Optical Flux Lattices for Cold Atom Gases

Nigel Cooper
Cavendish Laboratory, University of Cambridge

Artificial Magnetism for Cold Atom Gases
Collège de France, 11 June 2014

Jean Dalibard (Collège de France)
Roderich Moessner (MPIPKS Dresden)

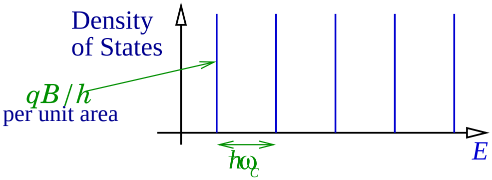
EPSRC
Engineering and Physical Sciences
Research Council

THE ROYAL SOCIETY

Motivation: fractional quantum Hall regime

2D charged particle in magnetic field ⇒ Landau levels

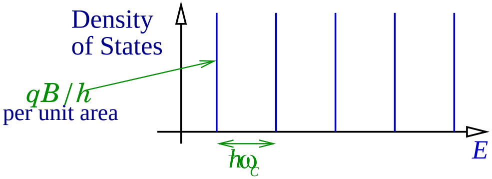
Flux density $n_\phi = \dfrac{qB}{h}$



Motivation: fractional quantum Hall regime

2D charged particle in magnetic field $\Rightarrow$ Landau levels

Flux density $n_\phi = \dfrac{qB}{h}$

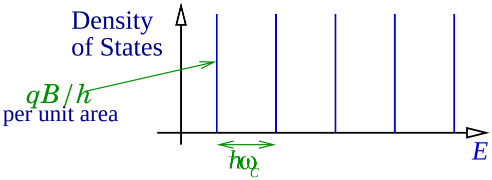


e-e repulsion $\Rightarrow$ fractional quantum Hall states, at certain $\nu \equiv \dfrac{n_{2\mathrm{D}}}{n_\phi}$

Motivation: fractional quantum Hall regime

2D charged particle in magnetic field ⇒Landau levels

Flux density $n_\phi = \dfrac{qB}{h}$



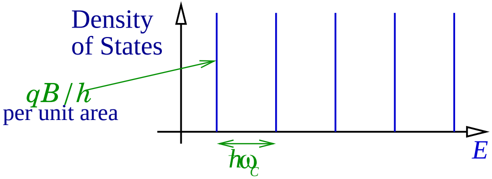
e-e repulsion ⇒fractional quantum Hall states, at certain $\nu \equiv \dfrac{n_{2D}}{n_\phi}$

Bosons? (contact repulsion)

Motivation: fractional quantum Hall regime

2D charged particle in magnetic field ⇒Landau levels

Flux density $n_\phi = \frac{qB}{h}$



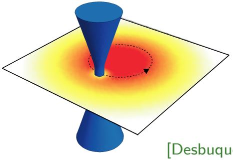
e-e repulsion ⇒fractional quantum Hall states, at certain $\nu \equiv \frac{n_{2D}}{n_\phi}$

Bosons? (contact repulsion)

- $\nu > \nu_c \simeq 6$: vortex lattice (BEC)
  [NRC, Wilkin & Gunn '01; Sinova, Hanna & MacDonald '02; Baym '04]
- $\nu < \nu_c$: FQH states (incl. "non-Abelian") [NRC, Wilkin & Gunn '01]

Synthetic Magnetic Field: Rotation

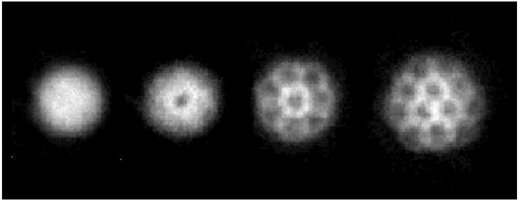
Stir the atomic gas



[Desbuquois *et al.* (2012)]

Rotating frame, angular velocity $\boldsymbol{\Omega}$

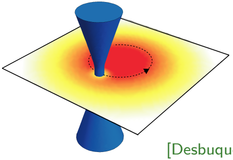
Coriolis Force ⇔ Lorentz Force $n_\phi \equiv \frac{qB}{h} = \frac{2M\Omega}{h}$



[K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000)]

Synthetic Magnetic Field: Rotation

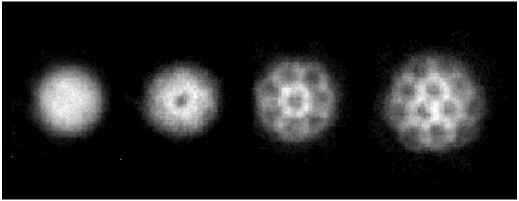
Stir the atomic gas



Rotating frame, angular velocity $\boldsymbol{\Omega}$

[Desbuquois *et al.* (2012)]

Coriolis Force $\Leftrightarrow$ Lorentz Force $\quad n_\phi \equiv \frac{qB}{h} = \frac{2M\Omega}{h}$

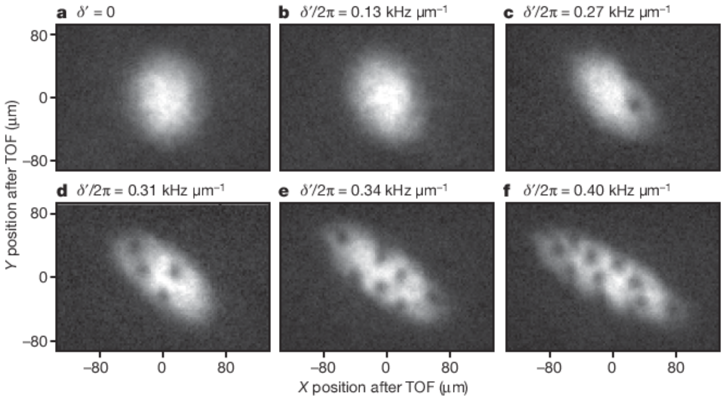


$\Omega \lesssim 2\pi \times 100\text{Hz}$
$\Rightarrow n_\phi \lesssim 2 \times 10^7 \text{cm}^{-2}$

[K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000)]

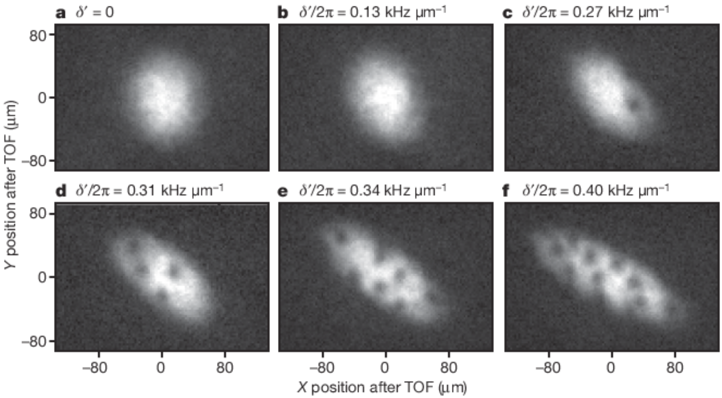
Synthetic Magnetic Field: Optically Dressed States

[Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto & I.B. Spielman, Nature **462**, 628 (2009)]



Synthetic Magnetic Field: Optically Dressed States

[Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto & I.B. Spielman, Nature **462**, 628 (2009)]



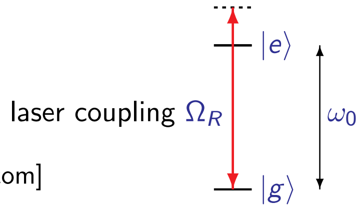
But... $n_\phi \lesssim \frac{1}{R\lambda} \sim 2 \times 10^7 \mathrm{cm}^{-2}$ [R cloud size]

Outline

Optically Dressed States

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, RMP **83**, 1523 (2011)]

Coherent optical coupling of N internal atomic states



[e.g. $^1S_0$ and $^3P_0$ for Yb or alkaline earth atom]

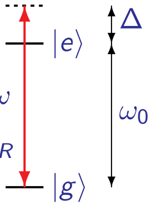
Forms the local “dressed state” of the atom

$$|0_{\mathbf{r}}\rangle = \alpha_{\mathbf{r}}|g\rangle + \beta_{\mathbf{r}}|e\rangle = \begin{pmatrix} \alpha_{\mathbf{r}} \\ \beta_{\mathbf{r}} \end{pmatrix}$$

optical coupling $\hat{V}(\mathbf{r})$

laser frequency ω

coupling strength Ω_R



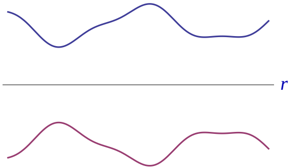
Rotating Wave Approximation $\omega \gg \Delta, \Omega_R$

$$\hat{V}(\mathbf{r}) \to \frac{\hbar}{2} \begin{pmatrix} \Delta & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta \end{pmatrix}$$

In general $\hat{V} = \frac{\hbar}{2}\begin{pmatrix} \Delta(\mathbf{r}) & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta(\mathbf{r}) \end{pmatrix}$

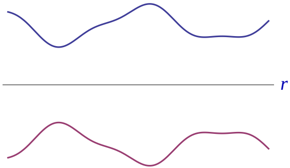
$E_1(\mathbf{r}),\ |1_{\mathbf{r}}\rangle$

$E_0(\mathbf{r}),\ |0_{\mathbf{r}}\rangle$



In general $\hat{V} = \frac{\hbar}{2}\begin{pmatrix} \Delta(\mathbf{r}) & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta(\mathbf{r}) \end{pmatrix}$

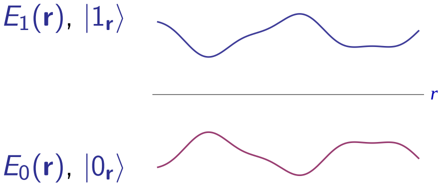
$E_1(\mathbf{r})$, $|1_\mathbf{r}\rangle$



$E_0(\mathbf{r})$, $|0_\mathbf{r}\rangle$

Adiabatic motion in lower energy dressed state, $|\Psi\rangle = \psi_0(\mathbf{r})|0_\mathbf{r}\rangle$

In general $\hat{V} = \frac{\hbar}{2}\begin{pmatrix} \Delta(\mathbf{r}) & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta(\mathbf{r}) \end{pmatrix}$



Adiabatic motion in lower energy dressed state, $|\Psi\rangle = \psi_0(\mathbf{r})|0_\mathbf{r}\rangle$

$$H_{\rm eff}\psi_0 = \langle 0_\mathbf{r}|\left[\frac{\mathbf{p}^2}{2M} + \hat{V}\right]\psi_0|0_\mathbf{r}\rangle \Rightarrow H_{\rm eff} = \frac{(\mathbf{p}-q\mathbf{A})^2}{2M} + V_0(\mathbf{r})$$

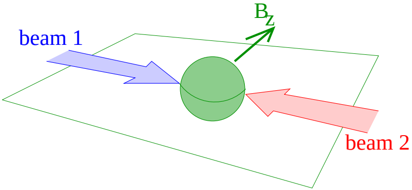
[“Berry connection” $\Rightarrow$ vector potential $q\mathbf{A} = i\hbar\langle 0_\mathbf{r}|\boldsymbol{\nabla} 0_\mathbf{r}\rangle$]

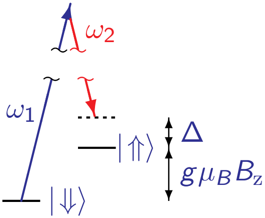
[J. Dalibard, F. Gerbier, G. Juzeliūnas & P. Öhberg, RMP **83**, 1523 (2011)]

Experimental Implementation

Rubidium BEC

[Lin, Compton, Jiménez-García, Porto & Spielman, Nature **462**, 628 (2009)]





$$\Delta k = k_1 - (-k_2) \simeq 2 \times \frac{2\pi}{\lambda}$$

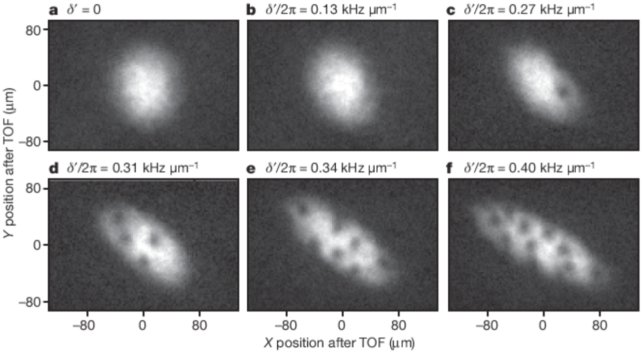
$$\hat{V} = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega_R e^{-i\Delta k\, x} \\ \Omega_R e^{i\Delta k\, x} & -\Delta \end{pmatrix}$$

field gradient $B_z(y) \Rightarrow \Delta \propto y$

Experimental Implementation

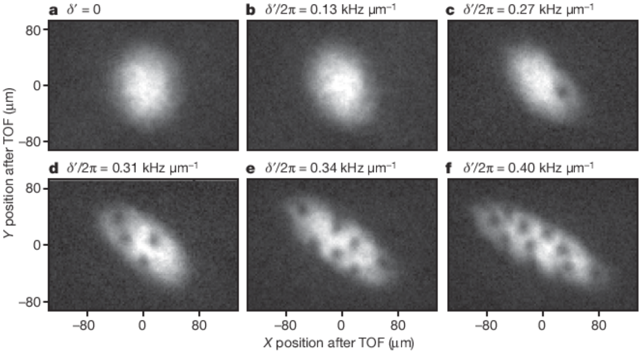
Rubidium BEC

[Lin, Compton, Jiménez-García, Porto & Spielman, Nature **462**, 628 (2009)]



Experimental Implementation

Rubidium BEC [Lin, Compton, Jiménez-García, Porto & Spielman, Nature **462**, 628 (2009)]



But... $n_\phi \lesssim \frac{1}{R\lambda} \sim 2 \times 10^7 \text{cm}^{-2}$ [R cloud size]

Maximum flux density: Back of the envelope

Vector potential $q\mathbf{A} = i\hbar\langle 0_{\mathbf{r}}|\boldsymbol{\nabla} 0_{\mathbf{r}}\rangle \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$

Cloud of radius $R \gg \lambda$

$$N_\phi \equiv \int n_\phi \, d^2 r = \frac{q}{h}\int \boldsymbol{\nabla}\times\mathbf{A}\cdot d\mathbf{S} = \frac{q}{h}\oint \mathbf{A}\cdot d\mathbf{r} \lesssim \frac{1}{\lambda}(2\pi R)$$

$$\Rightarrow \bar{n}_\phi \equiv \frac{N_\phi}{\pi R^2} \lesssim \frac{1}{R\lambda} \simeq 2\times 10^7 \mathrm{cm}^{-2} \qquad [R \simeq 10\mu\mathrm{m} \;\; \lambda \simeq 0.5\mu\mathrm{m}]$$

Maximum flux density: Carefully this time!

~~Vector potential $q\mathbf{A} = i\hbar\langle 0_{\mathbf{r}}|\boldsymbol{\nabla} 0_{\mathbf{r}}\rangle \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$~~

$\mathbf{A}$ can have *singularities* – if the optical fields have vortices

e.g. $\Omega_R(\mathbf{r}) \sim (x + iy)$

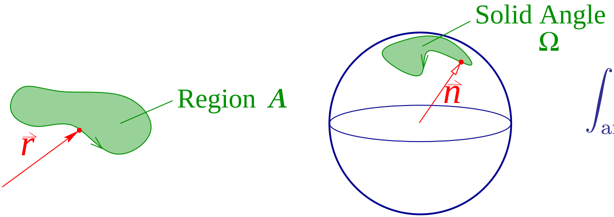
Vanishing net flux. Can be (re)moved by a gauge transformation.

[cf. "Dirac strings"]

Gauge-Independent Approach (two-level system)

Bloch vector $\vec{n}(\mathbf{r}) = \langle 0_\mathbf{r} | \hat{\vec{\sigma}} | 0_\mathbf{r} \rangle$

Flux density $n_\phi = \frac{1}{8\pi} \epsilon_{ijk}\epsilon_{\mu\nu} n_i \partial_\mu n_j \partial_\nu n_k$



$$\int_{\text{area } A} n_\phi d^2 r = \frac{\Omega}{4\pi}$$

The number of flux quanta in region A is the number of times the Bloch vector wraps over the sphere.

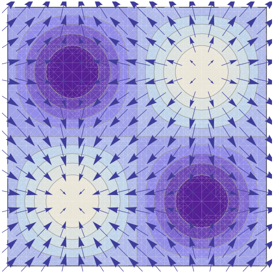
"Optical flux lattices"

[NRC, Phys. Rev. Lett. **106**, 175301 (2011)]

Spatially periodic light fields for which the Bloch vector wraps the sphere a nonzero integer number, N_ϕ, times in each unit cell.

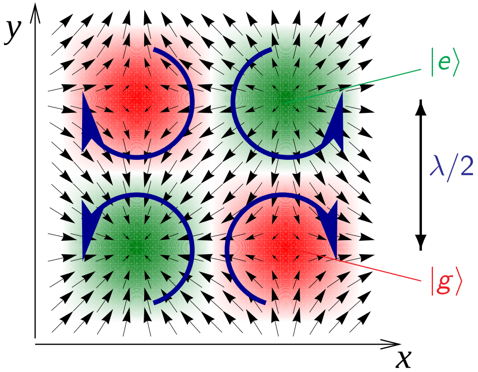
$$\bar{n}_\phi = \frac{N_\phi}{A_{\rm cell}} \sim \frac{1}{\lambda^2} \simeq 10^9 \mathrm{cm}^{-2}$$

vectors (n_x, n_y)
contours n_z
$N_\phi = 2$

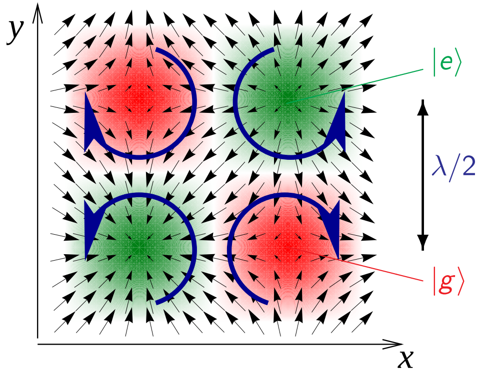


$$\hat{V} = \mathcal{V} \begin{pmatrix} \sin\kappa x \sin\kappa y & \cos\kappa x - i\cos\kappa y \\ \cos\kappa x + i\cos\kappa y & -\sin\kappa x \sin\kappa y \end{pmatrix}$$

Lorentz Force: Semiclassical Picture

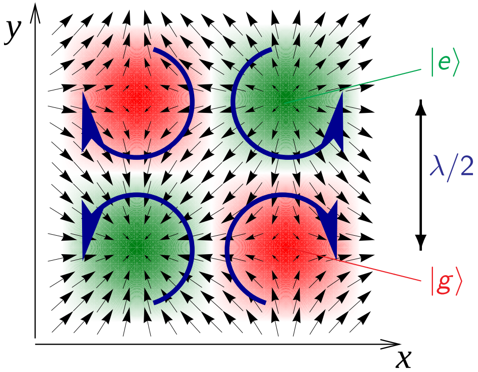


Lorentz Force: Semiclassical Picture



$$p_x \sim (\hbar k)\frac{y}{\lambda/2} \quad \Rightarrow \quad F_x \equiv \dot{p}_x \sim \frac{\hbar k}{\lambda/2} v_y$$

Lorentz Force: Semiclassical Picture

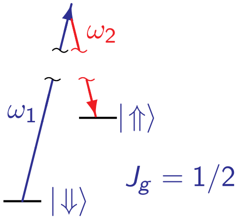


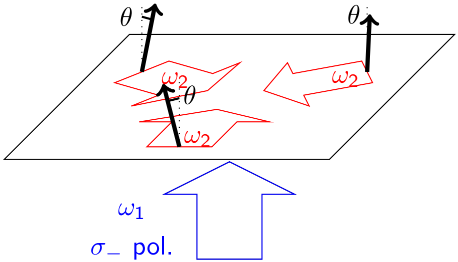
$$p_x \sim (\hbar k)\frac{y}{\lambda/2} \quad \Rightarrow \quad F_x \equiv \dot{p}_x \sim \frac{\hbar k}{\lambda/2} v_y$$

Lorentz force, with $qB \sim \frac{2h}{\lambda^2} \quad \Rightarrow \quad n_\phi \equiv \frac{qB}{h} \sim \frac{2}{\lambda^2}$

Example: Implementation for Hyperfine Levels

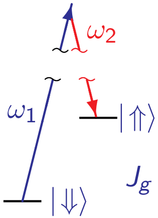
[NRC & Jean Dalibard, EPL **95**, 66004 (2011)]



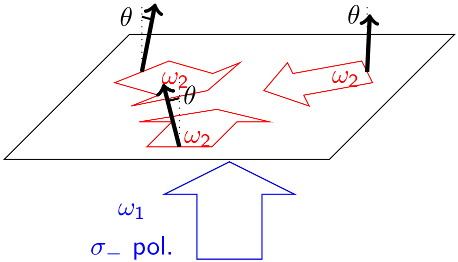


Example: Implementation for Hyperfine Levels

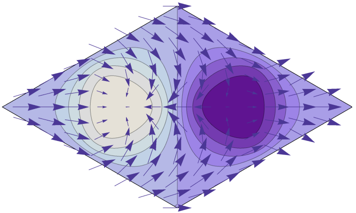
[NRC & Jean Dalibard, EPL **95**, 66004 (2011)]



$J_g = 1/2$



$$\hat{V} \propto \begin{pmatrix} |\mathcal{E}_2^+|^2 - |\mathcal{E}_2^-|^2 & \mathcal{E}_2^z \, \mathcal{E}_1^{-\,*} \\ \mathcal{E}_2^{z\,*} \mathcal{E}_1^- & -|\mathcal{E}_2^+|^2 + |\mathcal{E}_2^-|^2 \end{pmatrix}$$

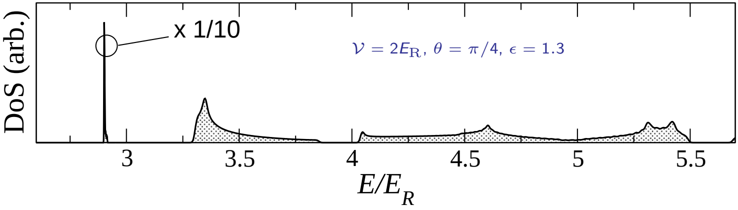


$N_\phi = 1$ (two level system)

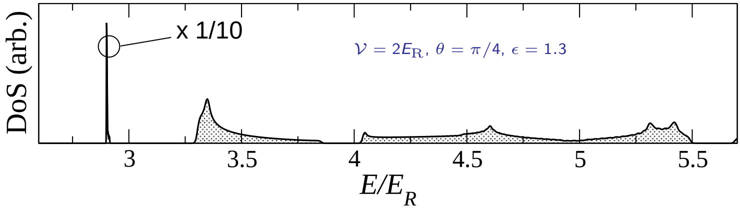
Bandstructure: $\frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$

Bandstructure: $\frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$

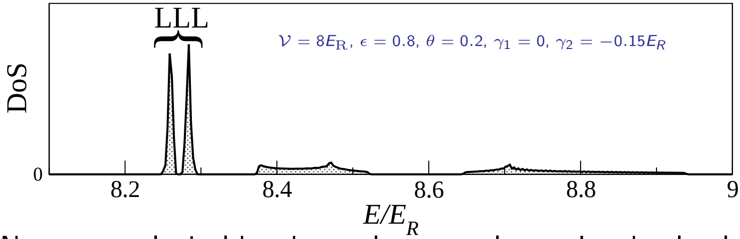
$J_g = 1/2$ (e.g. ^{171}Yb, ^{199}Hg, ^{6}Li) $\quad [E_R = \frac{h^2}{2M\lambda^2}]$



$J_g = 1/2$ (e.g. ^{171}Yb, ^{199}Hg, ^{6}Li) $\quad [E_R = \frac{h^2}{2M\lambda^2}]$



$J_g = 1$ (e.g. ^{23}Na, ^{39}K, ^{87}Rb)



- Narrow topological bands: analogous to lowest Landau level

Designing Optical Flux Lattices

[NRC & Roderich Moessner, PRL **109** 215302 (2012)]

Designing Optical Flux Lattices

[NRC & Roderich Moessner, PRL **109** 215302 (2012)]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

Optical lattices are conveniently defined in reciprocal space

Couplings $V^{\alpha'\alpha}_{\mathbf{k}'-\mathbf{k}} \equiv \langle \alpha', \mathbf{k}' | \hat{V} | \alpha, \mathbf{k} \rangle$ of internal states $\alpha = 1, \ldots, N$

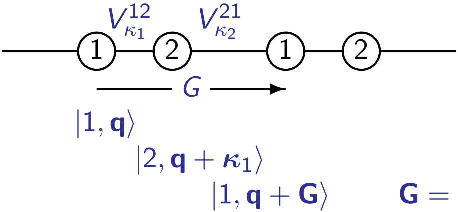
Designing Optical Flux Lattices

[NRC & Roderich Moessner, PRL **109** 215302 (2012)]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

Optical lattices are conveniently defined in reciprocal space

Couplings $V^{\alpha'\alpha}_{\mathbf{k}'-\mathbf{k}} \equiv \langle \alpha', \mathbf{k}' | \hat{V} | \alpha, \mathbf{k} \rangle$ of internal states $\alpha = 1, \ldots, N$



Reciprocal lattice $\{\mathbf{G}\}$

$$\mathbf{G} = \boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2$$

Bloch's theorem $|\psi^{n\mathbf{q}}\rangle = \sum_{\alpha,\mathbf{G}} c^{n\mathbf{q}}_{\alpha\mathbf{G}} |\alpha, \mathbf{q} + \mathbf{g}_\alpha + \mathbf{G}\rangle$

$$E_{n\mathbf{q}} c^{n\mathbf{q}}_{\alpha\mathbf{G}} = \frac{\hbar^2|\mathbf{q}+\mathbf{G}+\mathbf{g}_\alpha|^2}{2M} c^{n\mathbf{q}}_{\alpha\mathbf{G}} + \sum_{\alpha',\mathbf{G}'} V^{\alpha\alpha'}_{\mathbf{G}+\mathbf{g}_\alpha-\mathbf{G}'-\mathbf{g}_{\alpha'}} c^{n\mathbf{q}}_{\alpha'\mathbf{G}'}$$

Bloch's theorem $|\psi^{n\mathbf{q}}\rangle = \sum_{\alpha,\mathbf{G}} c^{n\mathbf{q}}_{\alpha\mathbf{G}} |\alpha, \mathbf{q} + \mathbf{g}_\alpha + \mathbf{G}\rangle$

$$E_{n\mathbf{q}} c^{n\mathbf{q}}_{\alpha\mathbf{G}} = \frac{\hbar^2 |\mathbf{q} + \mathbf{G} + \mathbf{g}_\alpha|^2}{2M} c^{n\mathbf{q}}_{\alpha\mathbf{G}} + \sum_{\alpha',\mathbf{G}'} V^{\alpha\alpha'}_{\mathbf{G}+\mathbf{g}_\alpha - \mathbf{G}' - \mathbf{g}_{\alpha'}} c^{n\mathbf{q}}_{\alpha'\mathbf{G}'}$$

Adiabatic limit (K.E. $\to 0$)

$$E_{n\mathbf{q}} c^{n\mathbf{q}}_{\alpha\mathbf{G}} = \sum_{\alpha',\mathbf{G}'} V^{\alpha\alpha'}_{\mathbf{G}+\mathbf{g}_\alpha - \mathbf{G}' - \mathbf{g}_{\alpha'}} c^{n\mathbf{q}}_{\alpha'\mathbf{G}'}$$

Tight-binding model in reciprocal space

Tight-binding model in reciprocal space

$$E_{n\mathbf{q}} c^{n\mathbf{q}}_{\alpha\mathbf{G}} = \sum_{\alpha',\mathbf{G}'} V^{\alpha\alpha'}_{\mathbf{G}+\mathbf{g}_\alpha-\mathbf{G}'-\mathbf{g}_{\alpha'}} c^{n\mathbf{q}}_{\alpha'\mathbf{G}'}$$

Bandstructure determines the dressed states in *real space*

$[\hat{H} = \hat{V}(\mathbf{r})]$

Tight-binding model in reciprocal space

$$E_{n\mathbf{q}} c^{n\mathbf{q}}_{\alpha\mathbf{G}} = \sum_{\alpha',\mathbf{G}'} V^{\alpha\alpha'}_{\mathbf{G}+\mathbf{g}_\alpha-\mathbf{G}'-\mathbf{g}_{\alpha'}} c^{n\mathbf{q}}_{\alpha'\mathbf{G}'}$$

Bandstructure determines the dressed states in *real space*

$[\hat{H} = \hat{V}(\mathbf{r})]$

conserved "momentum"	$\Leftrightarrow$	real space position, $\mathbf{r}$
Brillouin zone	$\Leftrightarrow$	real space unit cell
Bloch wavefunction	$\Leftrightarrow$	dressed state, $\lvert n_\mathbf{r}\rangle$
band energies	$\Leftrightarrow$	local dressed state energies, $E_n(\mathbf{r})$
Berry curvature	$\Leftrightarrow$	local flux density, n_ϕ
Chern number, $\mathcal{C}$	$\Leftrightarrow$	flux through unit cell, N_ϕ

Tight-binding model in reciprocal space

$$E_{n\mathbf{q}} c^{n\mathbf{q}}_{\alpha\mathbf{G}} = \sum_{\alpha',\mathbf{G}'} V^{\alpha\alpha'}_{\mathbf{G}+\mathbf{g}_\alpha-\mathbf{G}'-\mathbf{g}_{\alpha'}} c^{n\mathbf{q}}_{\alpha'\mathbf{G}'}$$

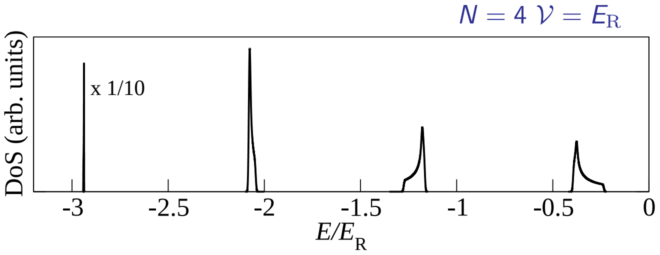
Bandstructure determines the dressed states in *real space*

$[\hat{H} = \hat{V}(\mathbf{r})]$

conserved "momentum"	$\Leftrightarrow$	real space position, $\mathbf{r}$
Brillouin zone	$\Leftrightarrow$	real space unit cell
Bloch wavefunction	$\Leftrightarrow$	dressed state, $\|n_\mathbf{r}\rangle$
band energies	$\Leftrightarrow$	local dressed state energies, $E_n(\mathbf{r})$
Berry curvature	$\Leftrightarrow$	local flux density, n_ϕ
Chern number, $\mathcal{C}$	$\Leftrightarrow$	flux through unit cell, N_ϕ

For an optical flux lattice, the lowest energy band of the reciprocal-space tight-binding model has non-zero Chern number

OFLs with uniform Magnetic Field and Scalar Potential



- Low energy spectrum closely analogous to Landau levels
- Narrow bands ⇒strongly correlated phases
- $N = 3$ scheme for ^{87}Rb shows robust Laughlin, CF/hierarchy and Moore-Read phases of bosons, even for weak two-body repulsion

[NRC & Jean Dalibard, PRL **110**, 185301 (2013)]

Outlook

- Strong magnetic field, $n_\phi \sim \dfrac{1}{\lambda^2}$
  - Novel FQH states of 2D bosons [NRC & J. Dalibard, PRL (2013)]
  - Correlated bosonic phases in 3D? [NRC, van Lankvelt, Reijnders & Schoutens, PRA '05]
  - Strong-coupling superconductivity vs. FQH, $\xi_{\rm pair} \sim \bar{a} \sim n_\phi^{-1/2}$
    [for a cuprate superconductor, would need $B \gtrsim 10^5$T!]

Outlook

- Strong magnetic field, $n_\phi \sim \frac{1}{\lambda^2}$

– Novel FQH states of 2D bosons [NRC & J. Dalibard, PRL (2013)]

– Correlated bosonic phases in 3D? [NRC, van Lankvelt, Reijnders & Schoutens, PRA '05]

– Strong-coupling superconductivity vs. FQH, $\xi_{\rm pair} \sim \bar{a} \sim n_\phi^{-1/2}$
[for a cuprate superconductor, would need $B \gtrsim 10^5$T!]

- Other Topological Bandstructures

– "Chern insulators" with $\mathcal{C} > 1$

– $\mathbb{Z}_2$ Topological Insulators in 2D and 3D

$\Rightarrow$Exploration of "fractional topological insulators"

Summary

- Coherent optical coupling of internal states provides a powerful way to create topological bands for cold atoms.

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

- Simple laser set-ups lead to "optical flux lattices": periodic magnetic flux with very high mean density, $n_\phi \sim 1/\lambda^2$.
- The bandstructure can be designed with significant control.
- These lattices offer a practical route to the study of novel strongly correlated topological phases in ultracold gases.