Non-equilibrium Bose-Einstein condensation phenomena in microcavity polariton systems

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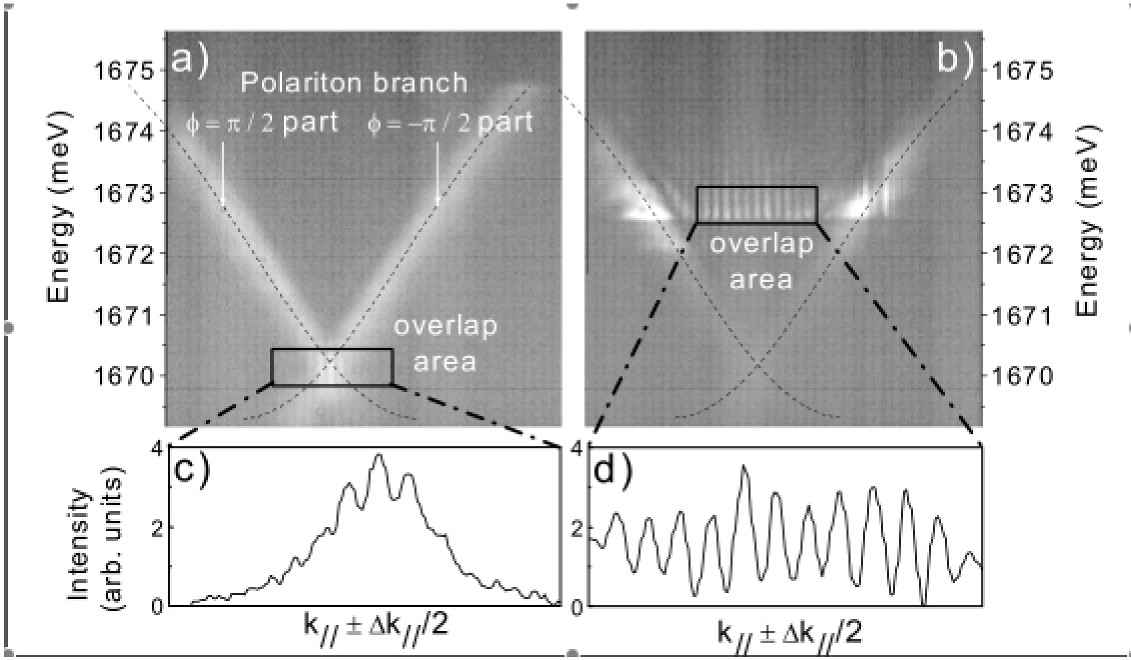
Many recent expts: macroscopic coherence in polariton systems

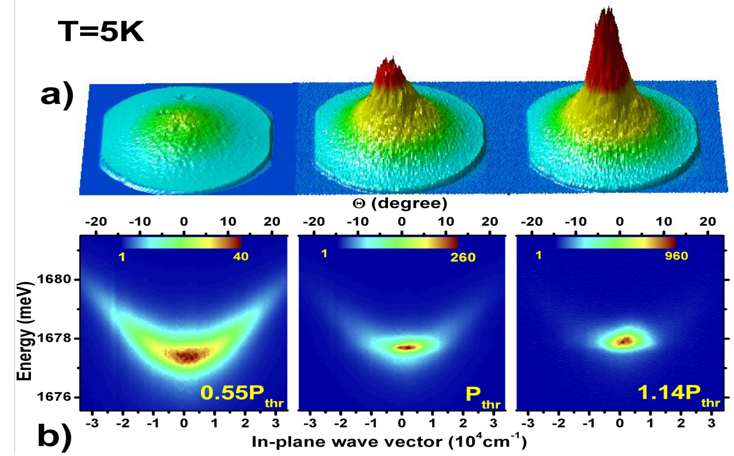
Young two-slit interference

Richard et al. PRL 2005, Grenoble group



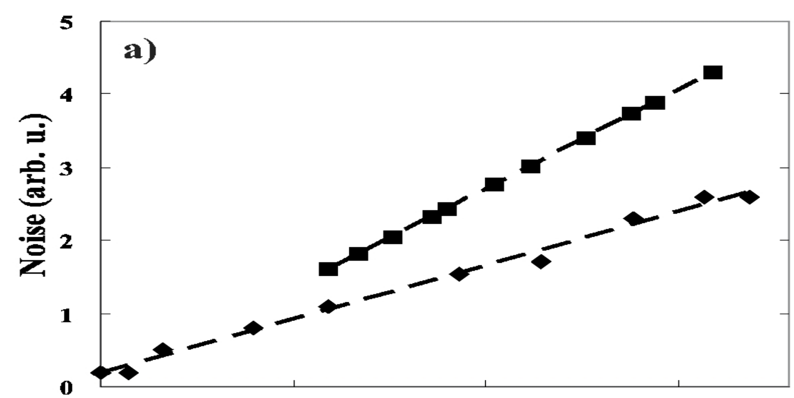
Line narrowing in k-space

Kasprzak et al., Nature 2006, Grenoble-Lausanne collaboration



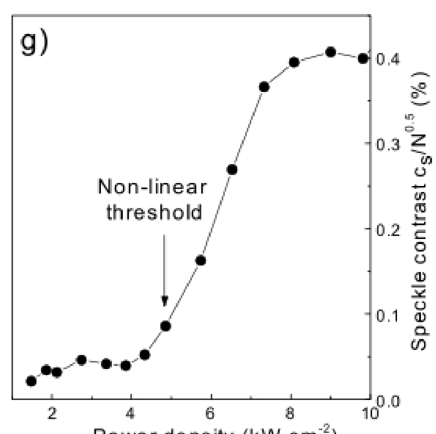
Noise reduction

Baas et al., PRL 2006, LKB-Paris



Threhold behaviour

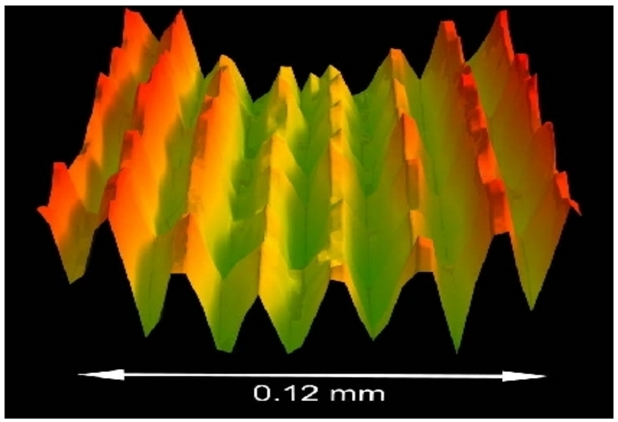
Richard et al. PRL 2005 Grenoble group



Same phenomenology as Bose-Einstein condensation in ultracold atom systems

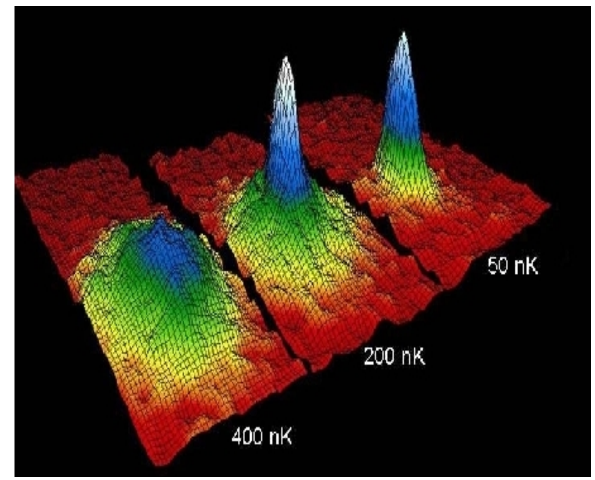
Interference and phase coherence

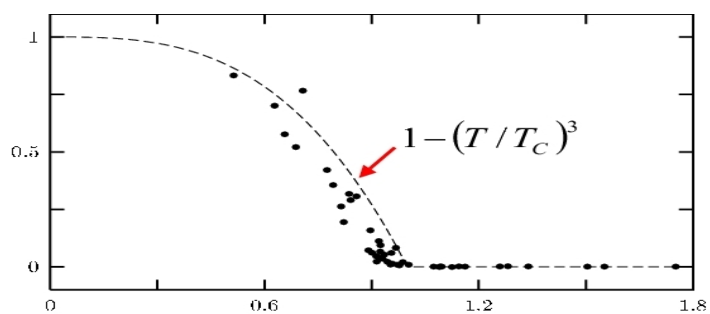
(MIT 1996)



Narrowing of momentum distribution

(JILA 1995)





Threshold behaviour at T_c

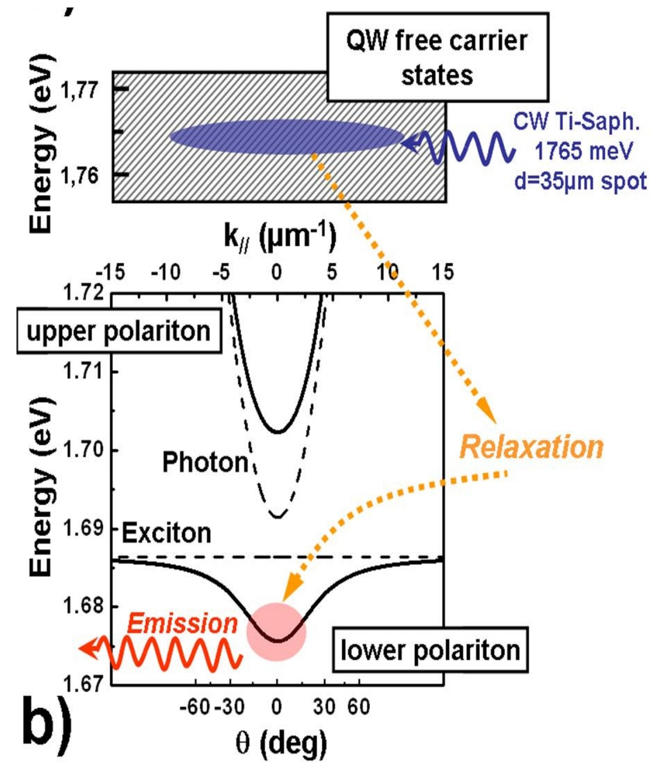
(JILA 1996)

Reduct. of 3-body recombination: proof of suppr. density fluctuations (MIT 1997)

A crucial difference: system is far from equilibrium

- Optical injection
- Relaxation: polariton-polariton and polariton-phonon scattering
- Stimulation of scattering to lowest states

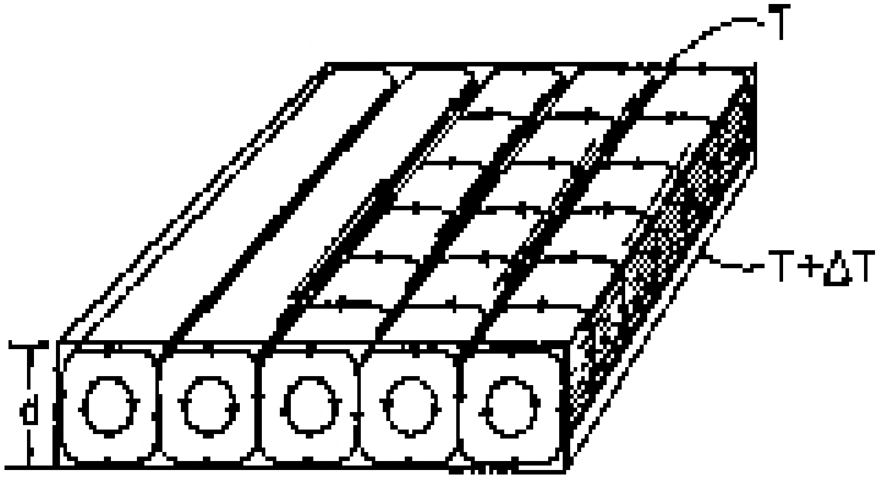
- Losses: particle number NOT conserved
- NO thermodynamical equilibrium
- Steady-state determined by dynamical balance of driving and dissipation



(Figure from Kasprzak et al., Nature 2006)

- Standard concepts of equilibrium statistical mechanics are not applicable
- Physics is different from usual equilibrium BEC, but...

Phase transitions in non-equilibrium systems as well !



Bénard cells in heat convection:

- dynamical equilibrium between driving (ΔT) and dissipation (viscosity)
- for $\Delta T > \Delta T_c$ translationally invariant state is dynamically unstable
- spontaneous breaking of translational symmetry: periodic pattern of convection rolls

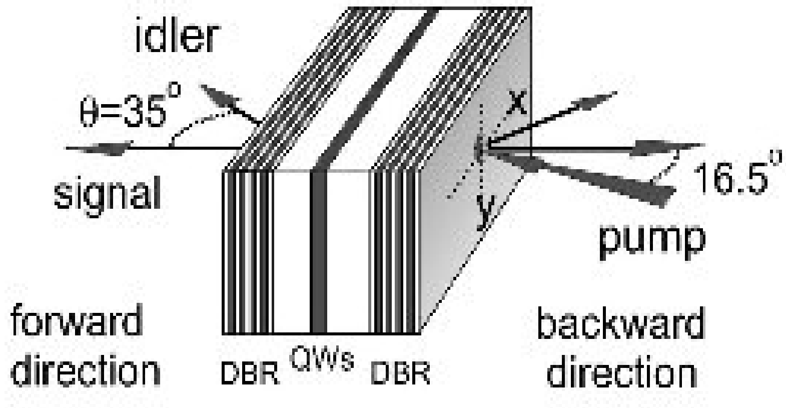
Other examples:

- Belousov-Zhabotinsky chemical reaction
- Coat patterns of mammalians
- Driven lattice gas

We therefore wonder...

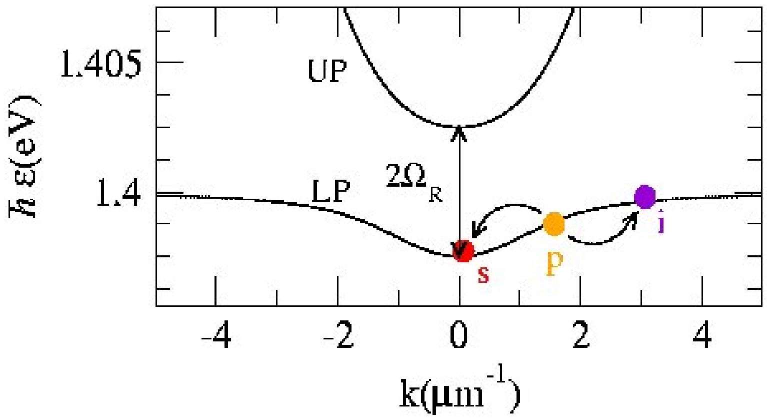
- To what extent can the observed macroscopic coherence be really considered as a Bose-Einstein condensation of polaritons ?
- If so, what new physics can be learnt from polaritons that was not possible with other "classical" systems such as liquid Helium and ultracold atoms ?
- Can it lead to completely new states of matter? If so, what are their properties? How can the new system lead to new fundamental physics?
- What are the consequences of the new physics for applications to optoelectronic devices?

The physical system: DBR microcavity with QWs



- DBR λ/4 GaAs/AlAs layers
- Cavity layer → confined photonic mode, delocalized along 2D plane
- In-plane photon dispersion:

$$\omega_C(\boldsymbol{k}) = \omega_C^0 \sqrt{1 + \boldsymbol{k}^2 / k_z^{\,2}}$$



- e and h confined in InGaAs QW
- e-h pair: sort of H atom. **Exciton**
- Excitons bosons if $n_{exc}\, a_{Bohr}^2 \ll 1$
- Excitons delocalized along cavity plane.

Flat exciton dispersion $\omega_X(\boldsymbol{k}) \approx \omega_X$

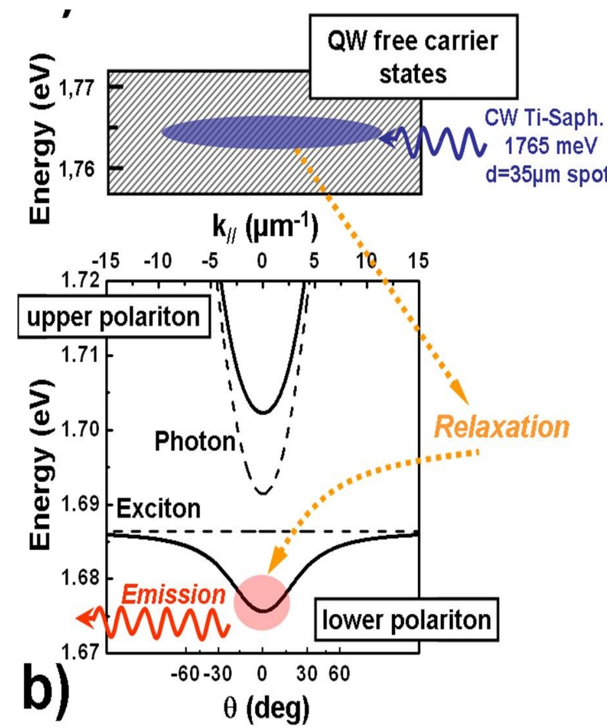
Exciton radiatively coupled to cavity photon at same in-plane k
No coupling to continuum, no spontaneous emission, Rabi oscillations at Ω_R
Bosonic superpositions of exciton and photon, called **polaritons**

Polariton mass $m_{pol} \approx 10^{-4}\, m_e$ → BEC expected at high "temperature" !!

Ways to generate macroscopic coherence

Direct injection by resonant pump laser

- coherence not spontaneous, imprinted by pump
- close relation with nonlinear optics, still interesting superfluidity properties



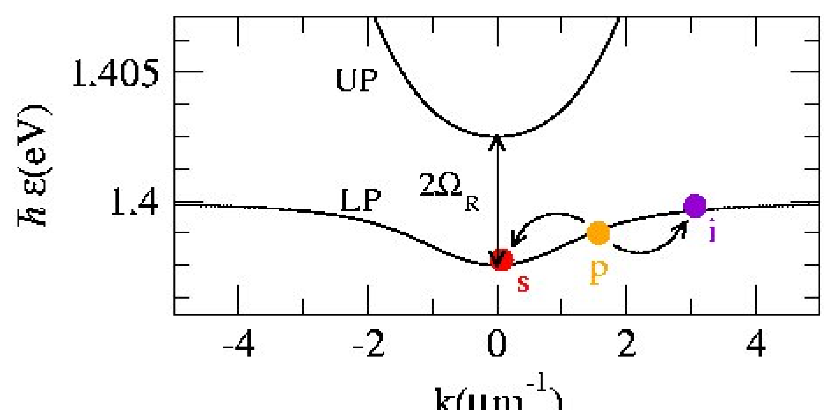
(Fig. from Kasprzak et al., Nature 2006)

Non-resonant pumping

- thermalisation due via polariton-polariton collisions, quasi-equilibrium condition
- coherence spontaneously created via BEC effect
- hard to theoretically model *ab initio*

OPO process

- stimulated scattering into signal/idler modes
- spontaneous coherence, not locked to pump
- same spontaneous symmetry breaking
- *ab initio* theoretical description by stochastic GPE



Wigner-QMC

Generalizes truncated-Wigner method for BECs (Lobo, Sinatra, Castin)

Time evolution: stochastic Gross-Pitaevskii equation

$$i\,d\begin{pmatrix}\psi_X(\mathbf{x},t)\\ \psi_C(\mathbf{x},t)\end{pmatrix}=\left[\mathbf{h}^0+\begin{pmatrix}V_X(\mathbf{x})+g(|\psi_X(\mathbf{x},t)|^2-1/dV)-i\gamma_X & 0\\ 0 & V_C(\mathbf{x})-i\gamma_C\end{pmatrix}\right]\begin{pmatrix}\psi_X(\mathbf{x},t)\\ \psi_C(\mathbf{x},t)\end{pmatrix}dt+$$

$$+\begin{pmatrix}0\\ k\,\mathcal{E}_p(\mathbf{x},t)\end{pmatrix}dt+\frac{1}{\sqrt{4\Delta V}}\begin{pmatrix}\sqrt{\gamma_X}\,dW_X(\mathbf{x},t)\\ \sqrt{\gamma_C}\,dW_C(\mathbf{x},t)\end{pmatrix}$$

Single particle Hamiltonian
$$\mathbf{h}^0=\begin{pmatrix}\omega_X(-i\nabla) & \Omega_R\\ \Omega_R & \omega_C(-i\nabla)\end{pmatrix}$$

Losses $\gamma_{X,C}$. Fluctuation-dissipation: white noise
$$\begin{aligned}\overline{dW_i(\mathbf{x},t)\,dW_j(\mathbf{x}',t)}&=0\\ \overline{dW_i(\mathbf{x},t)\,dW_j^*(\mathbf{x}',t)}&=2\,dt\,\delta_{\mathbf{x},\mathbf{x}'}\,\delta_{ij}\end{aligned}$$

Observables: MC averages over noise
$$\langle\psi_i^*(\mathbf{x})\,\psi_i(\mathbf{x})\rangle_W=\frac{1}{2}\left[\langle\hat{\Psi}_i^\dagger(\mathbf{x})\,\hat{\Psi}_i(\mathbf{x})\rangle+\langle\hat{\Psi}_i(\mathbf{x})\,\hat{\Psi}_i^\dagger(\mathbf{x})\rangle\right]$$

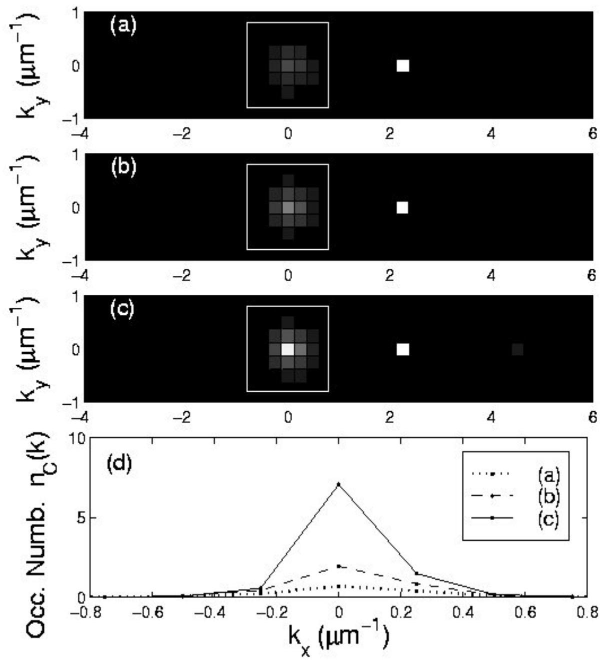
- not linearized theory, full account of large fluctuations around critical point
- any geometry can be simulated, full time-dynamics

→ Accurate *ab initio* description of OPO transition

IC and C. Ciuti, PRL, **93** 166401 (2004); PSSb **242**, 2224 (2005); PRB 72, 125335 (2005)

The parametric oscillation threshold

Pump beam close to magic angle for OPO process

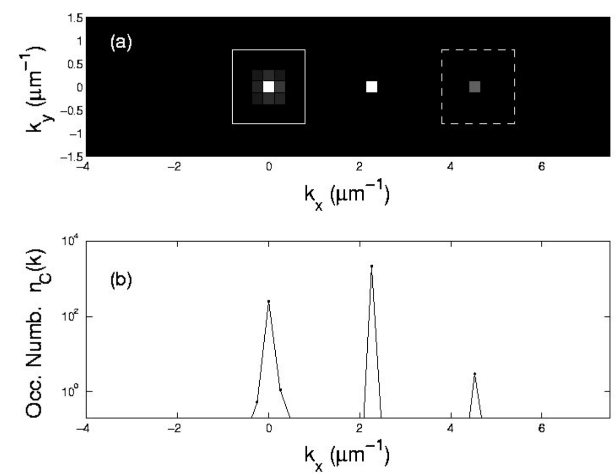


Below threshold:

- coherent emission from pump mode
- quantum fluctuations: many-mode incoherent luminescence
- strongest for signal and idler around phase-matching

Approaching threshold:

- signal/idler intensity increases, linewidth narrows



Above threshold:

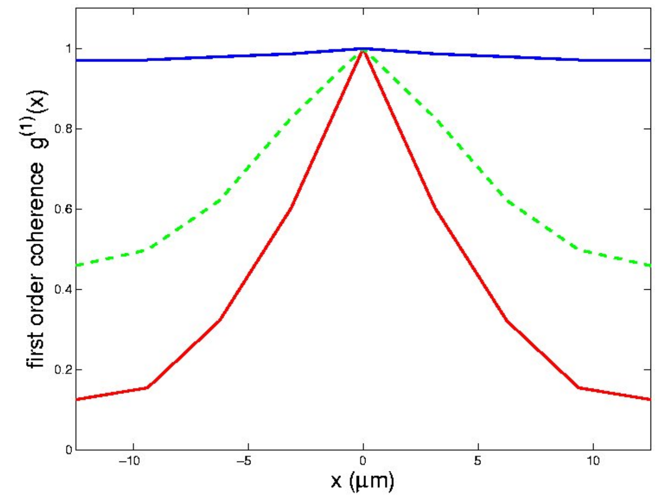
- single signal/idler pair selected
- emission becomes macroscopic
- signal/idler phases still random, only their sum fixed

IC and C. Ciuti, *Spontaneous microcavity-polariton coherence across the parametric threshold: Quantum Monte Carlo studies*, PRB 72, 125335 (2005)

Signal/idler coherence properties across threshold

First-order coherence $g^{(1)}(x)$

- approaching threshold from below: l_c diverges
- above threshold: long-range coherence
- BEC according to Penrose-Onsager criterion
- coherence NOT inherited from pump

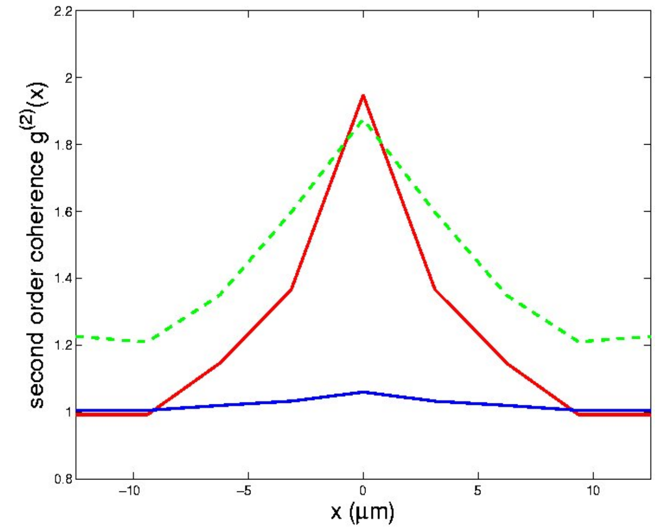


Second-order coherence $g^{(2)}(x)$

- below threshold: HB-T bunching

$$g^{(2)}(0)=2, \quad g^{(2)}(\text{large } x)\to 1$$

- above threshold: suppression of fluctuations: $g^{(2)}(x)=1$



Similar phenomena predicted and observed in atomic gases at equilibrium

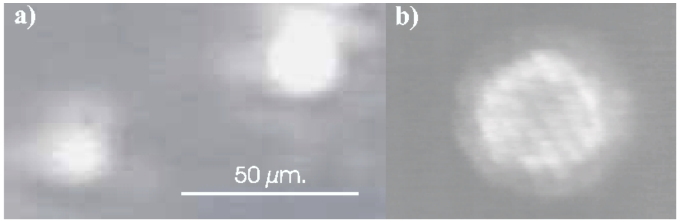
IC and C. Ciuti, *Spontaneous microcavity-polariton coherence across the parametric threshold: Quantum Monte Carlo studies*, PRB 72, 125335 (2005)

Experimental observations

Correlation functions of emission reproduce those of cavity polaritons

$g^{(1)}(x) \rightarrow$ Young-like experiment

- light from two paths interferes
- above threshold: fringes observed

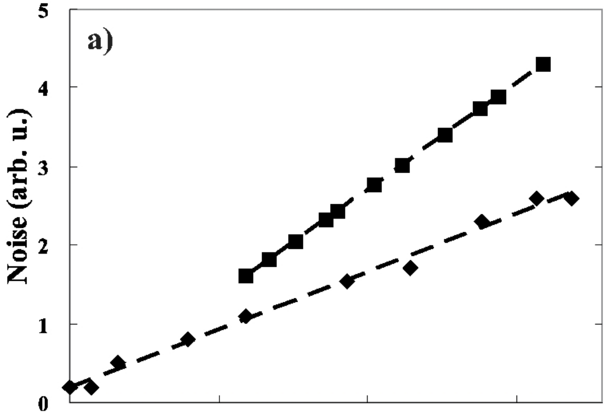


$g^{(2)}(x) \rightarrow$ noise-correlation experiment

- output beam cut by razor blade
- above threshold: linear dependence

  means single spatial mode
- slope means excess noise over standard

  quantum limit



(figs. from Baas et al., PRL 2006)

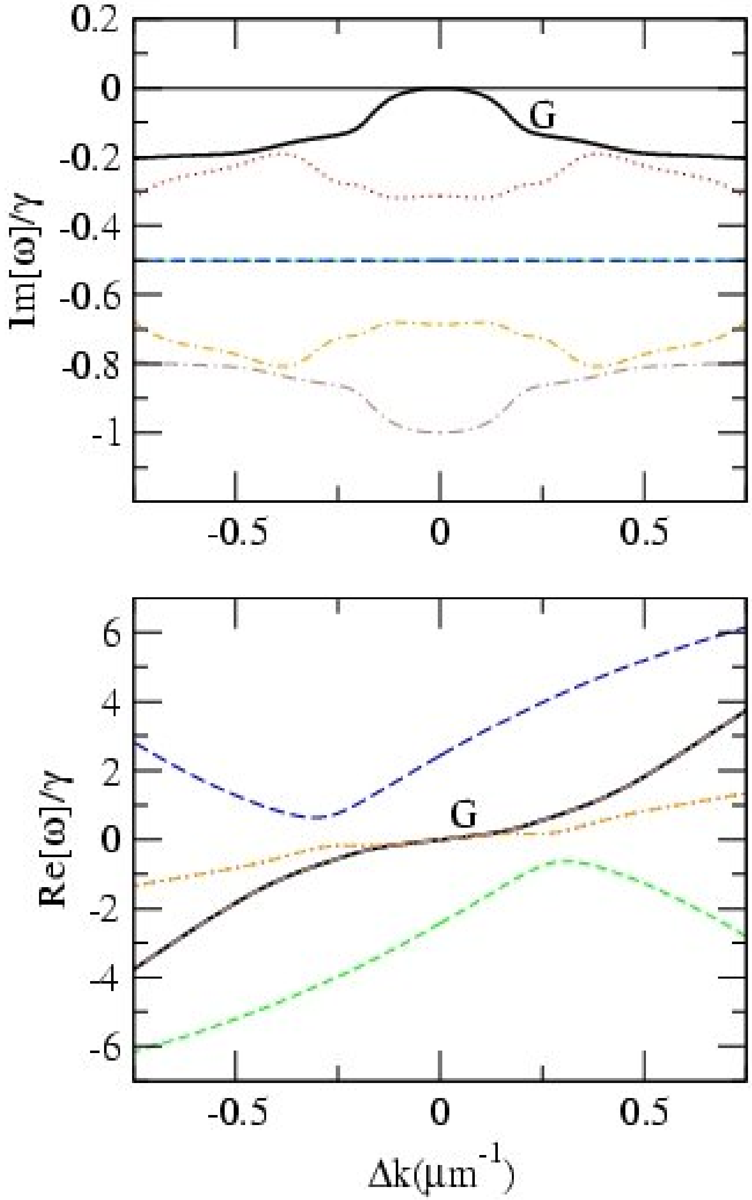
Good agreement with theory !!

Spontaneous symmetry breaking and Goldstone mode

Steady-state above threshold:

- coherent signal/idler beams
- U(1) symmetry spontaneously broken
- soft Goldstone mode $\omega_G(k) \to 0$ for $k \to 0$
- corresponds to slow signal-idler phase rotation

  $\to$ as Bogoliubov phonon at equilibrium !!!



Fundamental physical difference:

$\to$ Goldstone mode diffusive,

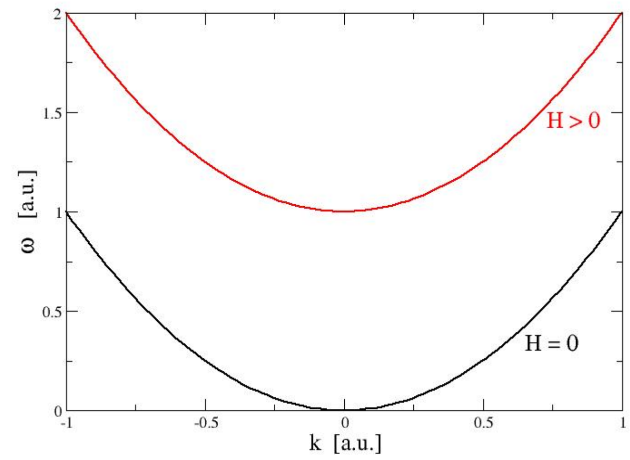
not propagating like sound

M. Wouters, IC, *The Goldstone mode of planar optical parametric oscillators*, cond-mat/0606755

Pinning the signal/idler phase

Goldstone mode in ferromagnets

- magnons: wavy oscillations in spin orientation
- spin orientation can be pinned by external B
- gap in Goldstone spectrum opens

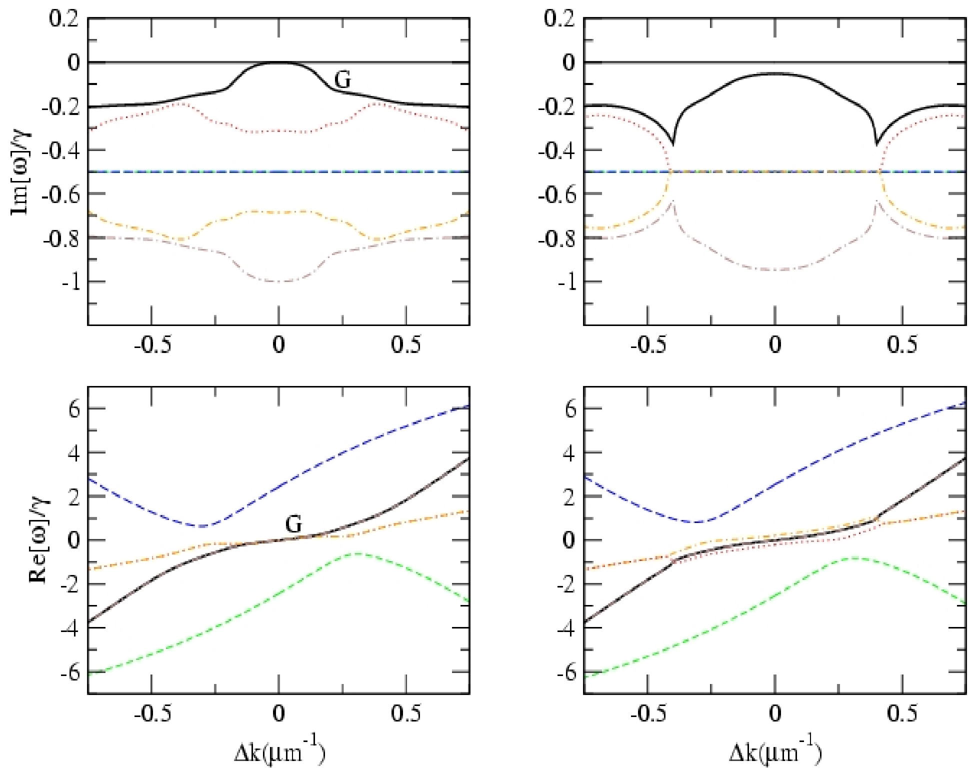


Goldstone mode of OPOs:

- slow rotation of signal/idler phases

Seed laser driving signal:

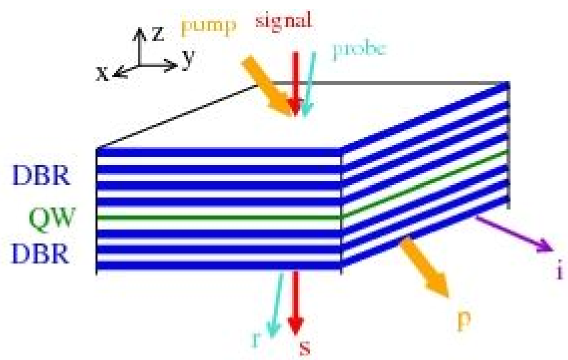
- stimulates signal emission, phase pinned
- phase symmetry explicitely broken
- gap opens in imaginary part of $\omega_G(k)$

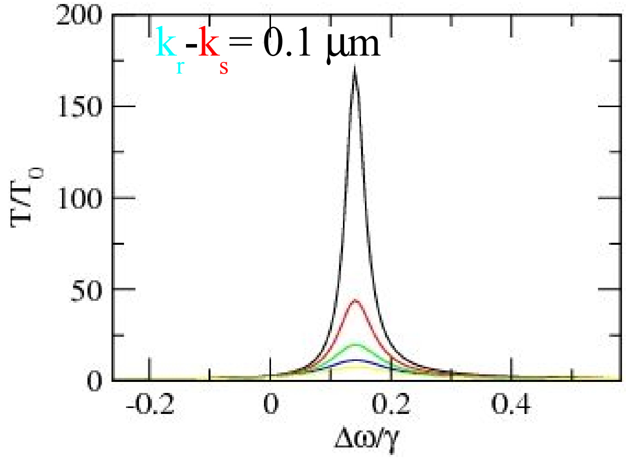


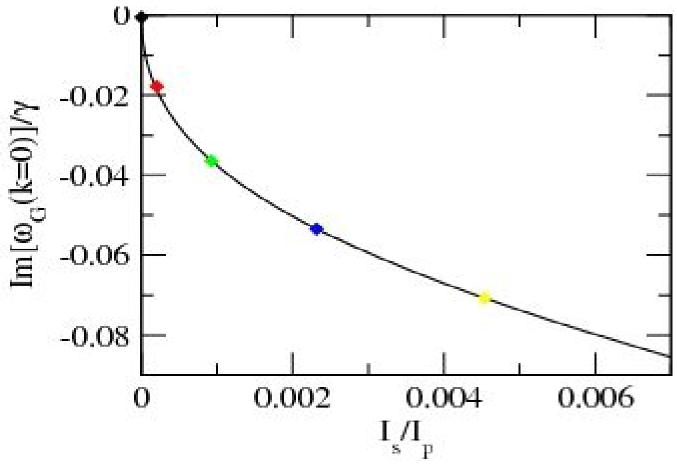
M. Wouters and IC, *The Goldstone mode of planar optical parametric oscillators*, cond-mat/0606755

Observing the Goldstone mode

- Goldstone mode: peak in probe transmission at angle close to signal
- amplified transmission w/r to unloaded cavity resonant transmission
- when phase pinned by signal laser: peak broadened and suppressed







Hard to do with atoms because of atom number conservation

M. Wouters and IC, *The Goldstone mode of planar optical parametric oscillators*, cond-mat/0606755

Simultaneously to our work:

M. H. Szymanska, J. Keeling, P. B. Littlewood, *Nonequilibrium Quantum Condensation in an Incoherently Pumped Dissipative System*, PRL 96, 230602 (2006)

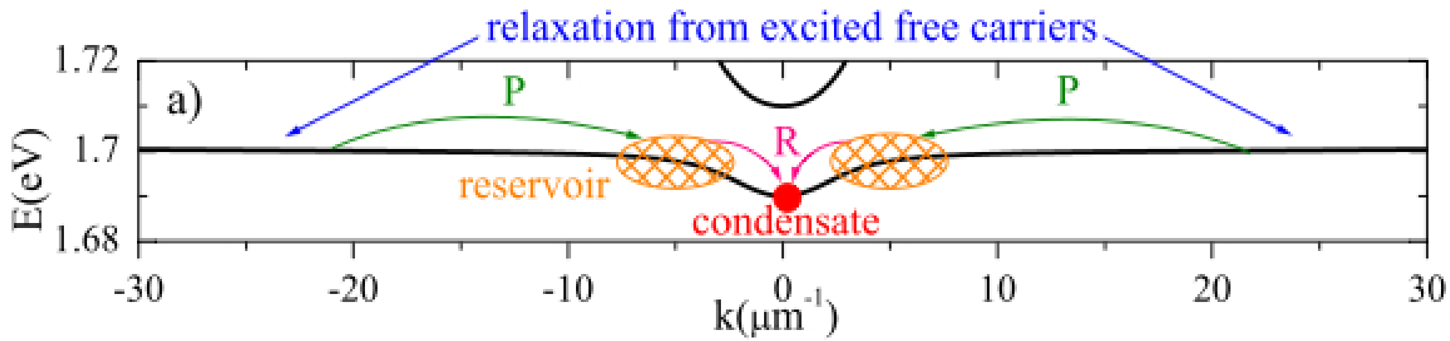
Calculate Goldstone mode dispersion under non-resonant pumping

also in this case: **diffusive Goldstone mode** !!

- Is this a general result of non-equilibrium systems ?
- Simple physical interpretation ?

A generalized GPE for non-resonantly pumped BECs

Inspired from "generic model of atom laser": Kneer et al., PRA 58, 4841 (1998)



- **Polariton condensate** : GPE with losses / amplification

$$i\frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2\nabla^2}{2m_{LP}} - i\gamma/2 + \frac{i}{2}R\left(n_B\right) + g\left|\psi\right|^2 + 2\tilde{g}n_B\right]\psi$$

macroscopic wavefunction ψ (x), loss rate γ, amplification R(n_B)

- **Incoherent reservoir** : rate equation for density n_B(x)

$$\frac{\partial}{\partial t}n_B = P - \gamma_B\bar{n}_B - R\left(n_B\right)\left|\psi\left(x\right)\right|^2 + \frac{D}{2}\nabla^2 n_B$$

pumping rate P, spatial diffusion D, thermalization rate γ_B

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, cond-mat/0702413

Bogoliubov theory of elementary excitations

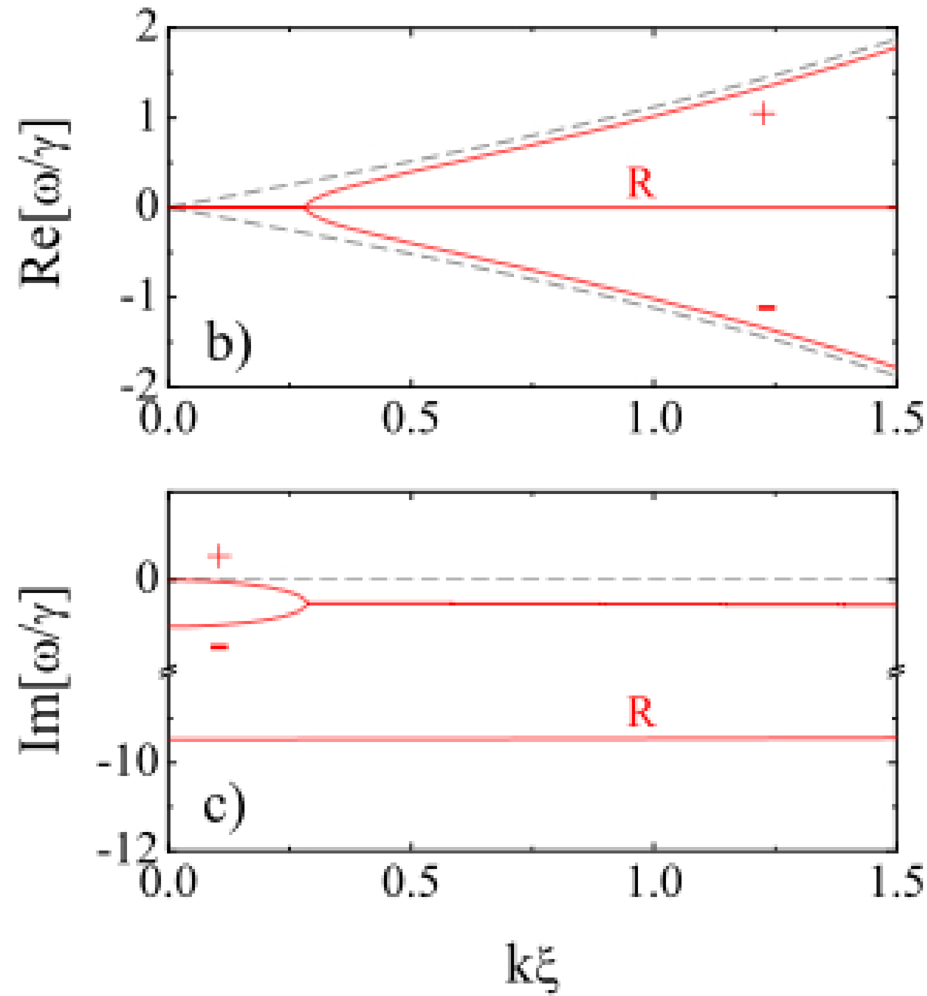
Linearize GPE around steady state:

- Reservoir R mode at $-i\gamma_R$
- Condensate modes ± at:

$$\omega_{\pm}(k) = -\frac{i\Gamma}{2} \pm \sqrt{[\omega_{Bog}(k)]^2 - \frac{\Gamma^2}{4}}$$

with:

$$\omega_{Bog}(k) = \sqrt{\frac{\hbar k^2}{2m_{LP}}\left(\frac{\hbar k^2}{2m_{LP}} + 2\,\mu\right)}$$



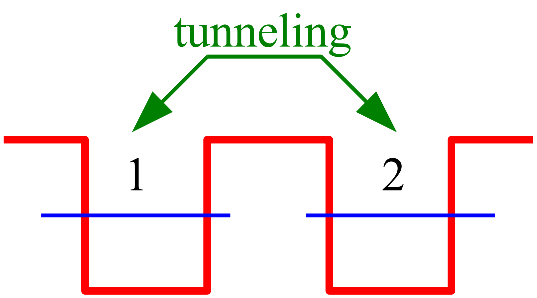
→ Goldstone mode is again diffusive !!!

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, cond-mat/0702413

Two-well geometry: Josephson effect

ψ_i → amplitude in i-th well; population $N_i = |\psi_i|^2$

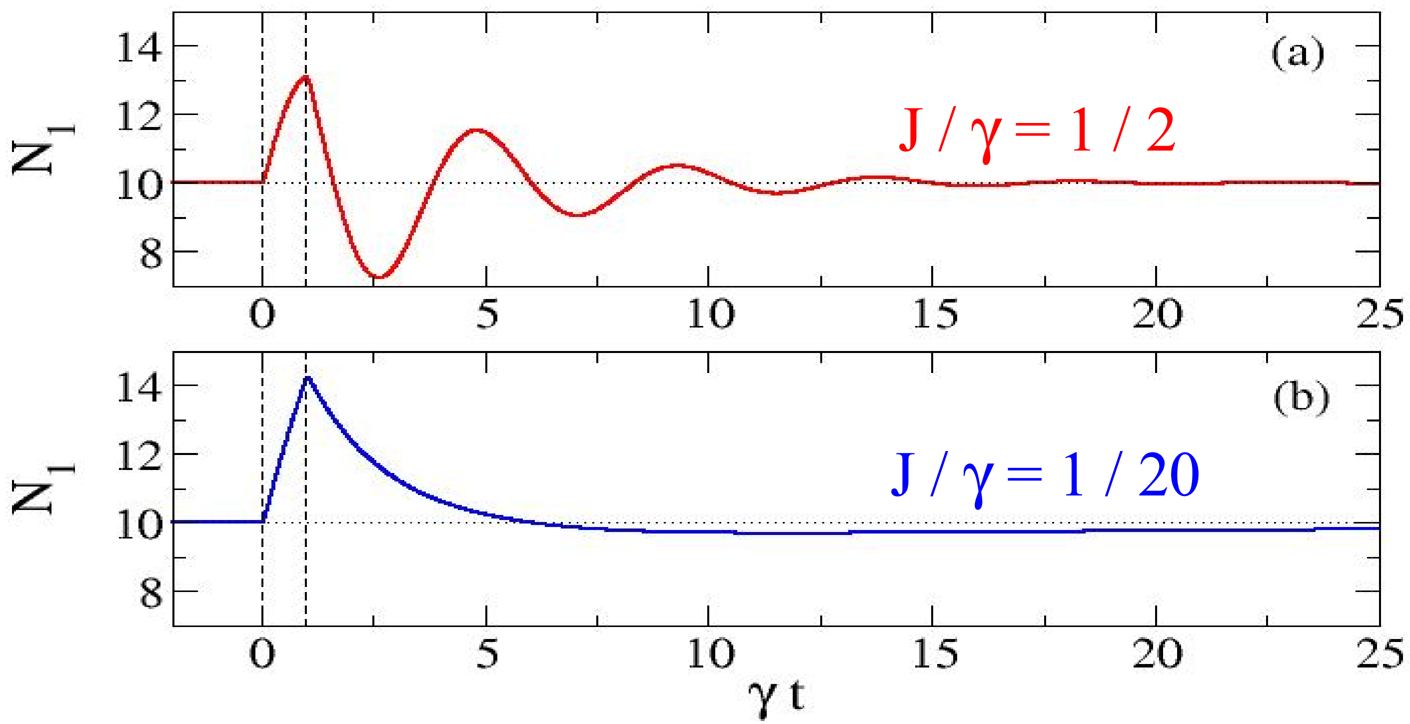
n_i → reservoir density behind i-th well



$$i\frac{d\psi_j}{dt} = -J\,\psi_{3-j} + U\,|\psi_j|^2\,\psi_j + \frac{i}{2}\left[R(n_j) - \gamma\right]\psi_j$$

$$\frac{dn_j}{dt} = P_j - \gamma_R\, n_j - R(n_j)|\psi_j|^2.$$

Exp. with polariton traps:
El Daif *et al.*, APL '06
Baas, Richard *et al.*, '07 (ICSCE-3)



Josephson oscillations

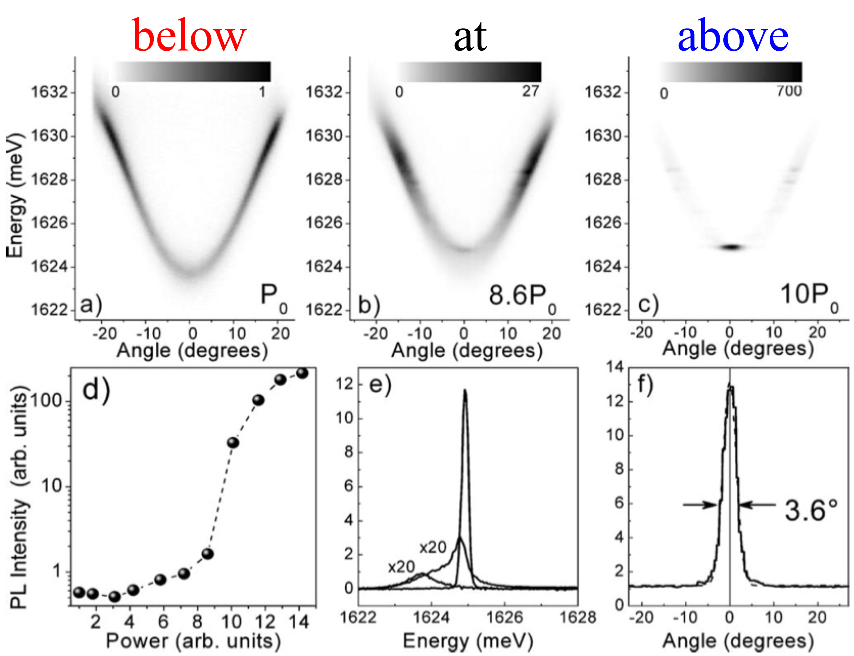
overdamped Josephson oscillations

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, cond-mat/0702413

BEC shape

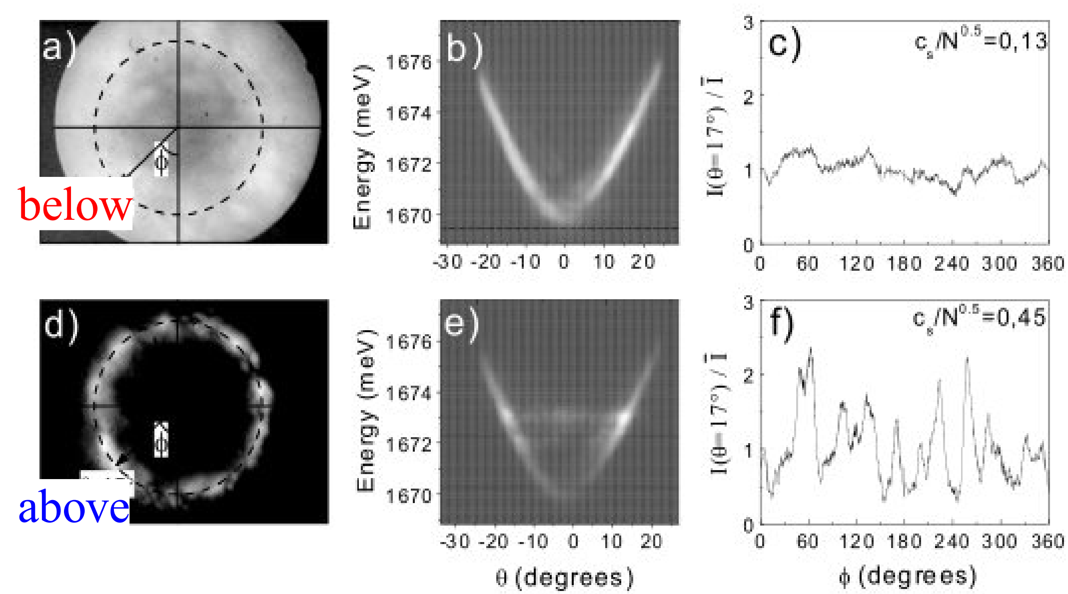
- Equilibrium, harmonic trap: Thomas-Fermi parabolic profile
- Non-equilibrium: dynamics affects shape. Stationary flow possible

Experimental observations: shape depends on pump spot size



pump spot: 20 µm

Richard et al., PRB **72**, 201301 (2005)



pump spot: 3 µm

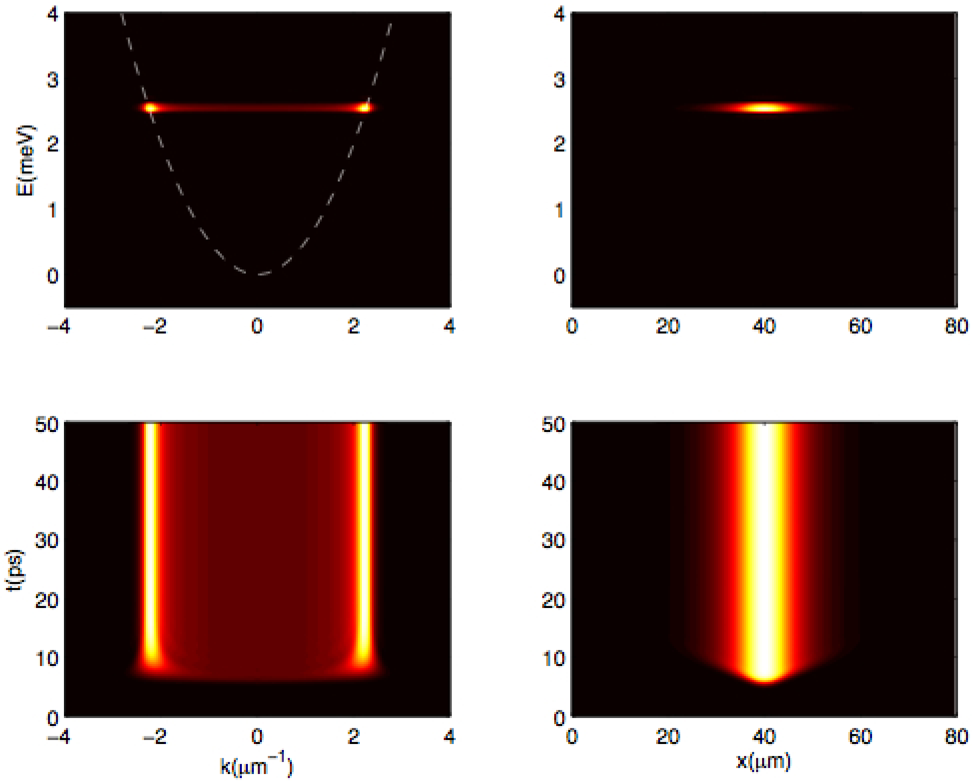
Richard et al., PRL **94**, 187401 (2005)

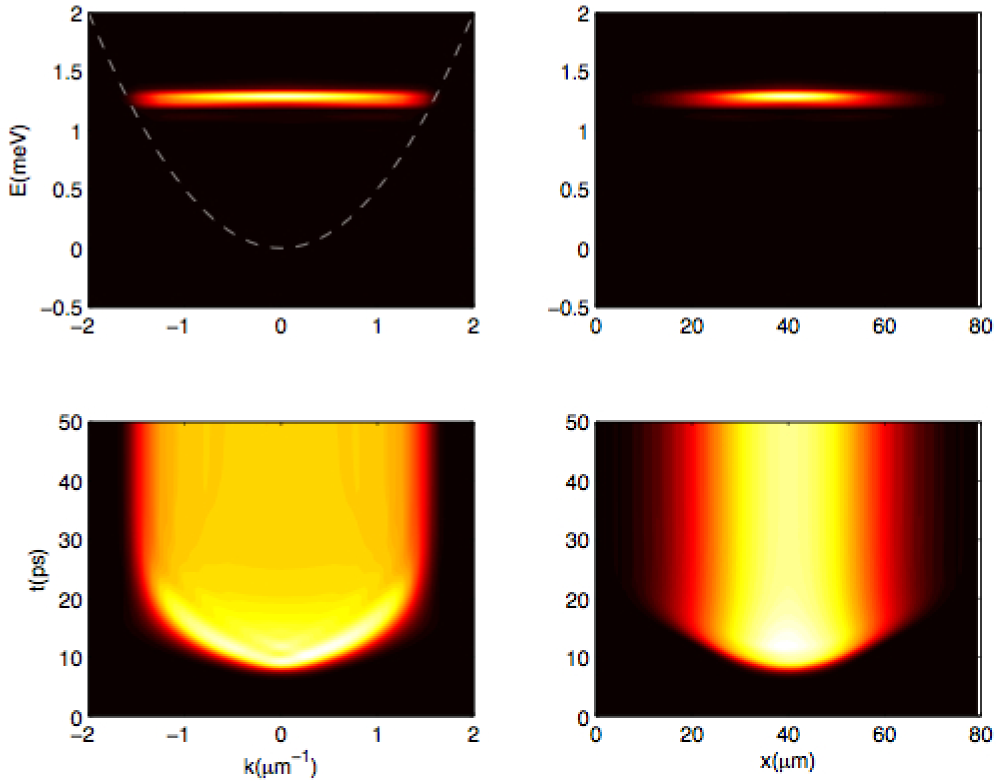
Numerical integration of non-equilibrium GPE

Stationary state under cw pumping

Narrow pump spot: σ = 5μm

Wide pump spot: σ = 20 μm



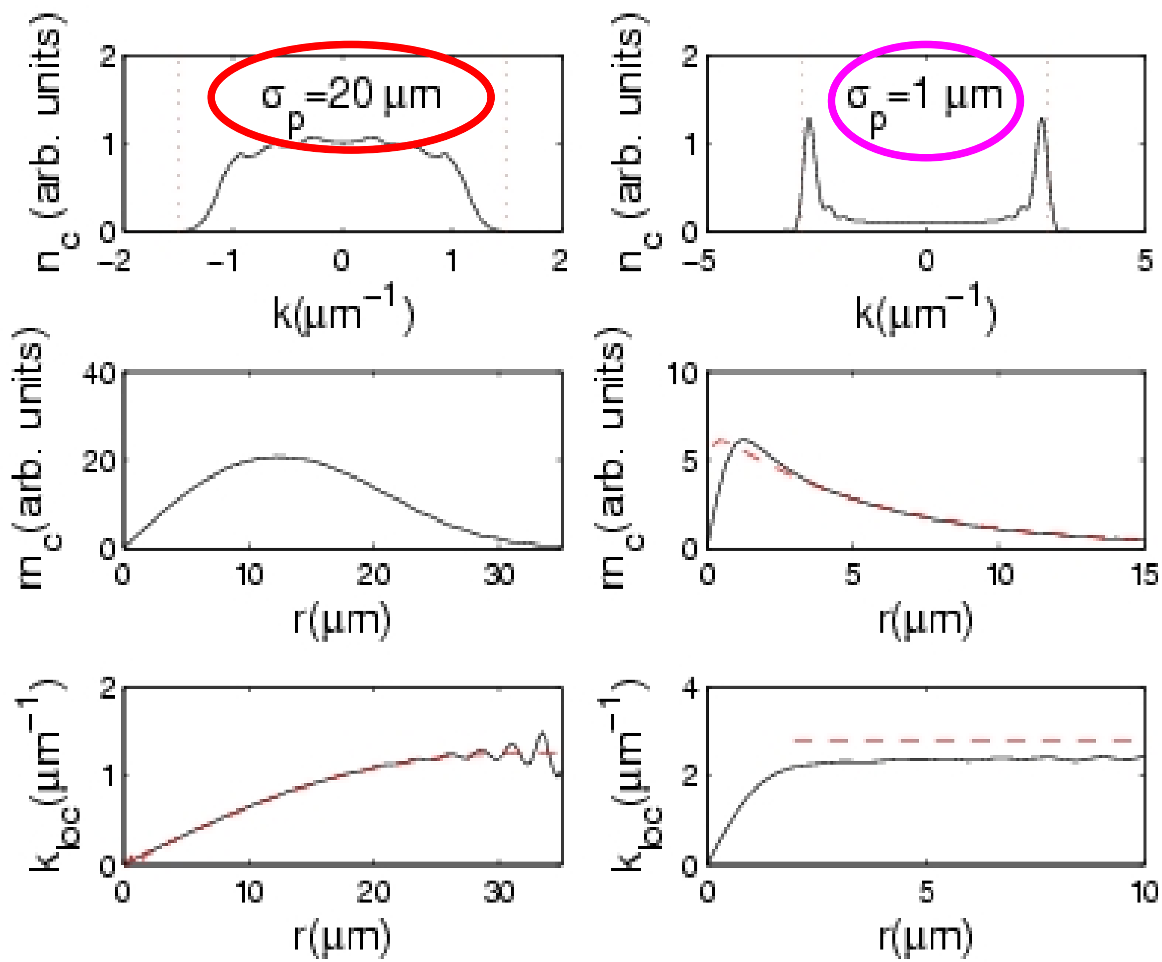


Emission on a ring at finite k
Spatially localized

Emission centered at k=0
Spatially localized

Good agreement with experiments !!

Physical interpretation of condensation at k≠0



Repulsive interactions

- outward radial acceleration
- energy conservation

$$E = k^2/2m + U_{int}(r)$$

→ local flow velocity

radially increasing !

Narrow spot:

- free flight outside pump spot $U_{int}(r)=0$, emission mostly on free particle disp.

M. Wouters, C. Ciuti, and IC, in preparation (2007)

Simulations for pulsed excitation

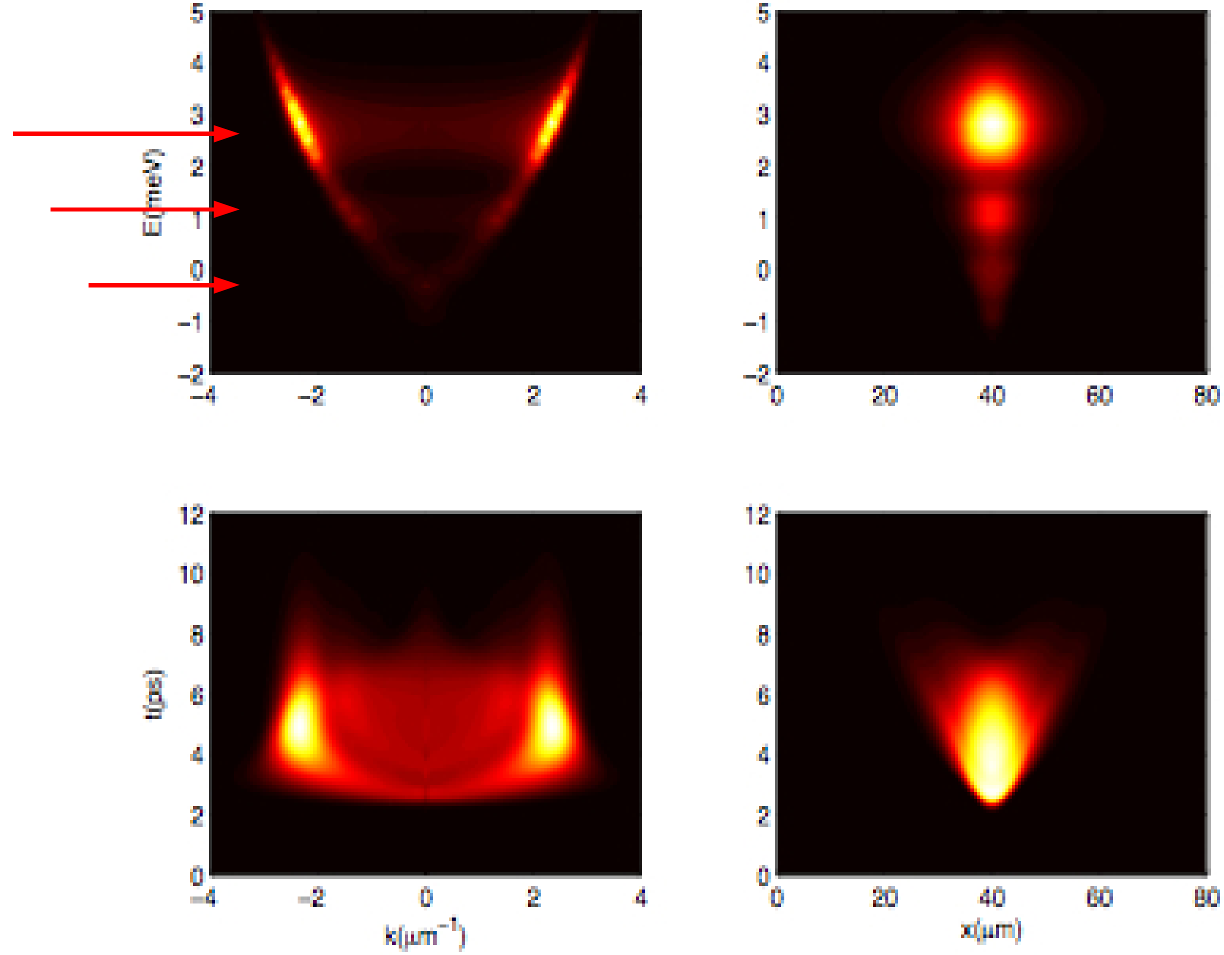
•Non-trivial time evolution:

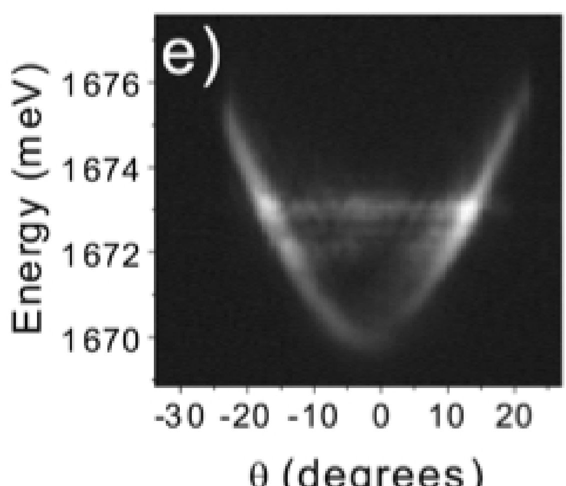
first k=0, then expands

•Emission concentrated

at several E's

Also in expt's !!!





M. Wouters, C. Ciuti, and IC, in preparation (2007)

Reduced dimensionality I: equilibrium

- 3D: BEC transition at finite T_c
- 2D: K-T transition at finite T_{KT} due to vortex pair unbinding :
  - algebraic decay of coherence for $T<T_{KT}$
  - exponential decay of coherence for $T>T_{KT}$
- 1D: exponential decay of coherence for $T\neq 0$
- Hohenberg-Mermin-Wagner theorem sets $d_c=2$ for U(1) SSB in the thermodynamical limit

Note: Finite-size effects: BEC possible in all dimensionality.

T_c depends on size L: as L^{-1} in 1D, logarithmically in 2D below T_{KT}

Reduced dimensionality II: non-equilibrium

- NO general Hohenberg-Mermin-Wagner-like theorem available

- **numerics**: Wigner-QMC calculations

- **analytics**: generalize modulus-phase Bogoliubov (Mora and Castin '03)

  accurate for small density fluctuations

  no condition on long range order

  quantum noise drives Bogoliubov modes

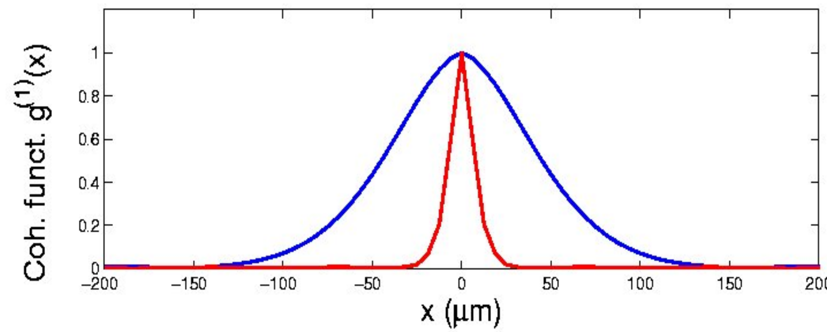
  effect strongest on Goldstone mode because of lowest damping

M. Wouters and IC, *Absence of Long-Range Coherence in the Parametric Emission from Photonic Wires*, PRB **74**, 245316 (2006)

Coherence in 1D OPOs: numerical QMC results

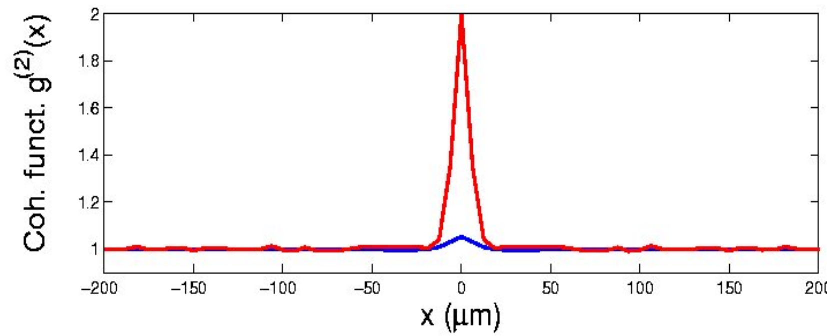
Below threshold:

- incoherent luminescence
- short range coherence



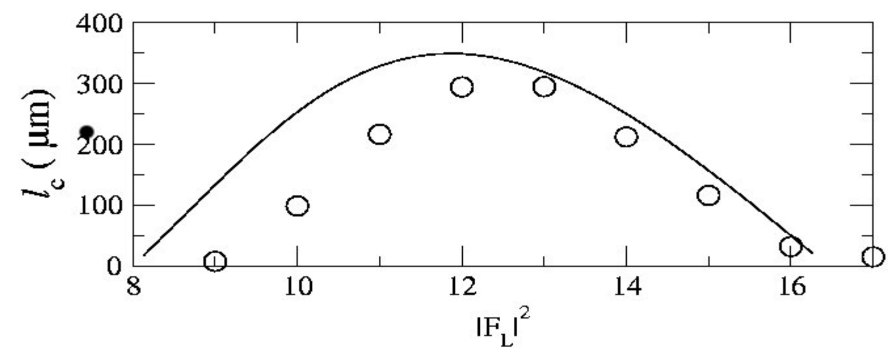
Above threshold:

- intensity fluctuations suppressed
- coherence length much longer
- but always finite



As a function of pump intensity:

- $l_c \rightarrow 0$ as threshold is approached
- reentrant behaviour due to blue-shift



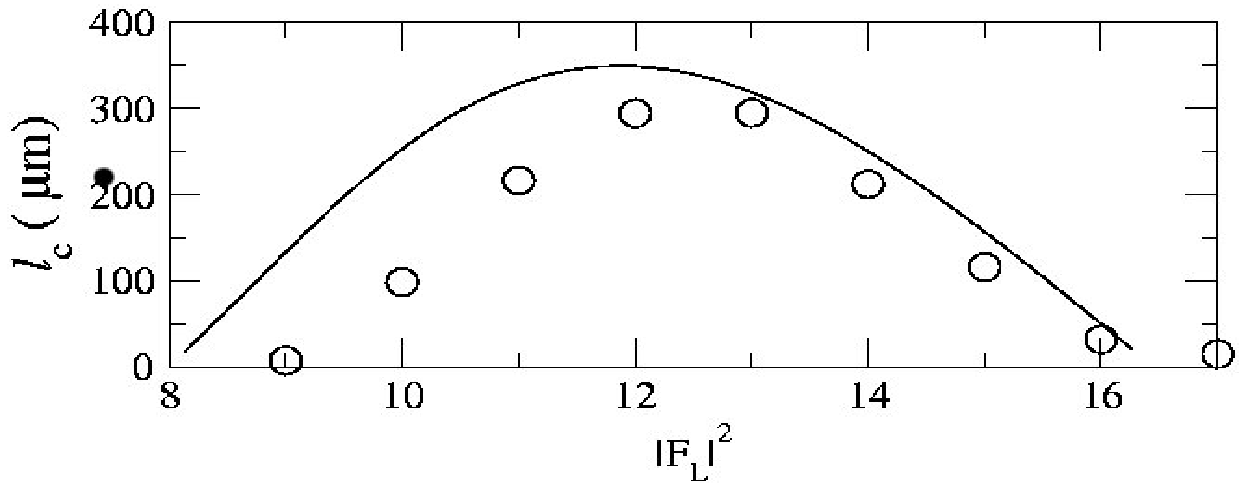
M. Wouters and IC, *Absence of Long-Range Coherence in the Parametric Emission from Photonic Wires*, PRB **74**, 245316 (2006)

Coherence in 1D OPOs: analytical results

Analytical integration of Wigner-Bogoliubov stochastic equations

Exponential decay of coherence. Coherence length:

- ➔ damping plays role of temperature
- ➔ bare boson mass replaced by imaginary mass of Goldstone mode



Strong nonlinearity of polariton system:

- ➔ l_c experimentally accessible, important in view of applications!!

Hic sunt leones...

- Effect of disorder and fluctuations on the transition:

  localized independent BECs and relative coherence of spots

  time-dependent correlation functions, phase diffusion rate

- Two-body physics of polariton-polariton scattering:

  possibility of Feshbach resonances on biexciton bound states

  (first results in: M. Wouters, PRB in the press)

- Critical properties:

  critical exponents as transition is approached; finite size effects

  effect of 2D geometry: vortex states, topological defects

  dynamics of phase transition, condensation kinetics

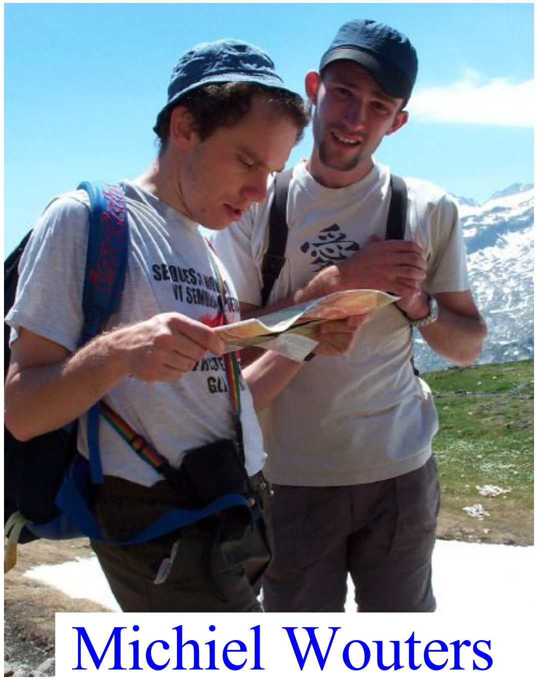
  superfluidity properties: Landau criterion and/or persistent currents

- Applications:

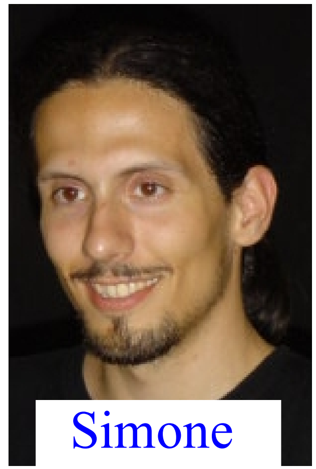
  studies of driven dissipative superfluid hydrodynamics, vortex dynamics

  quantum fluctuations in many-body systems

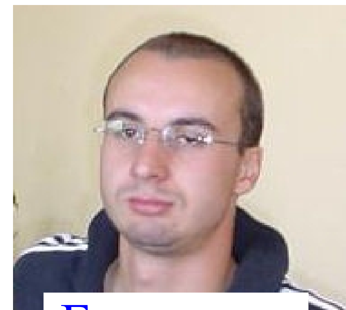
My brave polaritonic coworkers....



Michiel Wouters



Simone de Liberato



Francesco Bariani

free SOFTWARE FOUNDATION

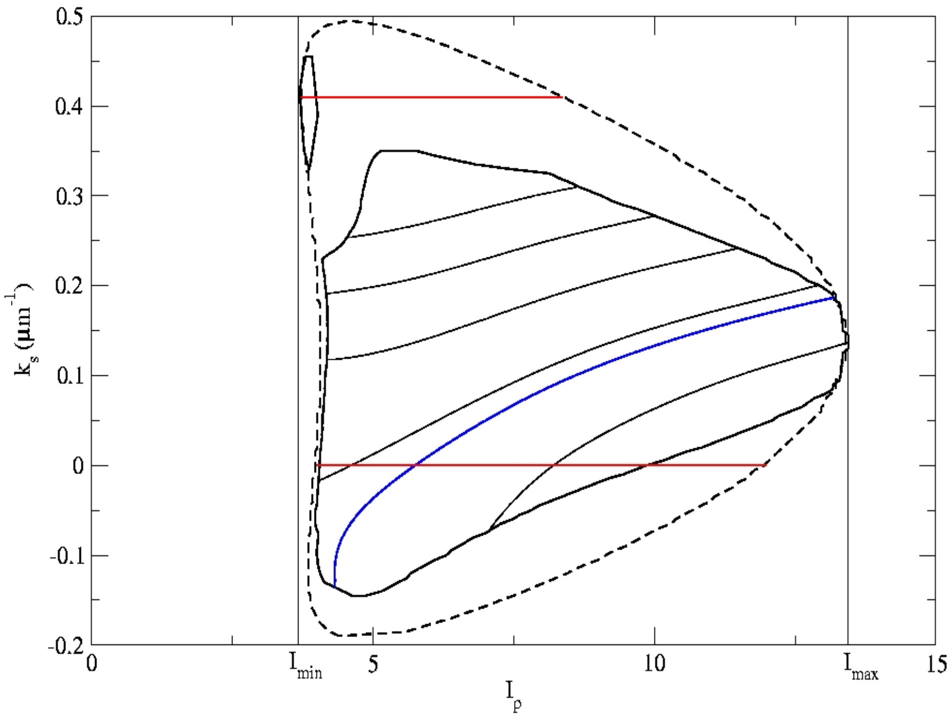
LINUX POWERED

Cristiano Ciuti

Arnaud Verger

and the Trento-BEC group

Finite spot effects



Equilibrium:

BEC in lowest energy state

Non-equilibrium:

- no free-energy available
- k_s dynamically selected
- methods of pattern formation in nonlinear dynamical systems

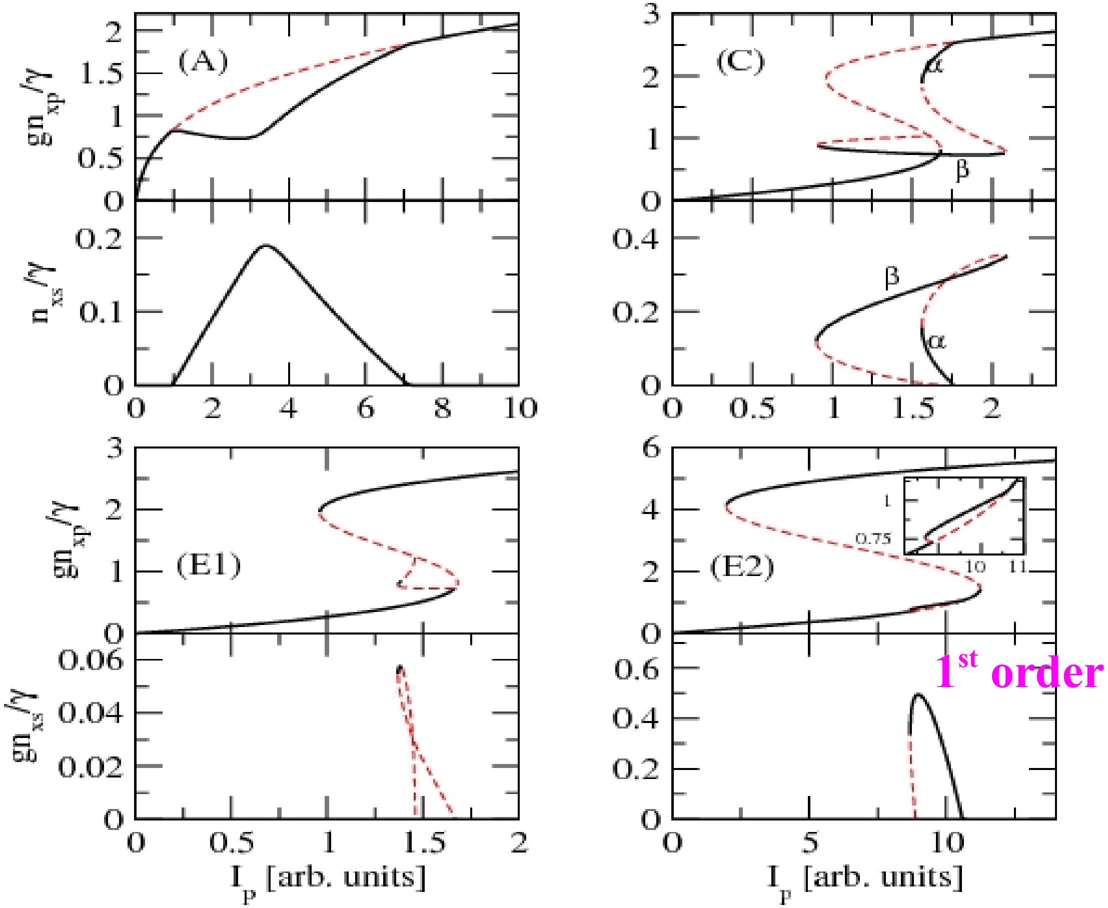
- Finite excitation spot: absolute vs. convective instability
- Single ω_s, inhomogeneous broadening of k_s due to spatially varying pump intensity profile: change in k_s

Richer physics than simple Thomas-Fermi profile of equilibrium BECs!!

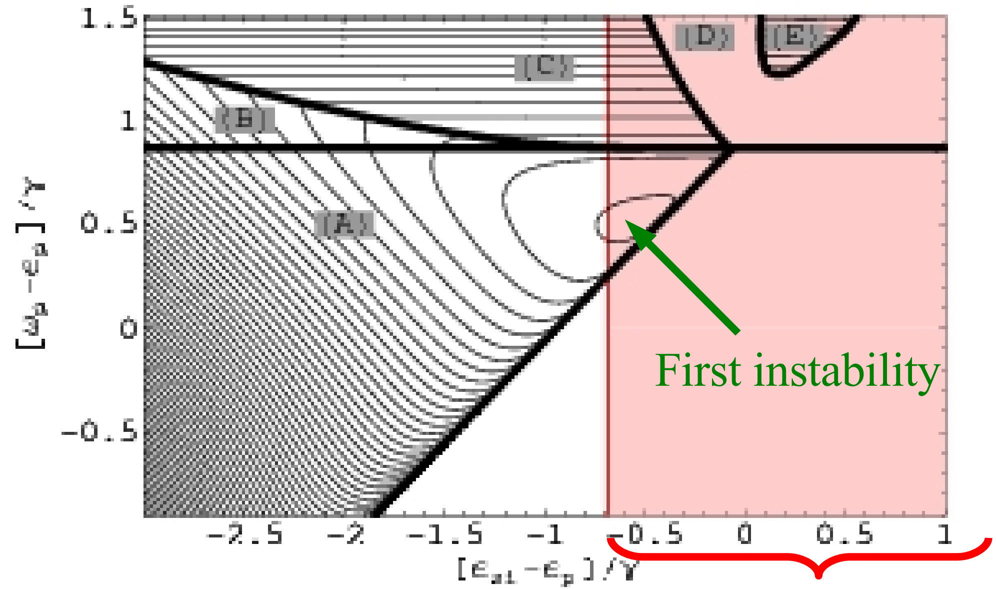
M. Wouters and IC, *Pattern formation effects in parametric oscillation in semiconductor microcavities*, in preparation

Not only second-order phase transition...

2nd order   mixed transition



Contours: value of threshold intensity



Available region depends on pump angle

"Magic angle" condition slightly modified !!

M. Wouters and IC, *The parametric oscillation threshold of semiconductor microcavities in the strong coupling regime*, PRB **75**, 075332 (2007)

Conclusions I: theoretical tools developed

Mean-field theory:
- polariton Gross-Pitaevskii equation developed
- pattern-formation techniques to find spatial profile

To include fluctuations:
- Wigner quasi-probability, stochastic Gross-Pitaevskii equation

Elementary excitations around stationary state:
- linearized Bogoliubov approach
- modulus-phase Bogoliubov for quasi-condensates

We now have a Wigner-MC numerical code able to:
- any geometry, any pulse shape, any applied potential
- no restriction to small fluctuation regime, critical points accessible
- take fully into account spatial dynamics of the field

Simple model of non-resonantly pumping:
- generalized GPE with loss and amplification

Conclusions II: Polaritons provide rich examples of non-equilibrium Bose-Einstein condensates

- coherence functions of luminescence characterized across OPO threshold:

  incoherent (below), diverging coherence length at threshold, long-range coherence (above)

- above threshold: spontaneous breaking of U(1) symmetry
- but: the Goldstone mode is diffusive rather then propagating

  can be optically probed, gap opens if signal phase pinned

- diffusive Goldstone mode also under incoherent pumping
- novel kinds of Josephson effect: standard and overdamped oscillations
- complex shape of non-equilibrium BEC as a function of spot size

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- effect of reduced dimensionality: exponential decay of coherence in 1D
- mean-field study of OPO critical point: first or second order phase transition