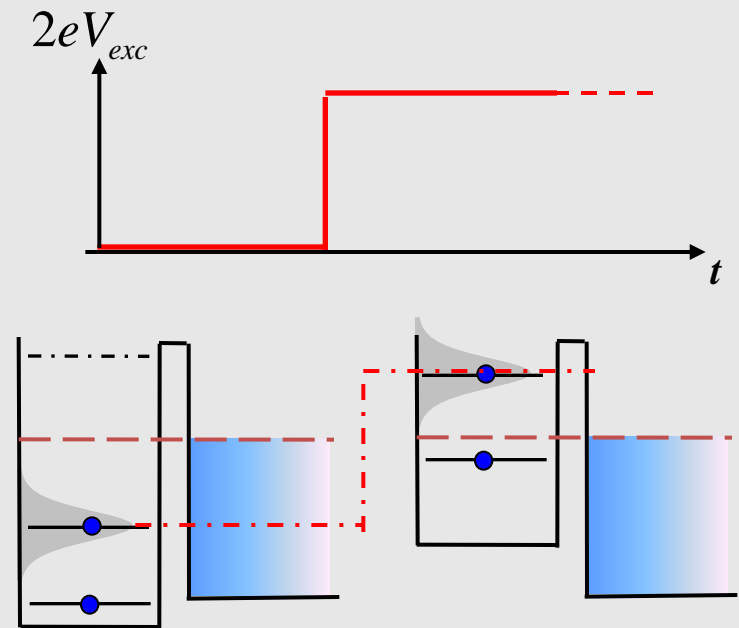
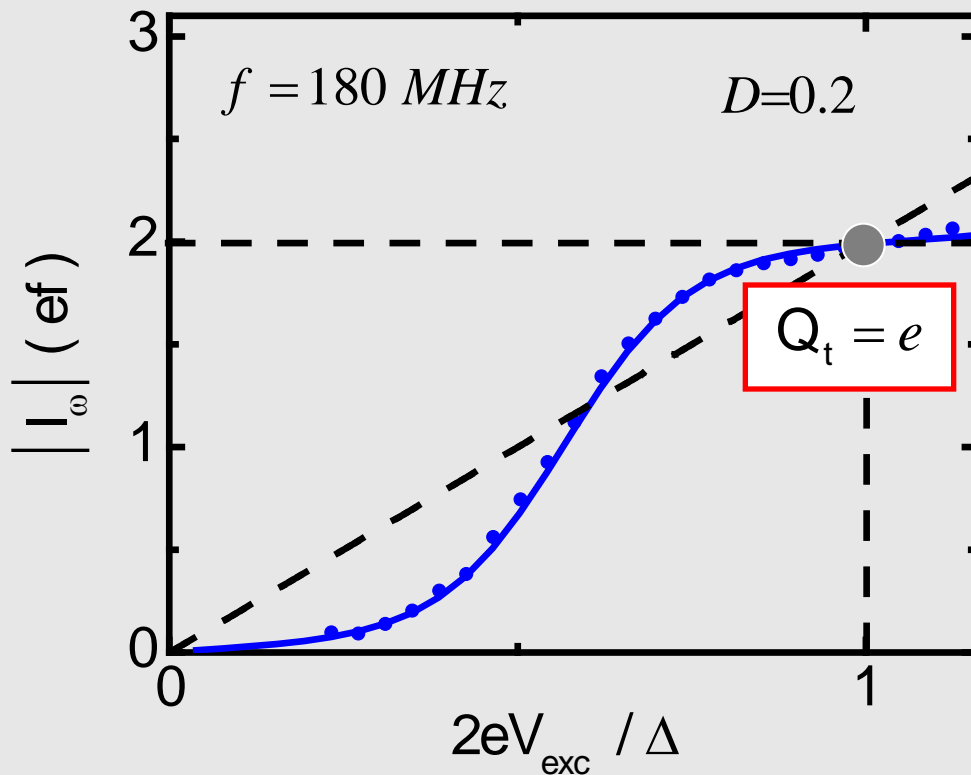


$$2eV_{exc} = \Delta$$



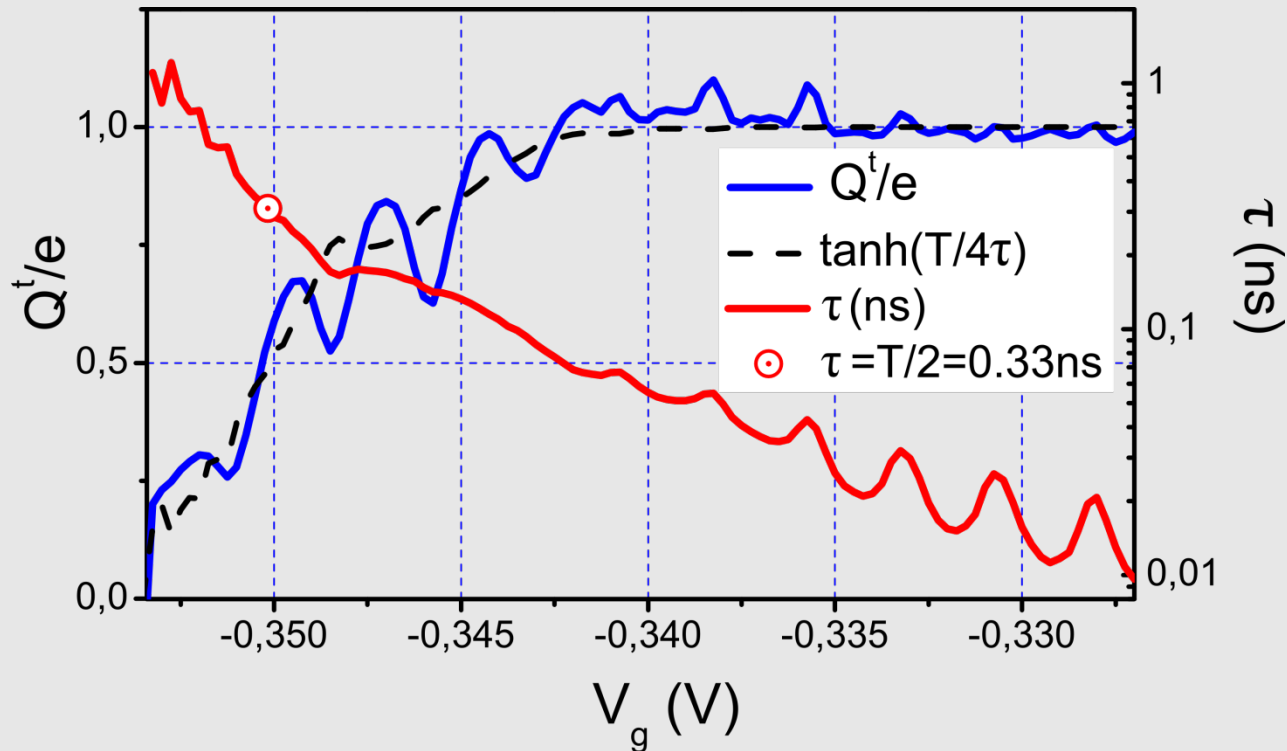
## Quantization of the AC current ?

Use homodyne detection at large clock frequency :  $\langle I_\omega \rangle = \frac{(2iQf)}{(1-i\omega\tau)}$



## Subnanosecond control of the emission time

Use homodyne detection at large clock frequency :  $\langle I_\omega \rangle = \frac{(2iQf)}{(1-i\omega\tau)}$



Single electron emission is characterized by current correlator

$$\overline{\langle I(t)I(t') \rangle} = \frac{Q^2}{T} \delta(t' - t)$$

AC noise correlations

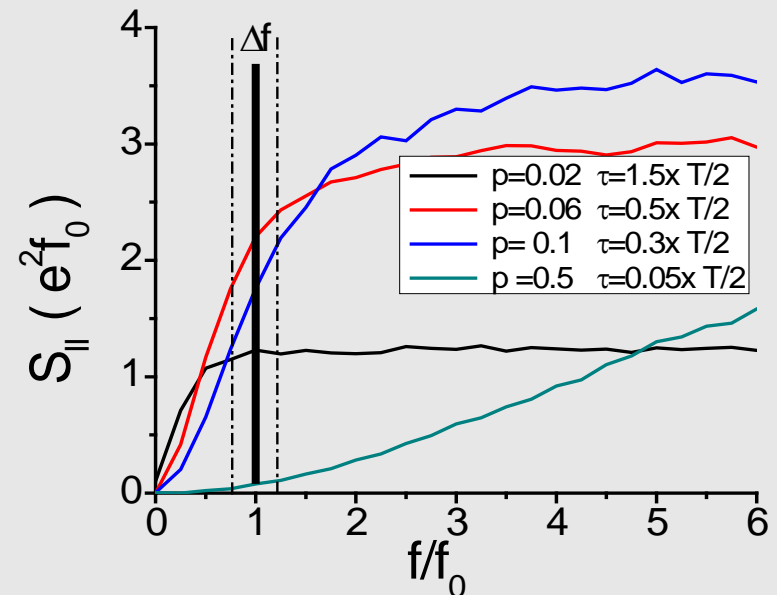
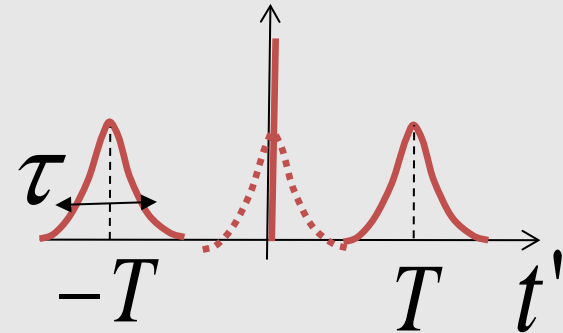
$$\overline{\langle \delta I(t) \delta I(t') \rangle} = \overline{\langle I(t) I(t') \rangle} - \overline{\langle I(t) \rangle \langle I(t') \rangle}$$

where  $\overline{\langle I(t) \rangle \langle I(t') \rangle} = \frac{Q^2}{T} \frac{e^{-(t'-t)/\tau}}{\tau}$

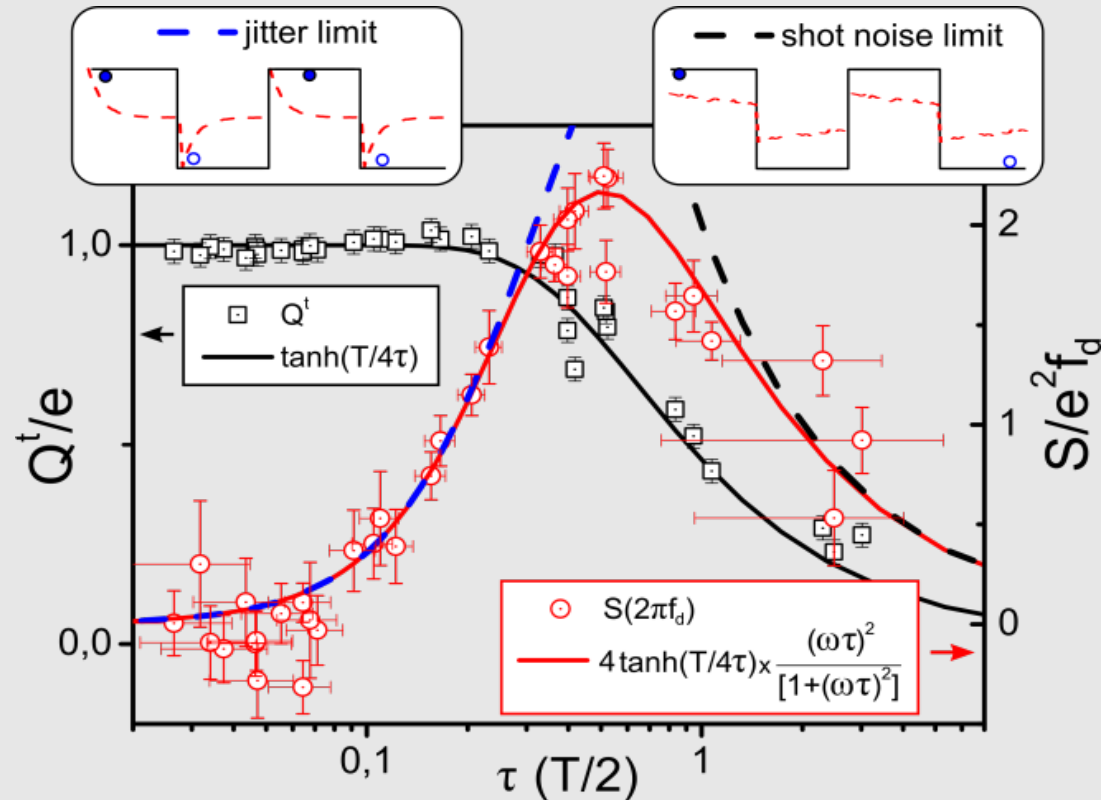
Whence the AC noise spectrum

$$S_{II}(\tau, \omega) = 2 \frac{e^2}{T} \frac{(\omega\tau)^2}{1 + (\omega\tau)^2}$$

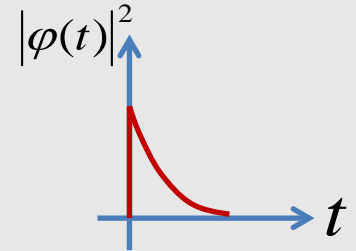
Experiment  $\rightarrow S_{II}(\tau, \omega = 2\pi f_0)$



Single electron source, beyond average current, average noise

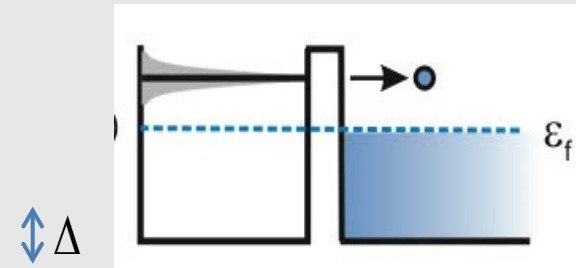


*average current* :  $\langle I(t) \rangle = e |\varphi(t)|^2$

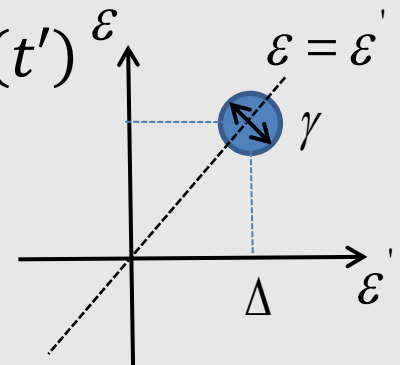


*current noise* :  $\langle I(t)I(t + \tau) \rangle = e^2 |\varphi(t)|^2 \delta(\tau)$

*sensitive to wave packet envelop* :  $|\varphi(t)|^2$



*How to access wave coherence?*  $\rho(t, t') = \varphi(t)\varphi^*(t')$



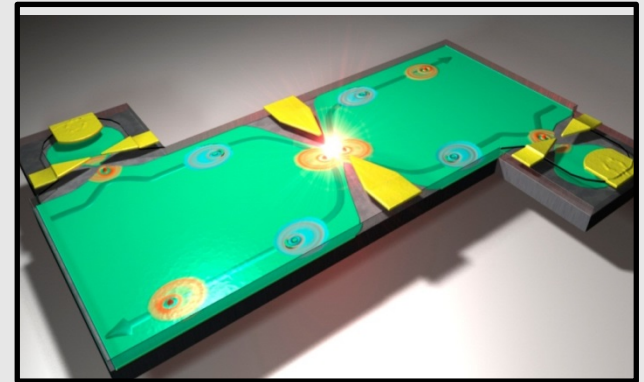
*Asses decoherence:*  $\rho(t, t') = \varphi(t)\varphi^*(t')D(t - t')$

### Part 3 : Introduction to electrons optics

- Electron optics principles
- Single electron sources
- Mesoscopic capacitor : average current and noise

### Part 4 : Quantum optics experiments

- Partitioning single electrons
- Colliding indistinguishable electrons
- Decoherence and charge fractionalization



## Input channel 1 noise

$$Q_1 = N_e - N_h \quad \overline{\langle \delta Q_1^2 \rangle} = 0 \quad S_{11}(\omega = 0) = 0$$

AC source is noise free at DC

## DC noise correlations at outputs

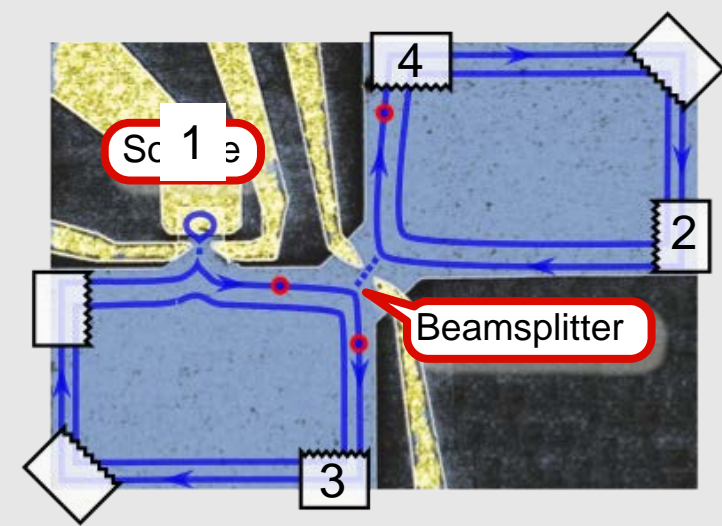
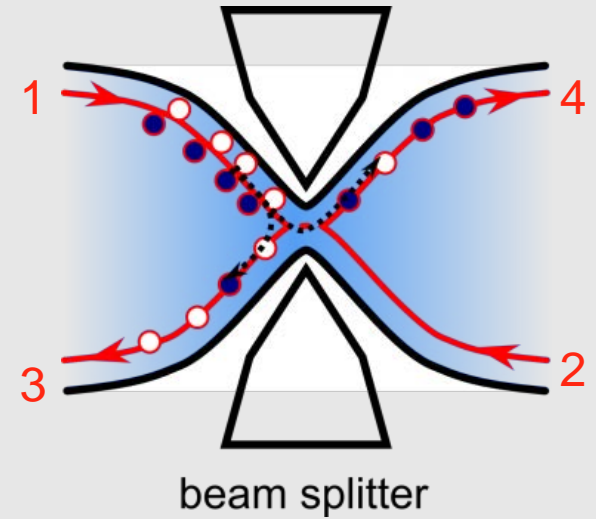
$$\overline{\langle \delta Q_3 \delta Q_4 \rangle} = -e^2 T(1 - T) \times (\langle N_e \rangle + \langle N_h \rangle)$$

Measures the average number of electron-hole pairs

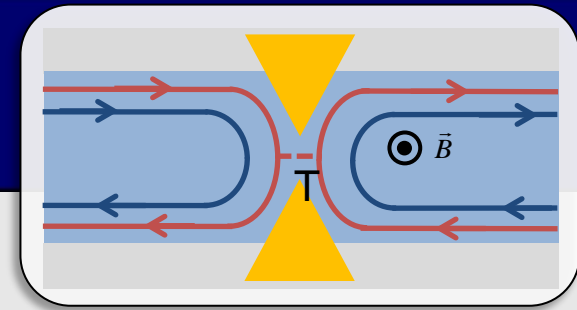
## Low frequency noise spectrum

$$S_{33}(\omega = 0) = -S_{34}(\omega = 0) = 4e^2 f T(1 - T) \times \langle N_{e/h} \rangle$$

Enough to measure noise at output 3

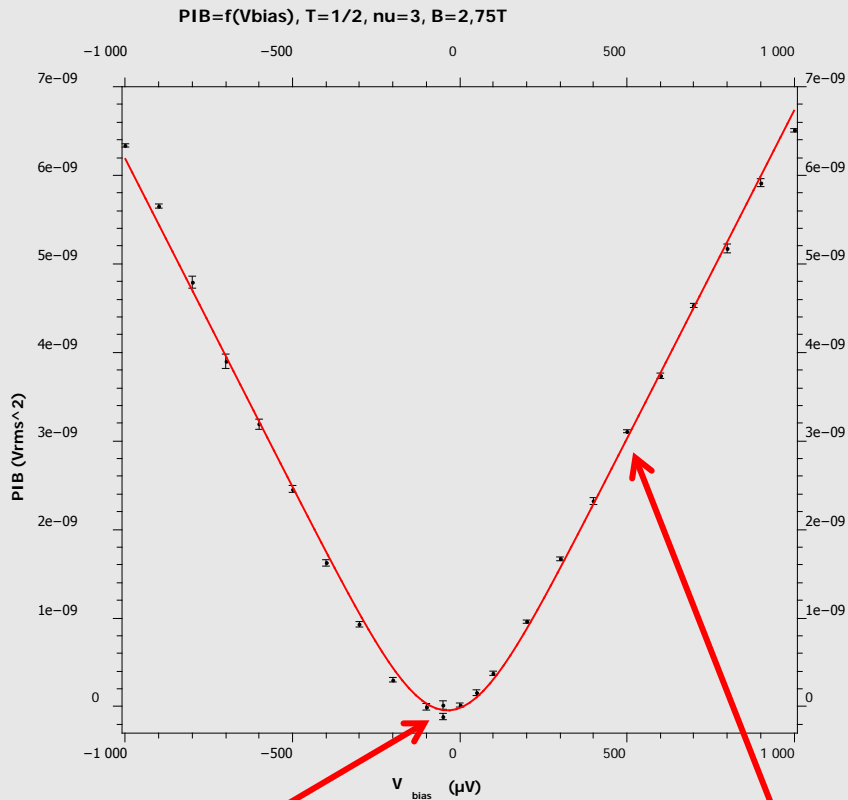






## Thermal crossover

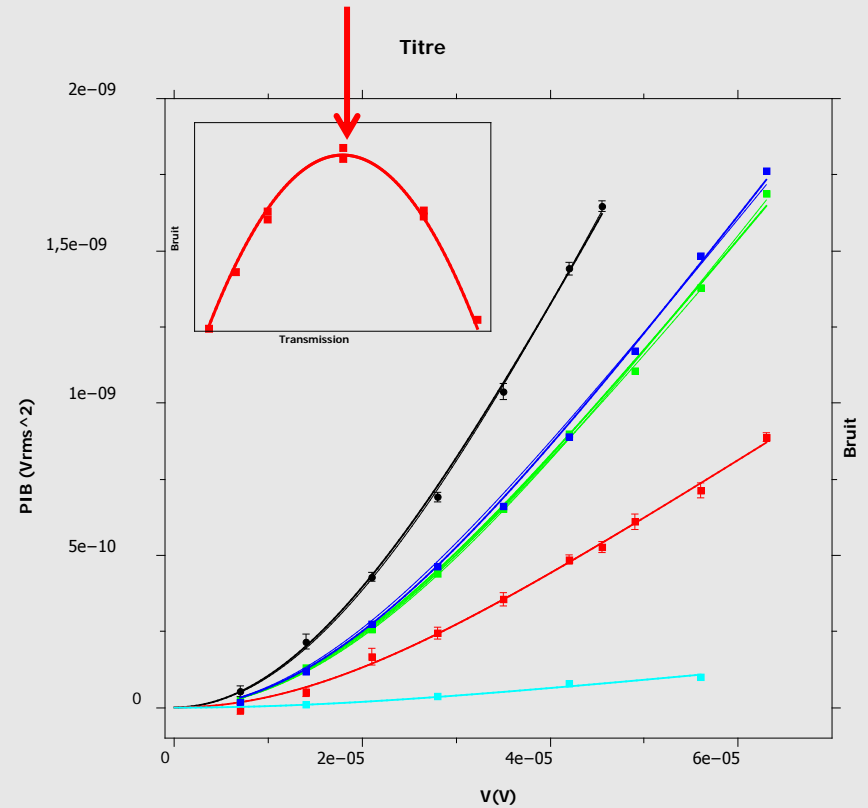
$$S_I = 2eI \tanh^{-1} \frac{eV}{2k\theta_e}$$



Thermal noise

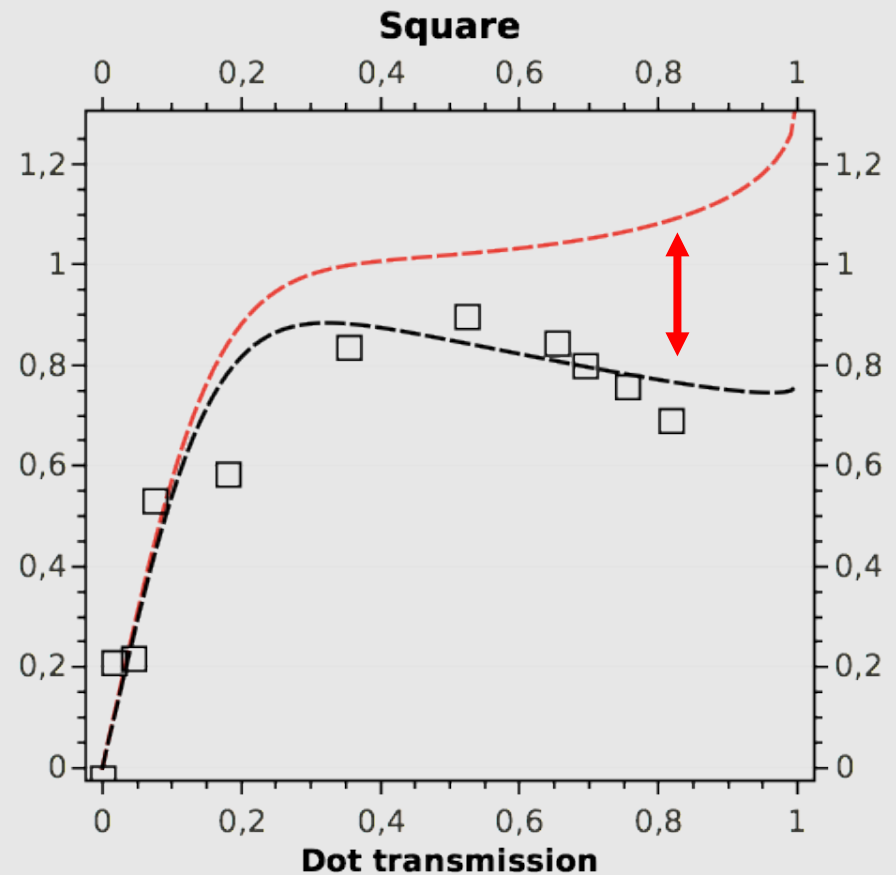
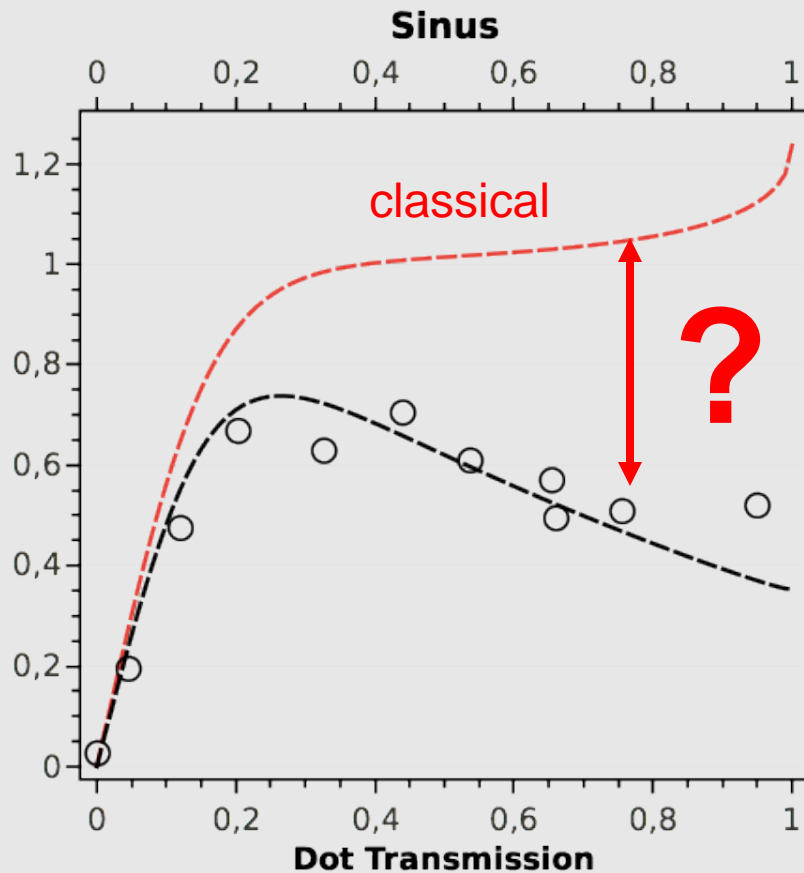
partition noise

partition noise :  $S_I \propto T(1 - T)$

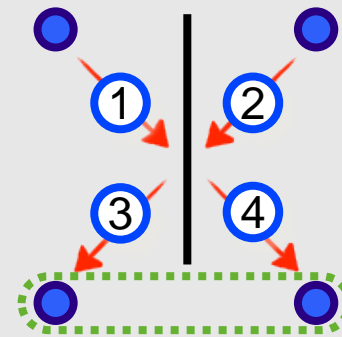
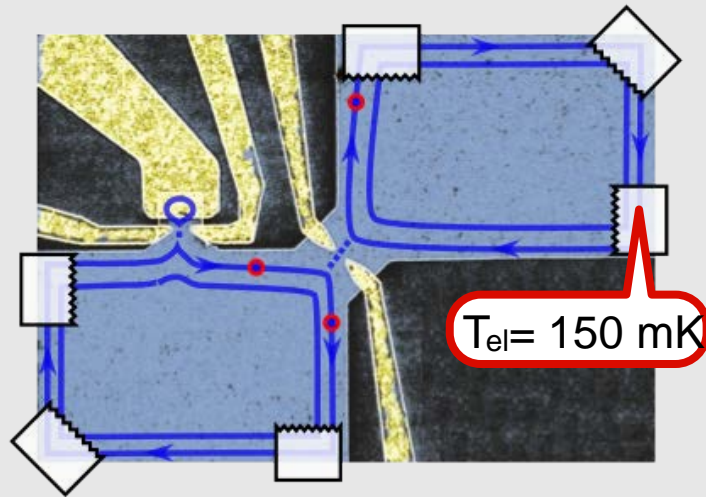


Bruit

HBT noise is smaller than classical prediction, why ????



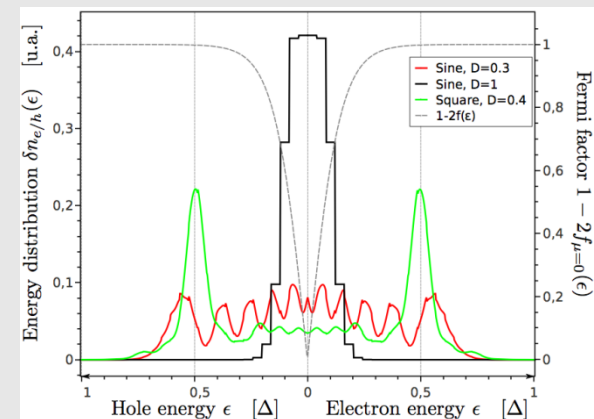
Antibunching with thermal excitations emitted by auxiliary entry 4 !!!!

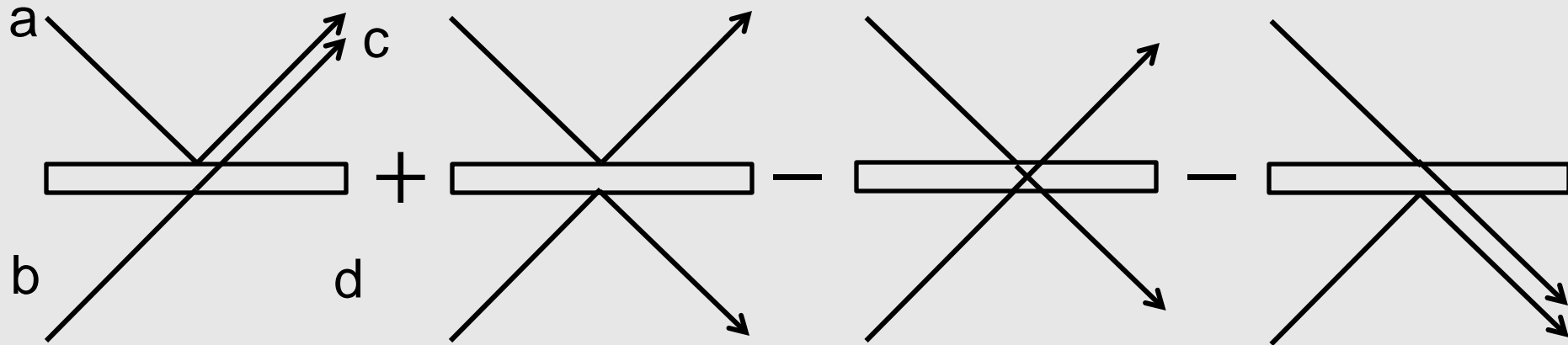


fermions

$$\delta N_{HBT} = \frac{\langle N_e \rangle + \langle N_h \rangle}{2} - \int_0^\infty d\epsilon [\delta n_e(\epsilon) + \delta n(\epsilon)] f(\epsilon)$$

$$\langle N_{e,h} \rangle = \int_0^\infty d\epsilon \delta n_{e,h}(\epsilon)$$



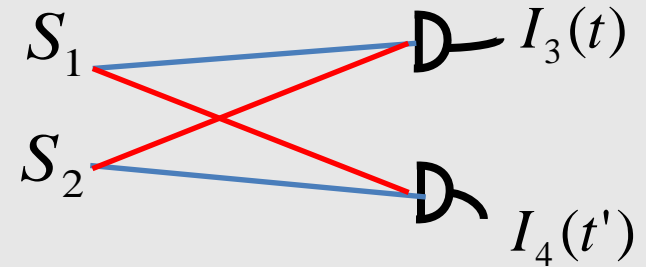
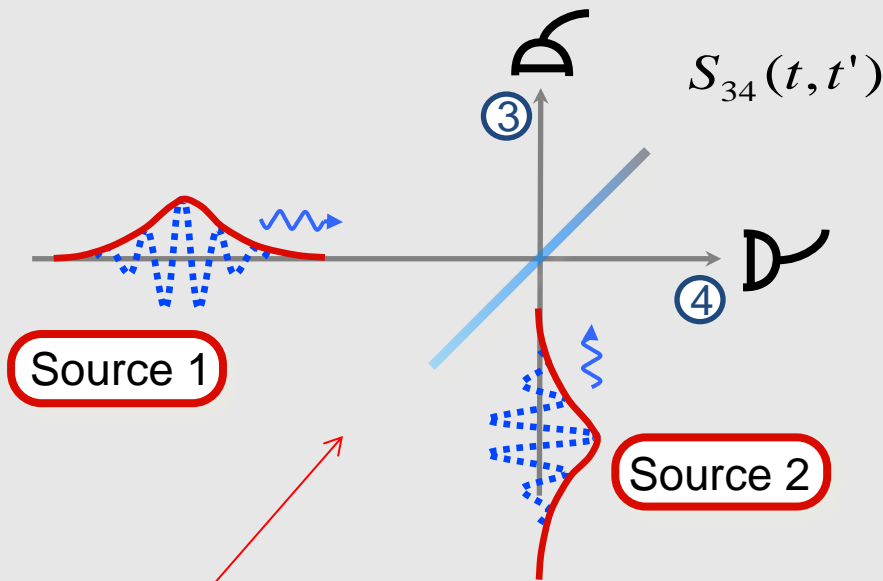


Splitter scattering matrix :  $\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} c \\ d \end{pmatrix}$

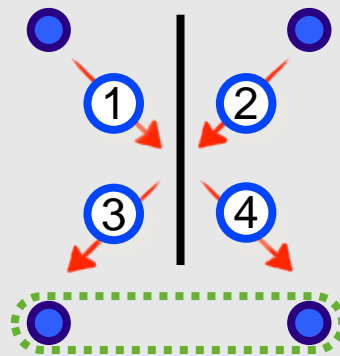
$$a^+ b^+ |0,0\rangle_{ab} = \frac{1}{2} (c^+ + d^+) (c^+ - d^+) |0,0\rangle_{cd}$$

bosons (destructive 2 – part. interference) :  $a^+ b^+ |0,0\rangle_{ab} = \frac{1}{2} (|2,0\rangle_{cd} - |0,2\rangle_{cd})$

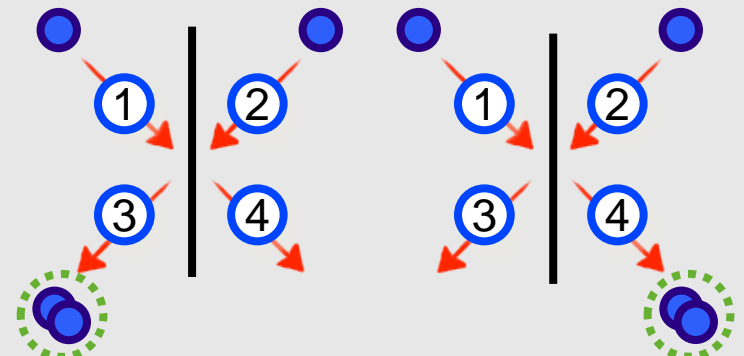
fermions (constructive 2 – part. interference):  $a^+ b^+ |0,0\rangle_{ab} = |1,1\rangle_{cd}$



Hanbury Brown & Twiss experiment

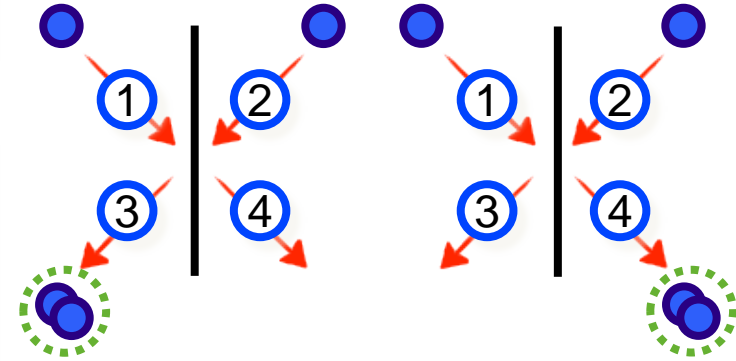
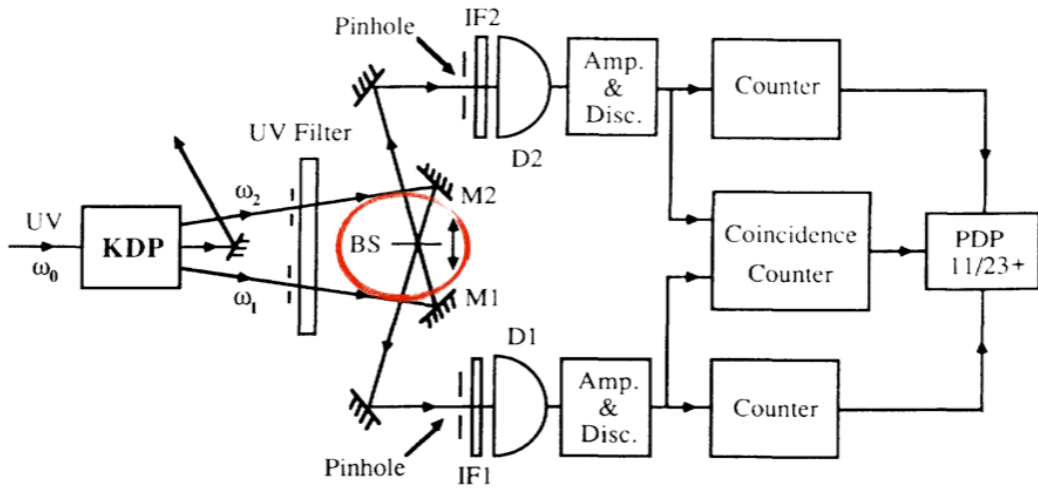


antibunching

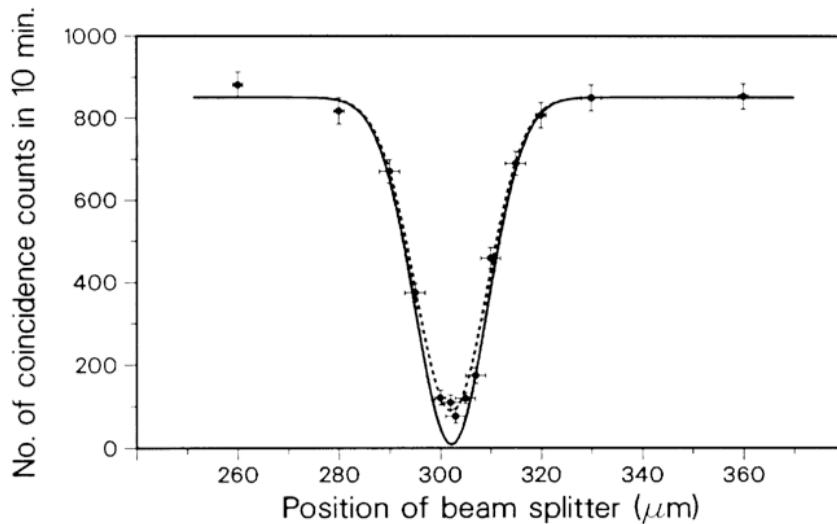


bunching

# How indistinguishable are photons ?



Undistinguishable photons



**Photons pairs :**

C. Hong *et al.*, PRL **59**(18), 2044 (1987)

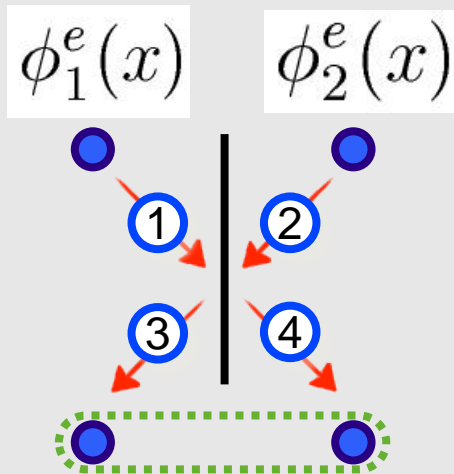
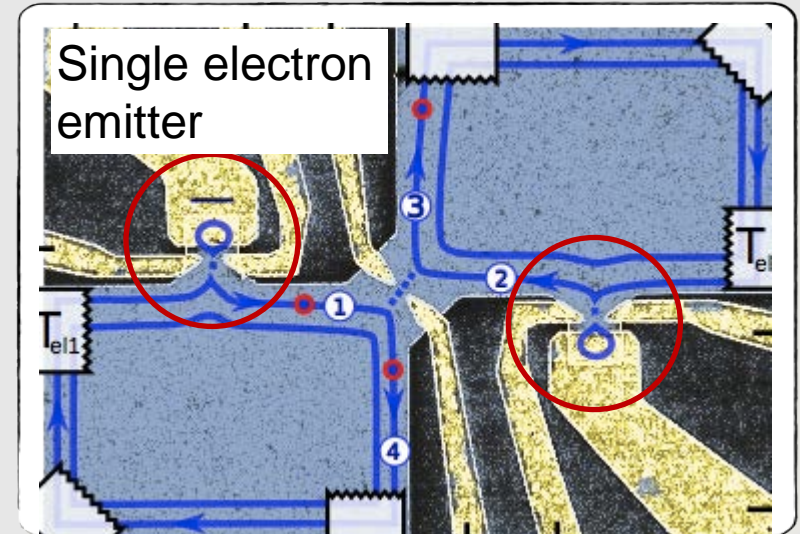
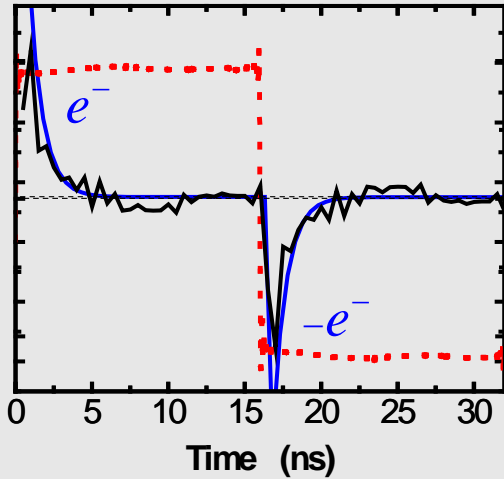
**Independent emitters :**

J. Beugnon *et al.*, Nature **440**, 779 (2006)

P. Maunz *et al.*, Nature Physics **3**, 538 (2007)

E. B. Flagg *et al.*, PRL **104**, 137401 (2010)

C. Lang *et al.*, Nature Physics **9**, 345 (2013)



*From joint probability :*

$$P(1,1) = \frac{1}{2} [1 + |\langle \varphi_1^e | \varphi_2^e \rangle|^2]$$

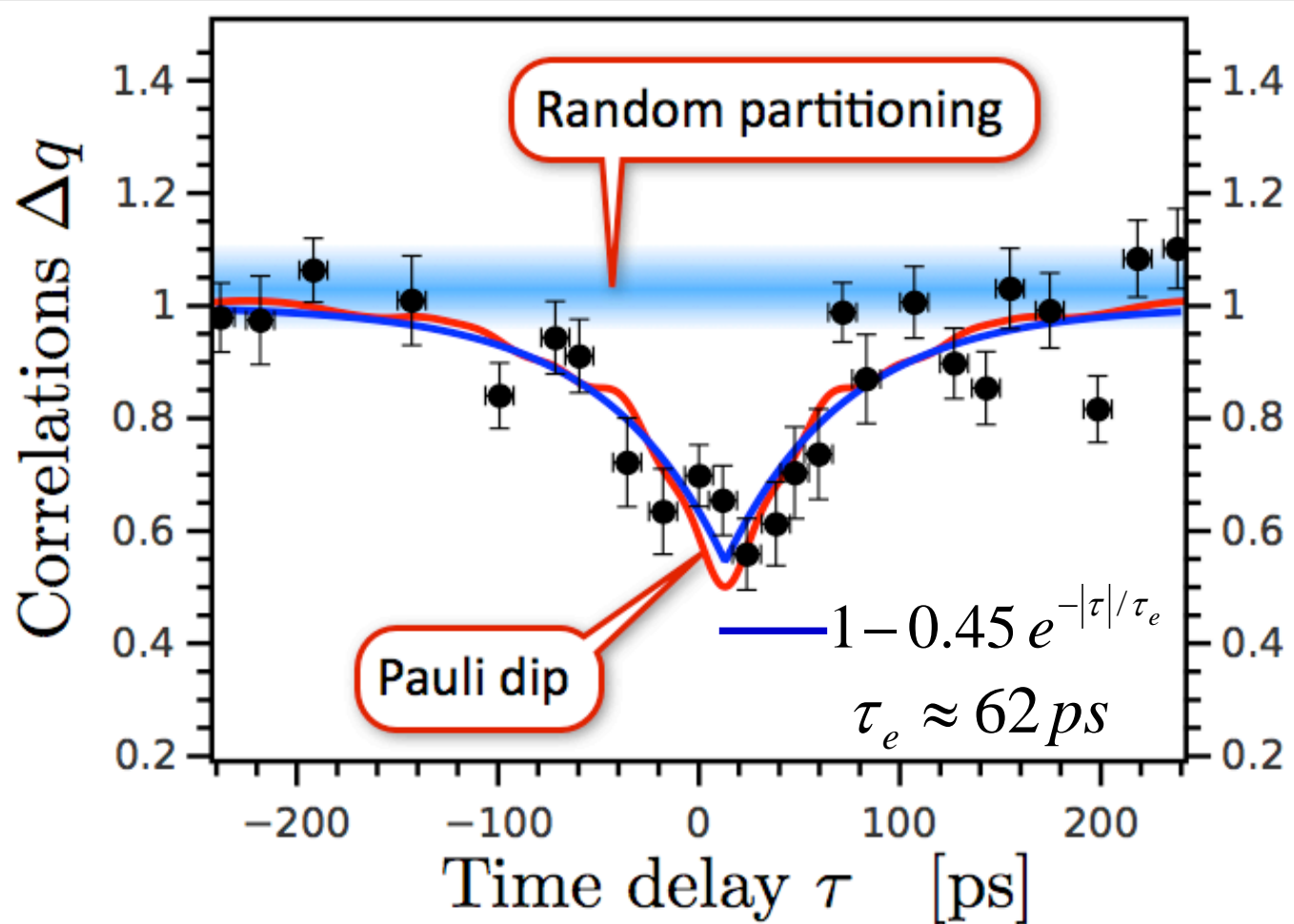
*To normalized noise :*

$$\begin{aligned} \frac{S_{HOM}}{S_{HBT}} &= 1 - \left| \int dt \varphi_1(t + \tau) \varphi_2^*(t) \right|^2 \\ &= 1 - e^{-|\tau|/\tau_e} \end{aligned}$$

First order coherence

$$D_1 = D_2 \approx 0.4$$

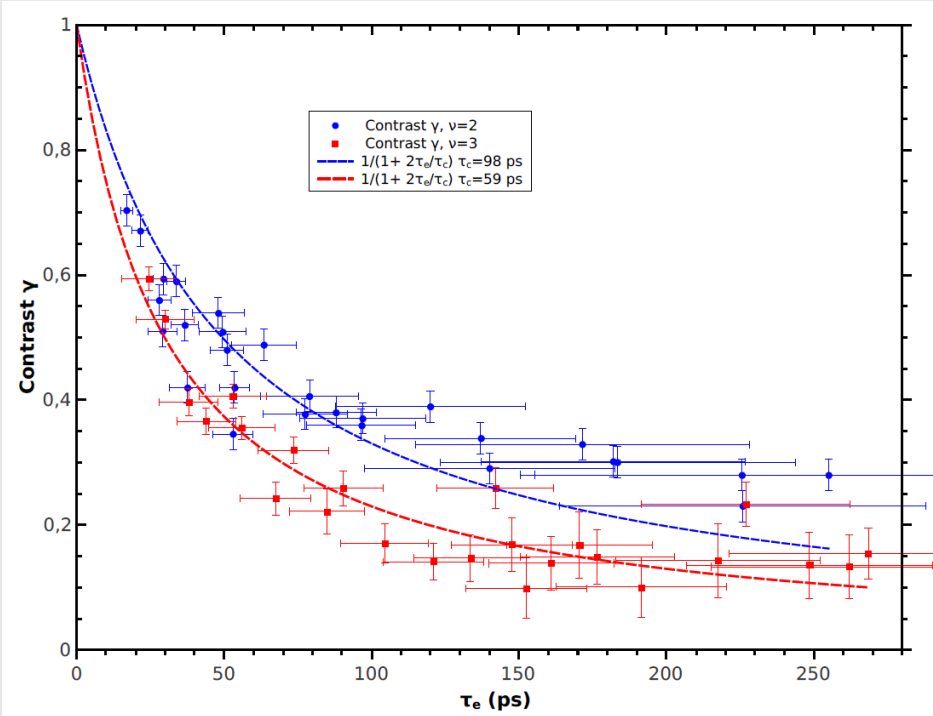
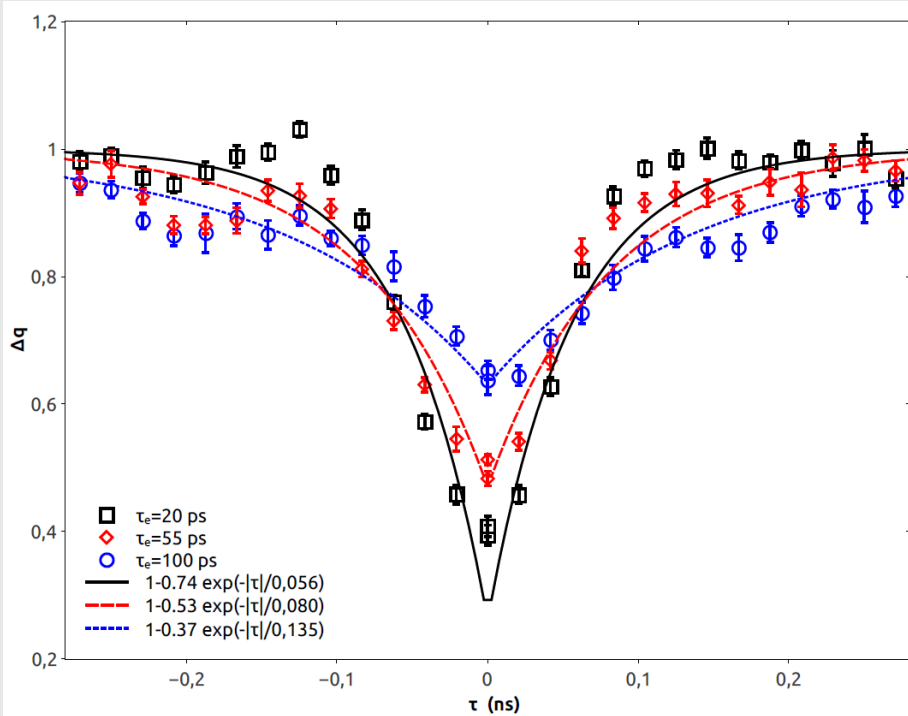
$$\tau_e \approx 58 \text{ ps}$$





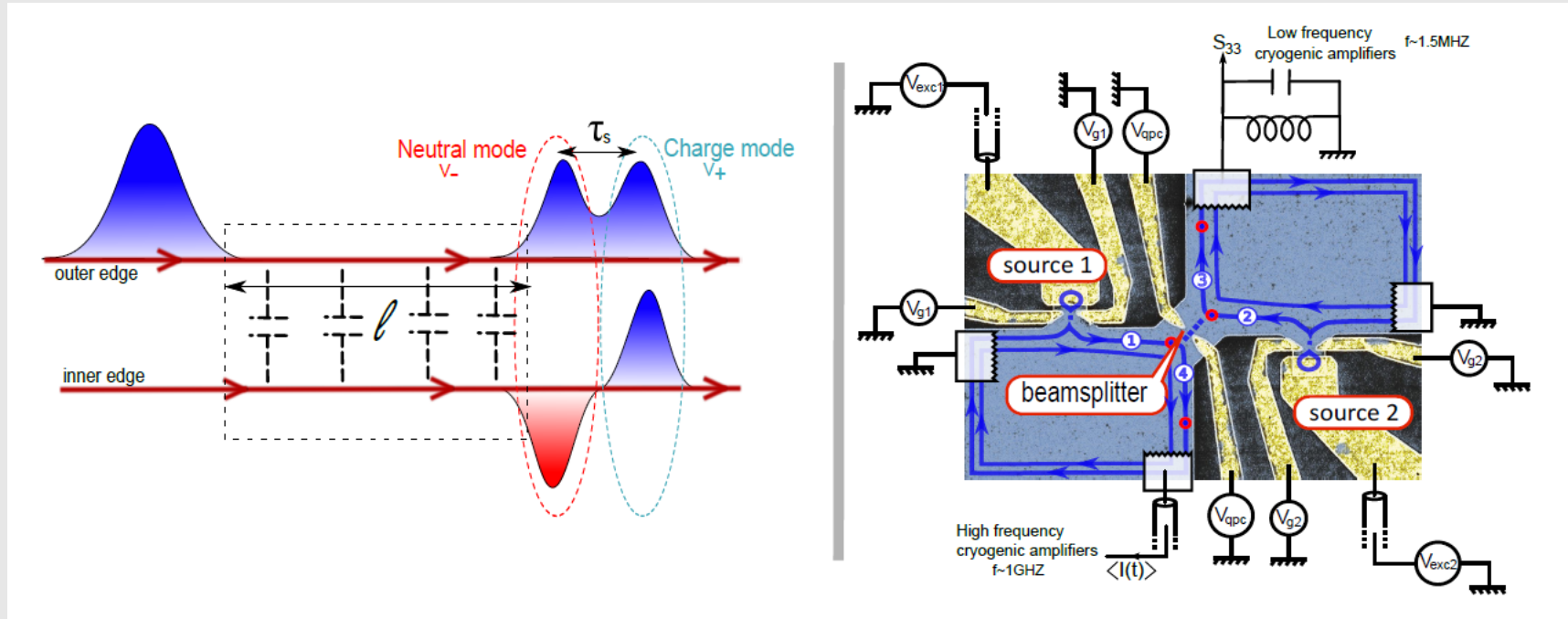
**!! HOM experiment combines particle and wave aspects !!**

$$\frac{S_{HOM}}{S_{HBT}} = 1 - \left| \int dt \varphi_1(t + \tau) D(\tau) \varphi_2^*(t) \right|^2 \approx (1 - e^{-|\tau|/\tau_e}) \times \left( \frac{1}{1 + 2\tau_e/\tau_\varphi} \right)$$



# Autopsy of the quasi-particle (QP) HOM in the environment

*QPs are not eigenstates in 1D; they degrade into collective  $e - h$  excitations*



*Input of the interaction region : single electron state in outer channel*

$$|\psi_{in}\rangle = \left( \int dx \varphi(x) \otimes |\lambda(x)\rangle_{outer} \right) \otimes |0\rangle_{inner}$$

*Output : the electron gets entangled with environment  $e - h$  excitations*

$$|\psi_{out}\rangle = \int dx \varphi(x) \otimes (|t \times \lambda(x)\rangle_{outer}) \otimes (|r \times \lambda(x)\rangle_{inner})$$

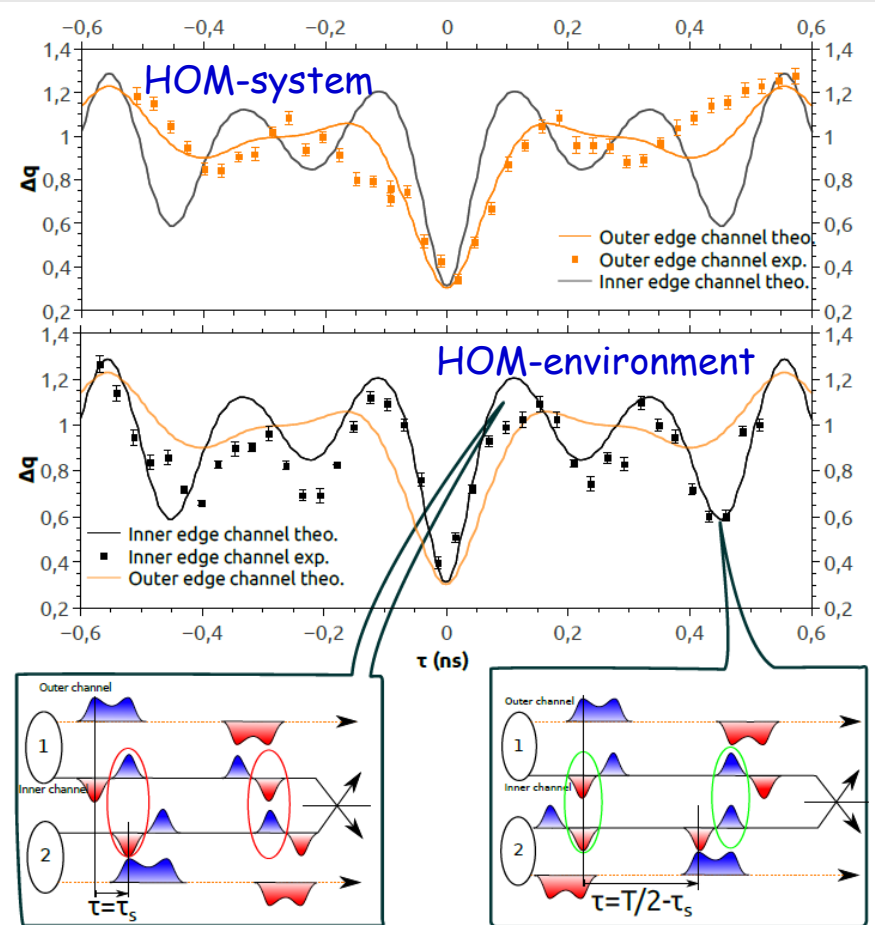
# HOM experiment in the environment evidence for electron-fractionalization

*Output : the electron gets entangled with environment excitations*

$$|\psi_{out}\rangle = \int dx \varphi(x) \otimes (|t \times \lambda(x)\rangle_{outer}) \otimes |r \times \lambda(x)\rangle_{inner}$$

*→ electron – hole antidips.*

*→ electron – electron satellites*



- Is single particle description relevant ? (YES, to some extent !)
- Learn about particle statistics and coherence
- Role of the Fermi sea
- Many-body effects

### perspectives

- Quantum tomography of the single particle states
- Implementation in other systems (graphene ? TI's ?)
- Use for quantum information (Flying qubits ?)