Cat States in Ultracold Atoms

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in collaboration with

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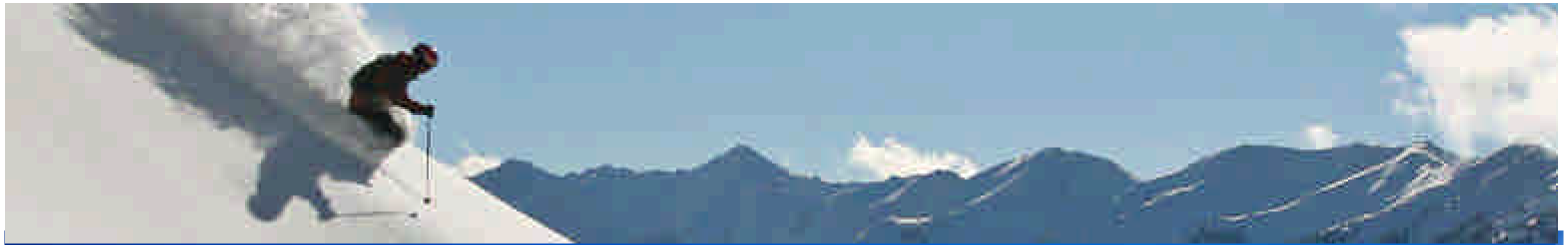
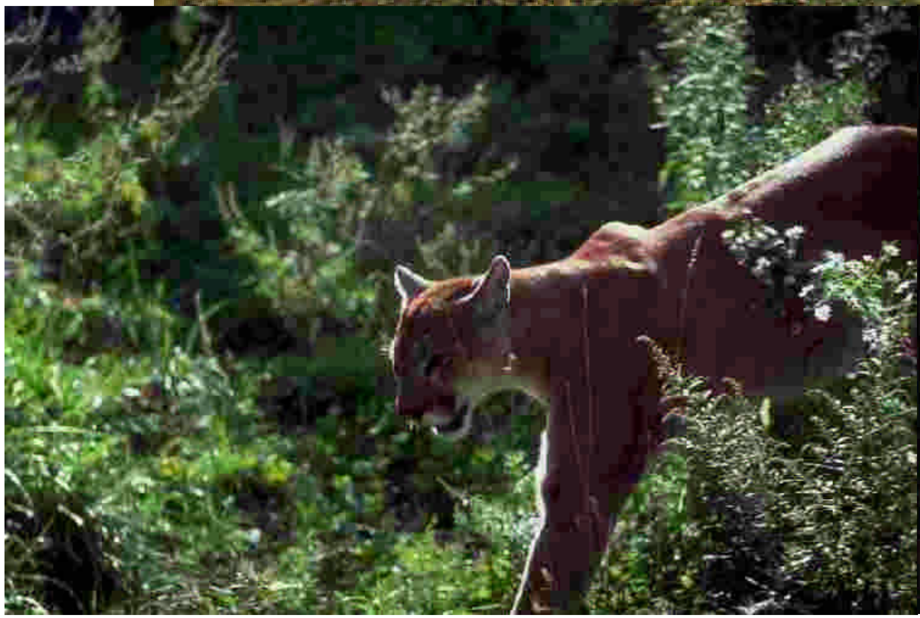
Carr Theoretical Physics Research Group at the Colorado School of Mines

Several groups at Colorado School of Mines looking for new Ph.D. students, including mine
50% of faculty condensed matter…





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Carr Theoretical Physics Group Research at Colorado School of Mines

- Quantum Many Body Physics
  - Designer solid state materials
  - Complex quantum dynamics
  - Macroscopic superposition and quantum tunneling
- Nonlinear Dynamics
  - Fractals in spin waves in thin films
  - Solitons and vortices
  - Ultrafast optical resonators
- Millimeter waves
  - Anderson localization
  - Negative index materials

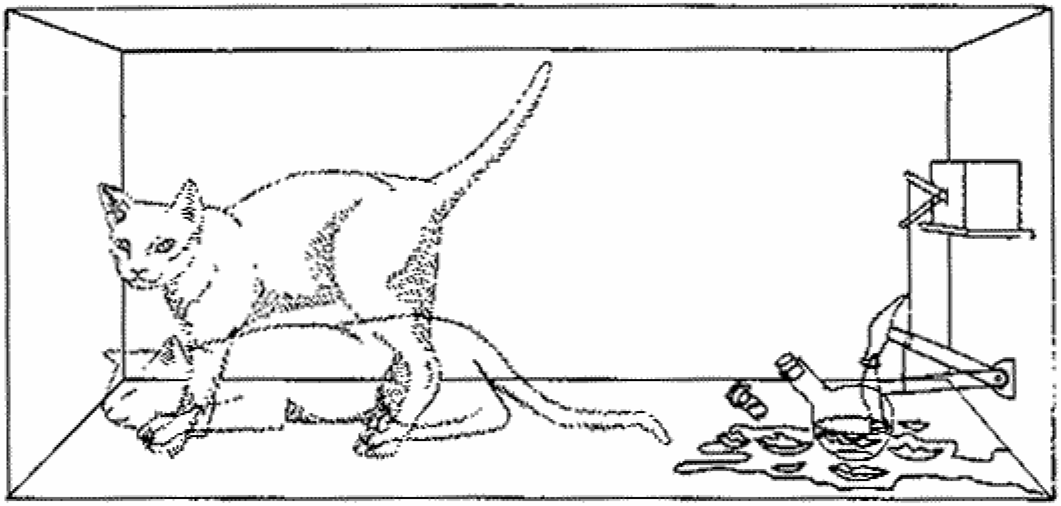
Outline and Major Results

- In classic N-body quantum problem in a double well,
- As realized in a Bose-Einstein condensate
- We will describe Macroscopic Superposition, i.e., Cat states

  - Two new energy scales required
  - Many body wavefunction protects Cats
  - Dynamic scheme to obtain Cat-like states

Schrodinger Cats

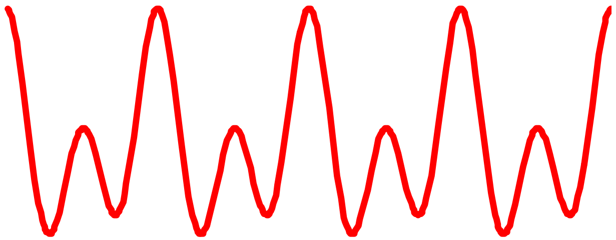
- Mesoscopic Quantum Mechanics: Breakdown?



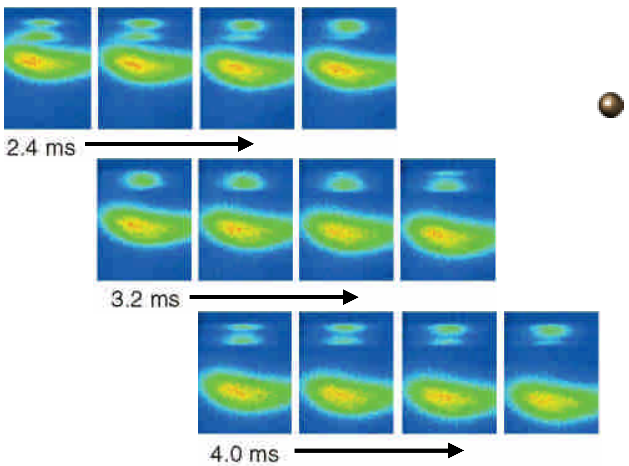
- NOON state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|N,0\rangle \pm |0,N\rangle\right)$$

Cold Atoms in Optical Lattices

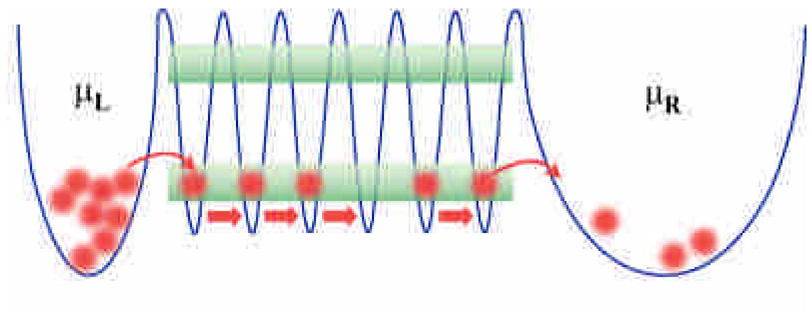


- Quantum information processing
  - Controlled-NOT gates
    - Brennen *et al.*, PRL **82**, 1060 (1999)
  - Lattice of tilted double-wells: 2-qubit gates
    - Calarco *et al.*, PRA **70**, 012306 (2004)
    - Sebby-Strabley *et al.*, PRA **73** , 033605 (2006)



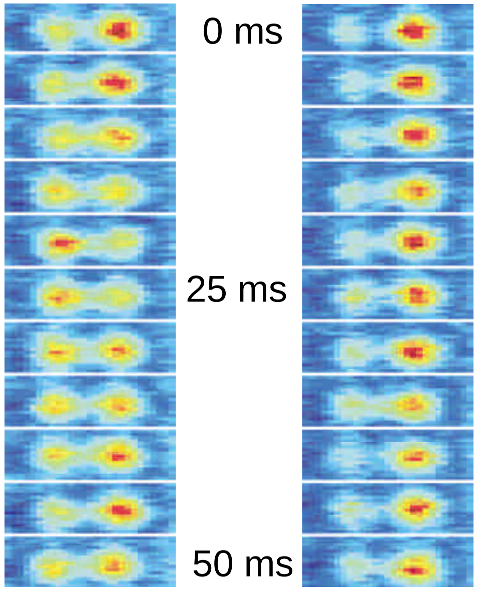
- Atom lasers & Gravitometry
  - Continuous source of coherent matter waves
    - Anderson & Kasevich, Science **282**, 1686 (1998)
  - Applied field → Bloch oscillations
    - Ferrari *et al.*, PRL **97**, 060402 (2006)

- Atomtronic materials and devices
  - P- and N-type materials
    - Seaman *et al.*, PRA, in press



Double-well Potentials

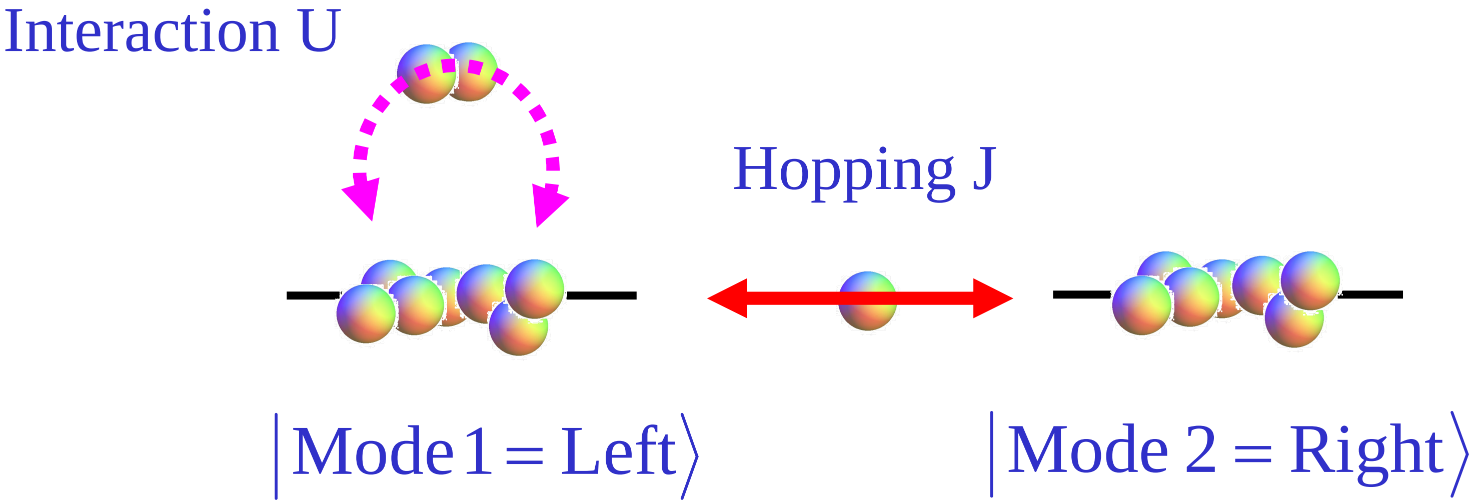
- Practical applications
  - Atom-chip based gravity sensors
    - Schumm *et al.*, Nature Phys. **1**, 57 (2005)
    - Hall *et al.*, cond-mat/0609014 (2006)
  - Primary noise thermometer
    - Gati *et al.*, NJP **8**, 189 (2006)
  - Storage and retrieval of optical information
    - Ginsberg *et al.*, Nature **445**, 605 (2007)



- Fundamental many-body quantum physics
  - Macroscopic quantum tunneling
    - Milburn *et al.*, PRA **55**, 4318 (1997)
    - Albiez *et al.*, PRL **95**, 010402 (2005)
  - Creation of Schrödinger cats
    - Mahmud *et* al., PRA **71**, 023615 (2005)
    - Huang *et al.*, PRA **73**, 023606 (2006)

Usual Approach: 2 Parameters

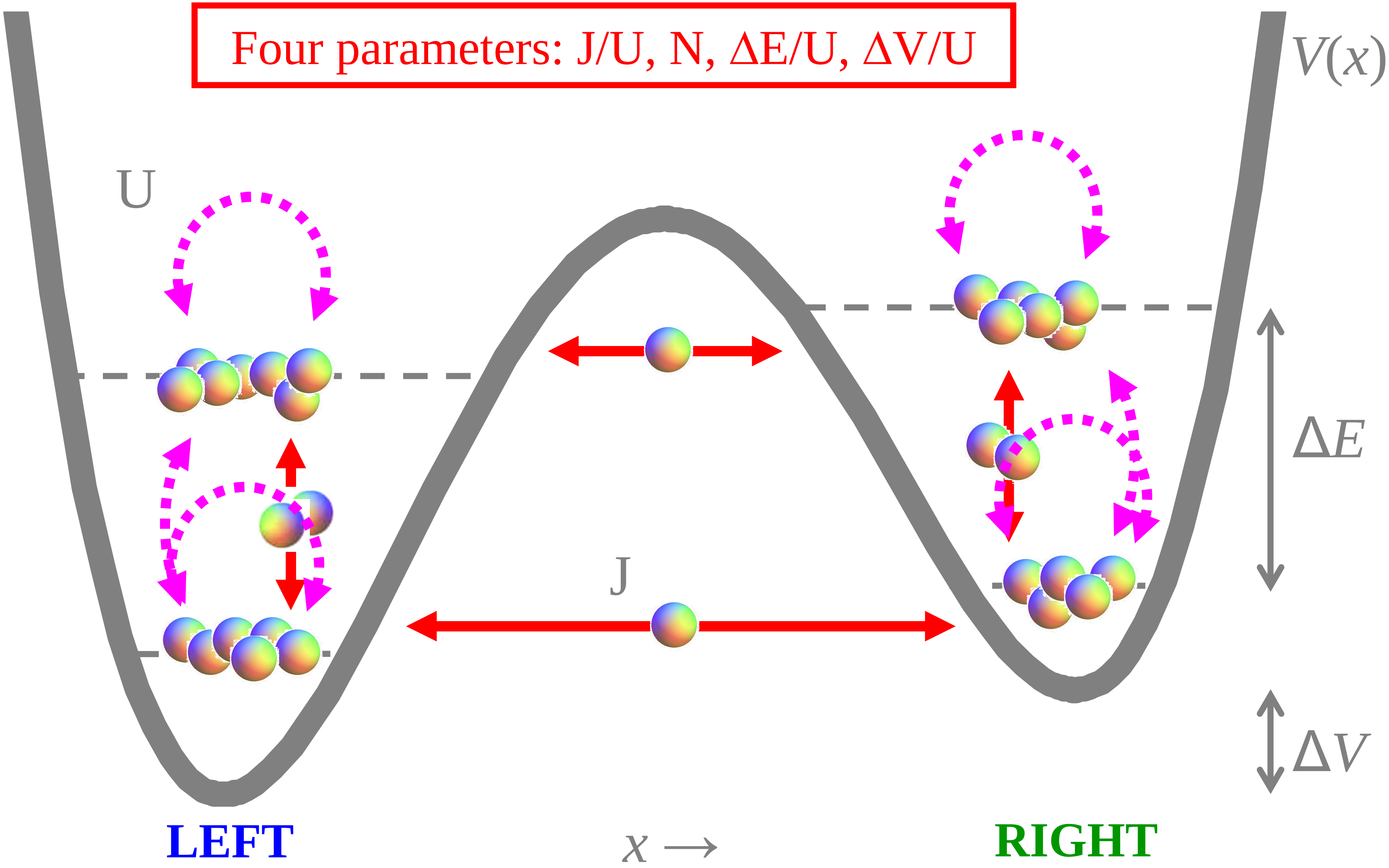
- Originated with Lepkin-Meshkov-Glick model
  - H. J. Lipkin, N. Meshkov, and A. J. Glick, Nucl. Phys. 62, 188 (1965)
  - Nuclear Spin ensemble translates to N-body problem in 2 modes



  - Two parameters: J/U, N

Our Physical Model

Four parameters: J/U, N, ΔE/U, ΔV/U



1D Hamiltonian, 4 Modes

- $\hat{H} = \hat{H}^0 + \hat{H}^1 + \hat{H}^{01}$
- $\hat{H}^\ell \equiv -J^\ell \sum_{j\neq j'} \hat{b}_j^{\ell\dagger}\hat{b}_{j'}^\ell + U^\ell \sum_j \hat{n}_j^\ell\left(\hat{n}_j^\ell - 1\right) + (\Delta V/2)\left(\hat{n}_L^\ell - \hat{n}_R^\ell\right) + E^\ell\left(\hat{n}_L^\ell + \hat{n}_R^\ell\right)$ } Single Level
- $\hat{H}^{01} \equiv U^{01} \sum_{j,\ell\neq\ell'}\left(2\hat{n}_j^\ell\hat{n}_j^{\ell'} + \hat{b}_j^{\ell\dagger}\hat{b}_j^{\ell\dagger}\hat{b}_j^{\ell'}\hat{b}_j^{\ell'}\right)$ } Inter-level

- J = Hopping
- U = Interaction
- ΔV = Tilt
- E = Energy level offset

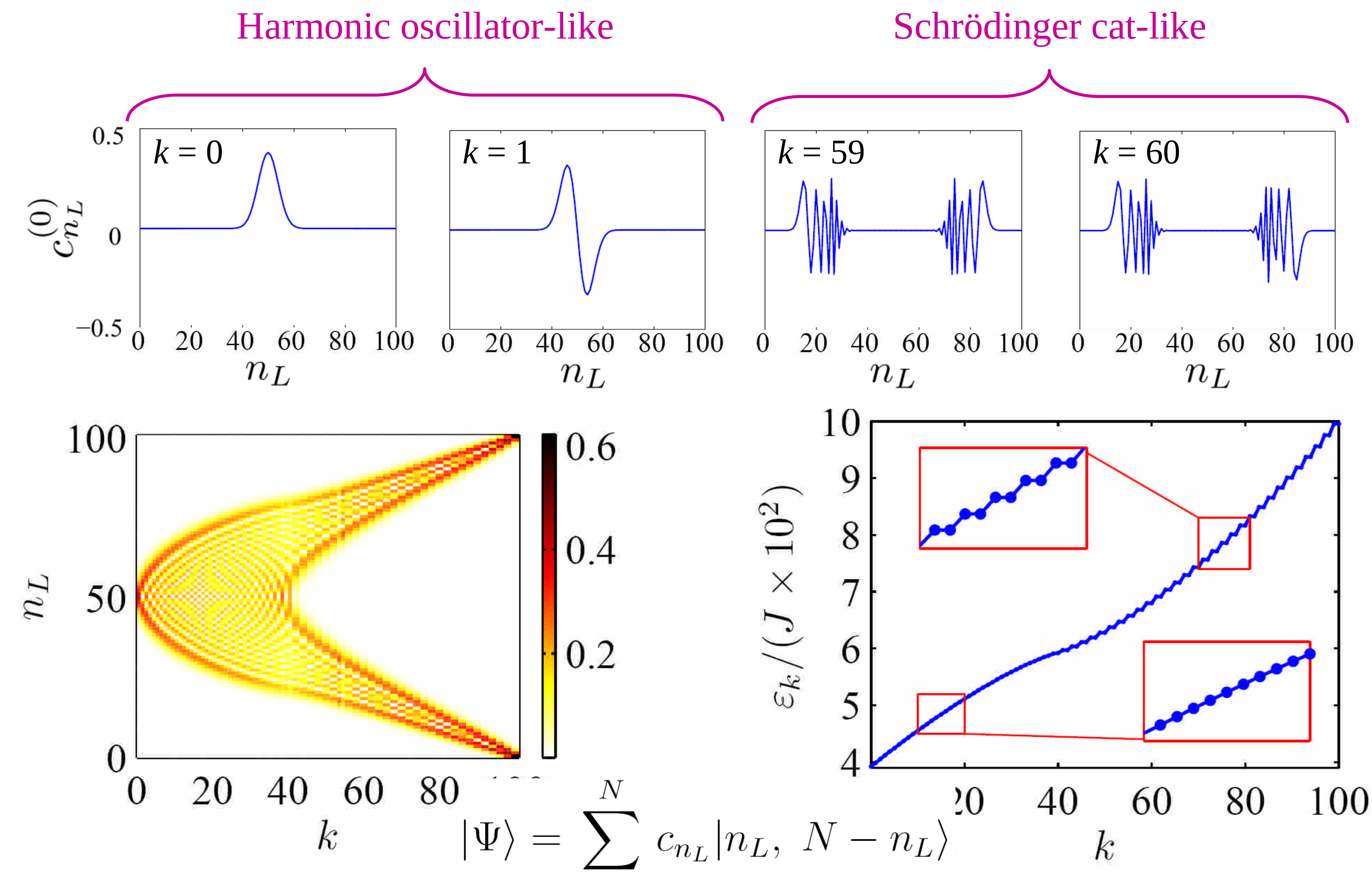
} → Three parameters + N

$\ell, \ell' \in \{0, 1\}$ are the energy level indices

$j, j' \in \{L, R\}$ are the well or site indices

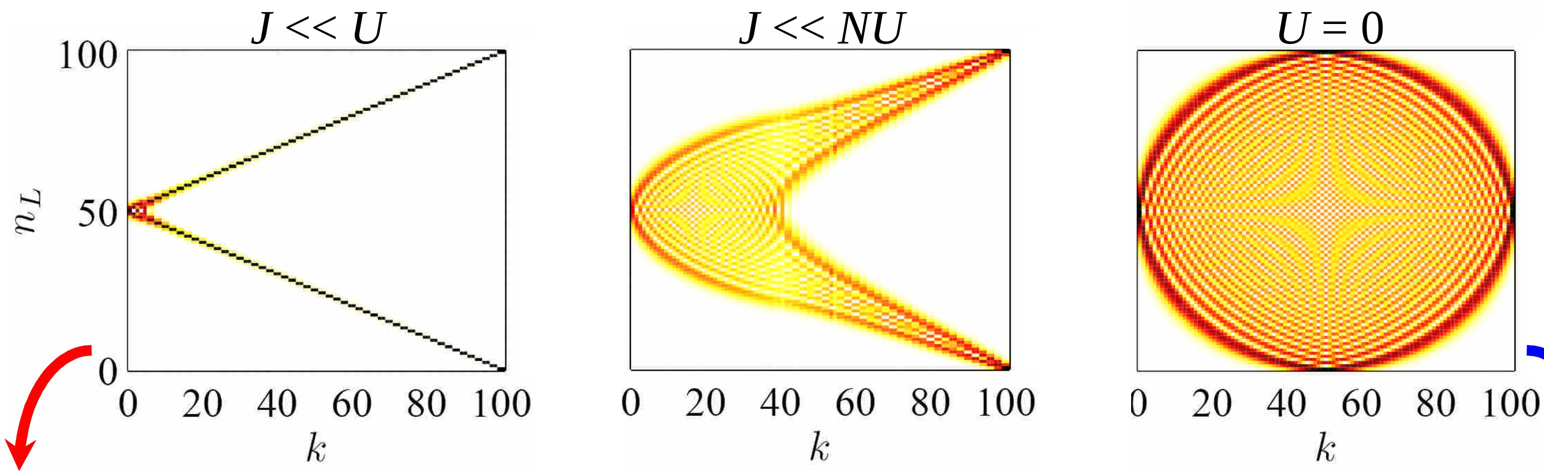
$J^0 \ll J^1 \ll E^1$, $U^1 = (1/2)U^0$ and $U^{01} = (3/4)U^0$

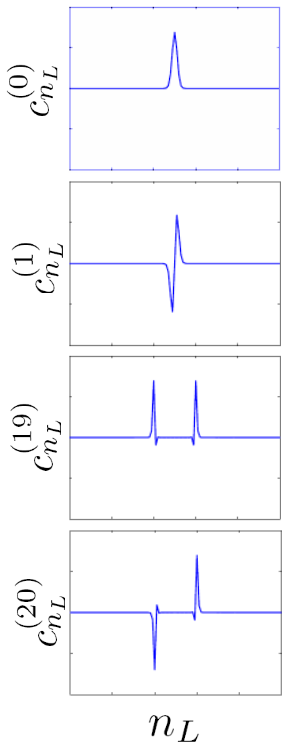
Stationary states of the one-level Hamiltonian



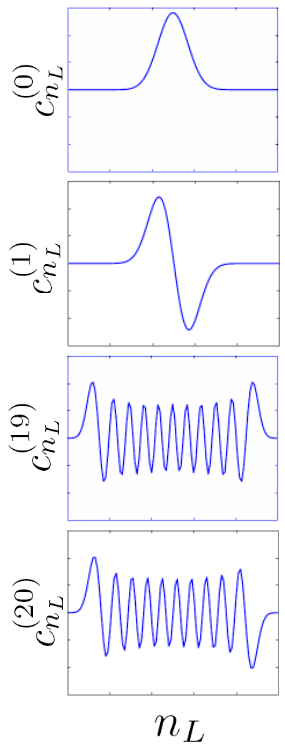
$$|\Psi\rangle = \sum_{n_L=0}^{N} c_{n_L} |n_L,\ N-n_L\rangle$$

Eigenstates: Effect of Barrier Size



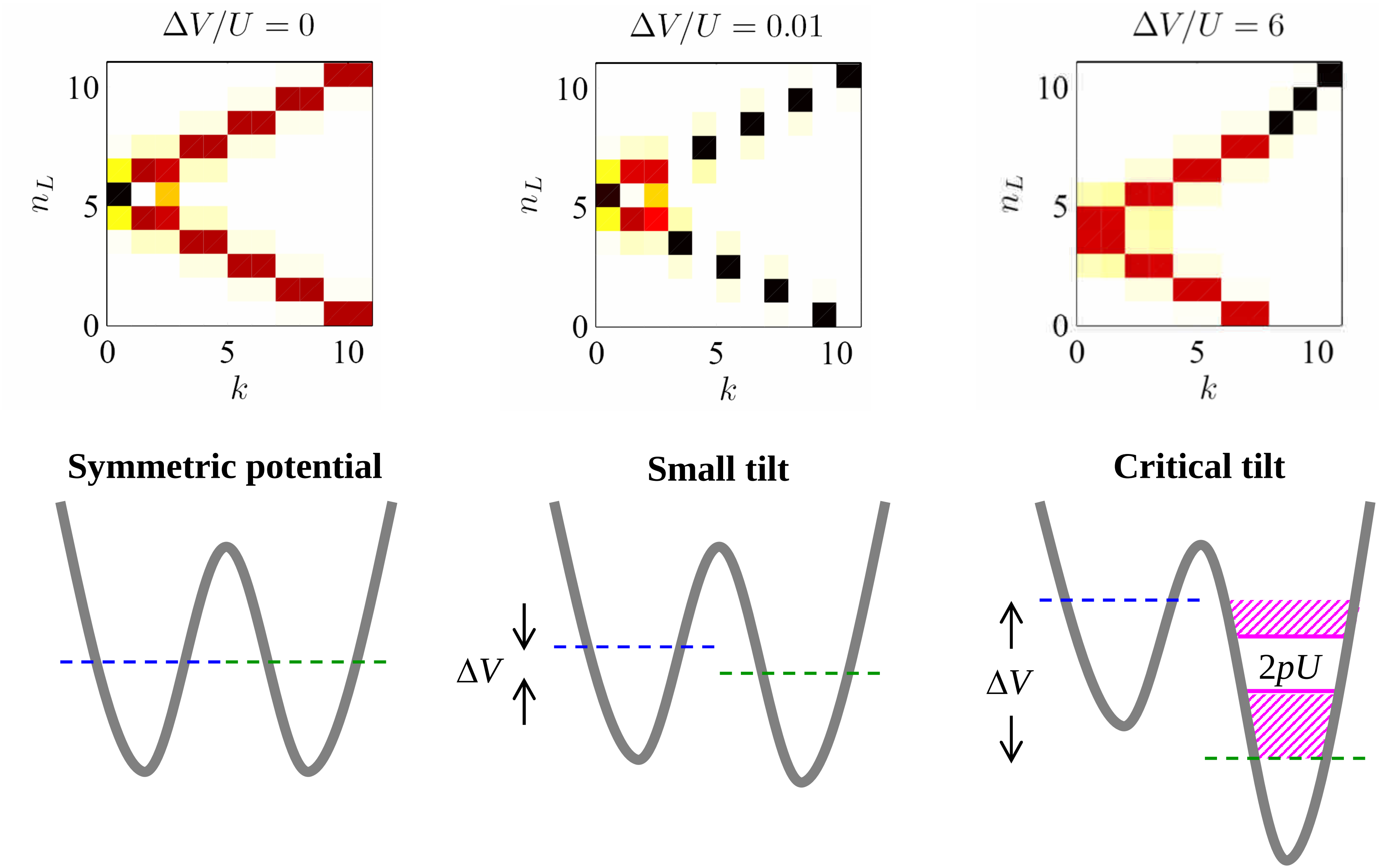


Decreasing barrier (more tunneling)

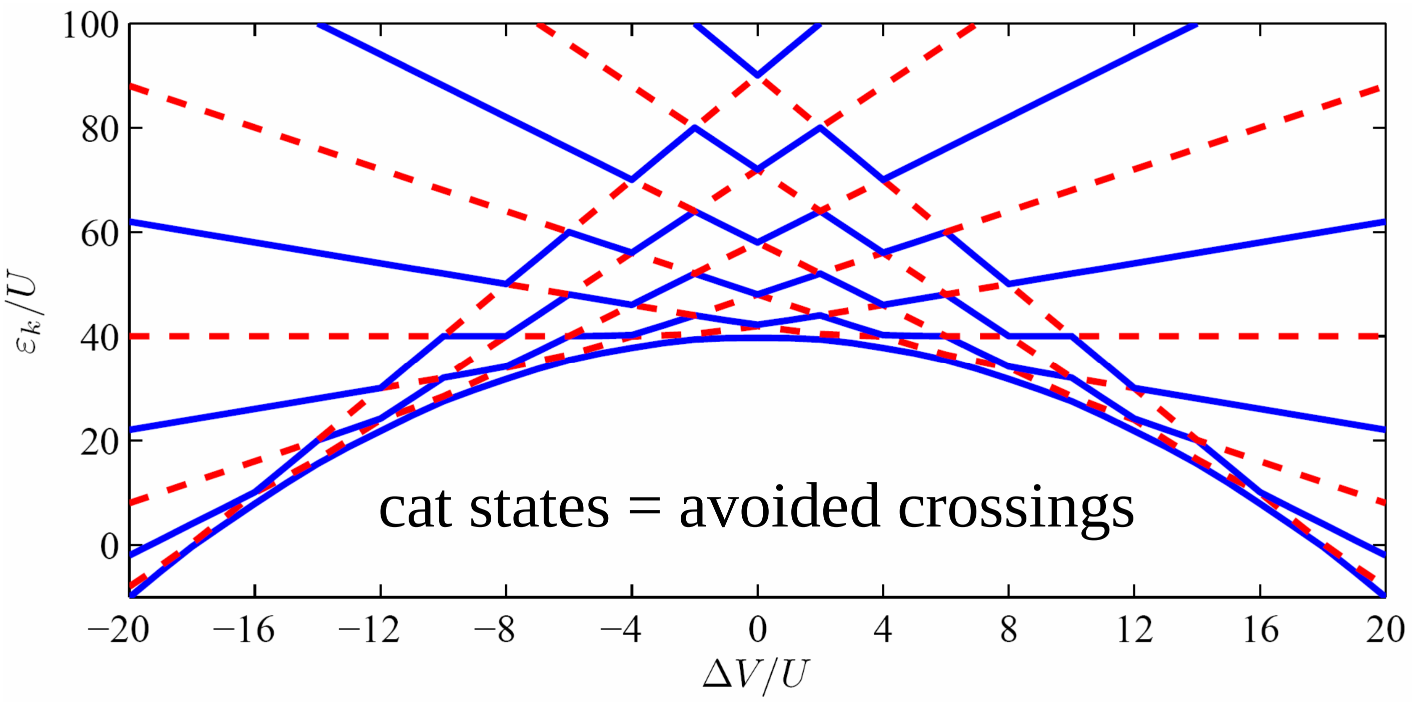


- High barrier, $J << U$
  - Schrödinger cat-like states
- Medium barrier, $J << NU$
  - Both types of states
- Low barrier, $J >> NU$
  - Oscillator-like states

Tilt and Potential Decoherence



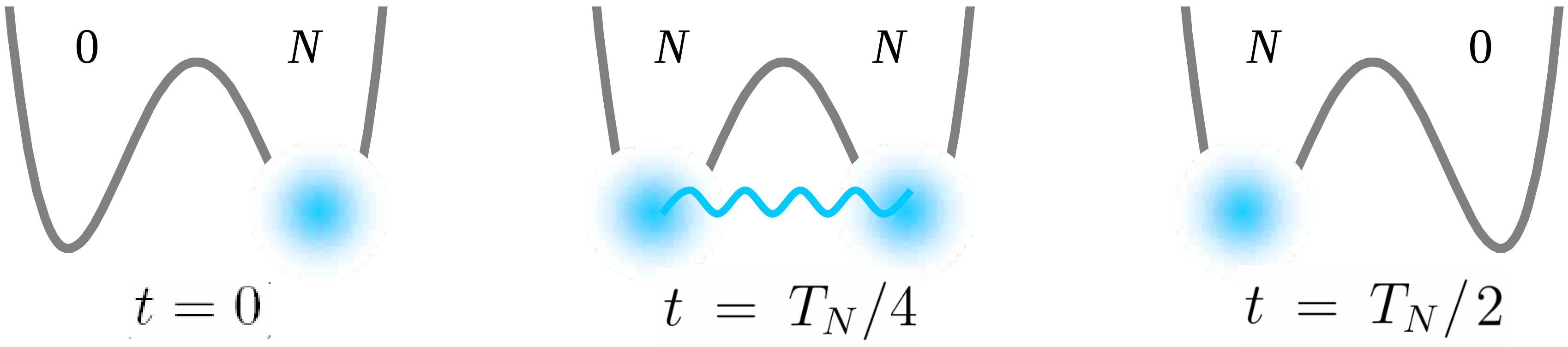
Energy Level Diagram: Effect of Tilt



- Collapse of cat states when $\Delta V > \dfrac{2\Delta\varepsilon_{N-n_L}}{N-2n_L}$
  - Potential decoherence induced by slight misalignment of energy levels

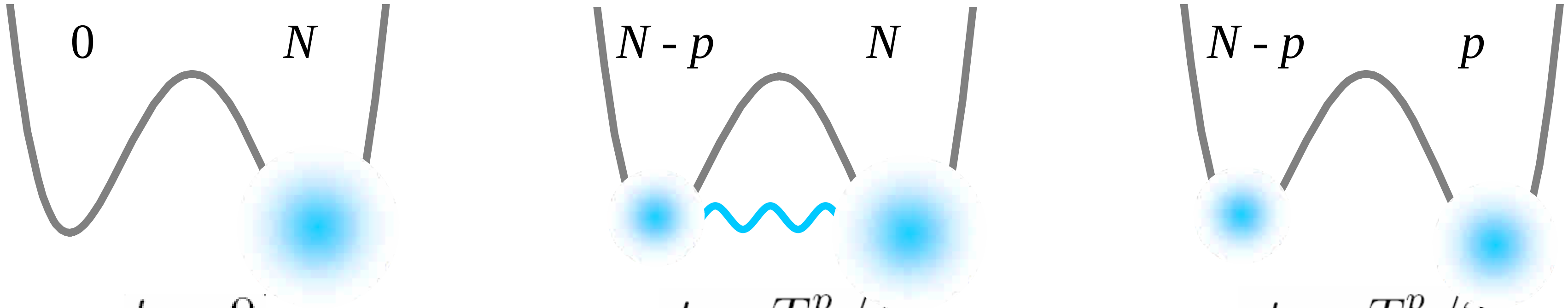
- Cat-like states reappear when $\Delta V = \Delta V_p \equiv 2pU, \quad p \in \{1, 2, \ldots, N-1\}$
  - Cause tunneling resonances in high barrier dynamics

Dynamic Scheme to Obtain Schrodinger Cats



- All atoms initially occupy right well: $|\psi(t=0)\rangle = |0,\ N\rangle$
- Two-state system: $|\psi(0)\rangle = \left(|\phi_+;0\rangle - |\phi_-;0\rangle\right)/\sqrt{2}$
- Tunneling frequency and period of oscillation:
  $\omega_N = \Delta\varepsilon_N/\hbar$ and $T_N \equiv 2\pi/\omega_N$
- At quarter period, system in cat state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|N,0\rangle \pm |0,N\rangle\right) = |\phi_\pm;0\rangle$

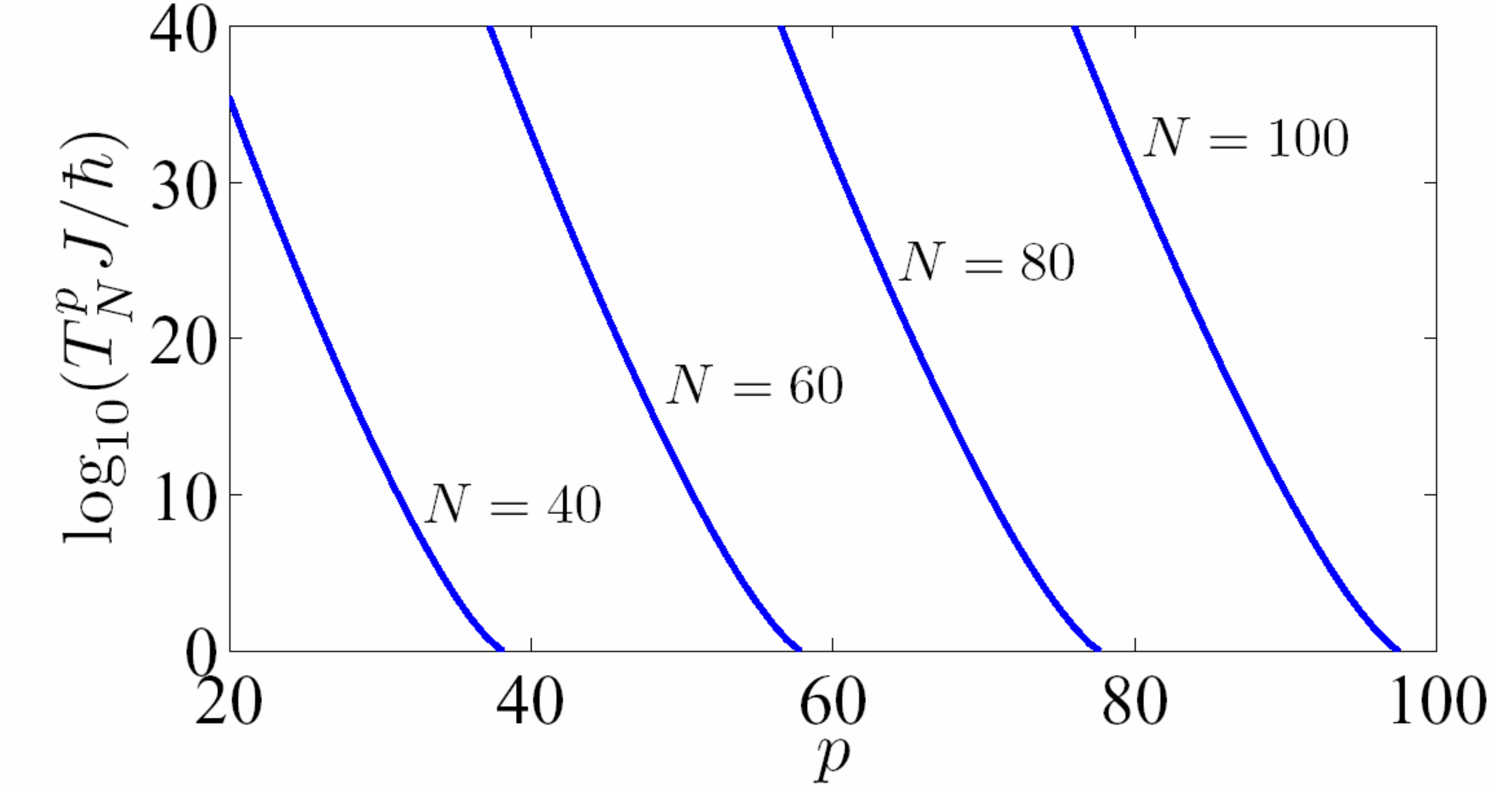
Tunneling in a Tilted Potential



$t = 0$ $\quad$ $t = T_N^p/4$ $\quad$ $t = T_N^p/2$

- Initial state is superposition of $|\phi_\pm; 0, p\rangle \equiv (|N-p,\, p\rangle \pm |0,\, N\rangle)/\sqrt{2}$
  - Level splitting is $\Delta\varepsilon_N^p = \dfrac{4U(J/2U)^{N-p}(N-p)}{(N-p-1)}\sqrt{\dfrac{N!}{p!(N-p)!}}$
- Tunneling frequency: $\omega_N^p = \Delta\varepsilon_N^p/\hbar$
- Only *N* - *p* atoms tunnel between wells
  - *p* atoms remain in lower well to compensate tilt
- At quarter period, system in partial Schrödinger cat state

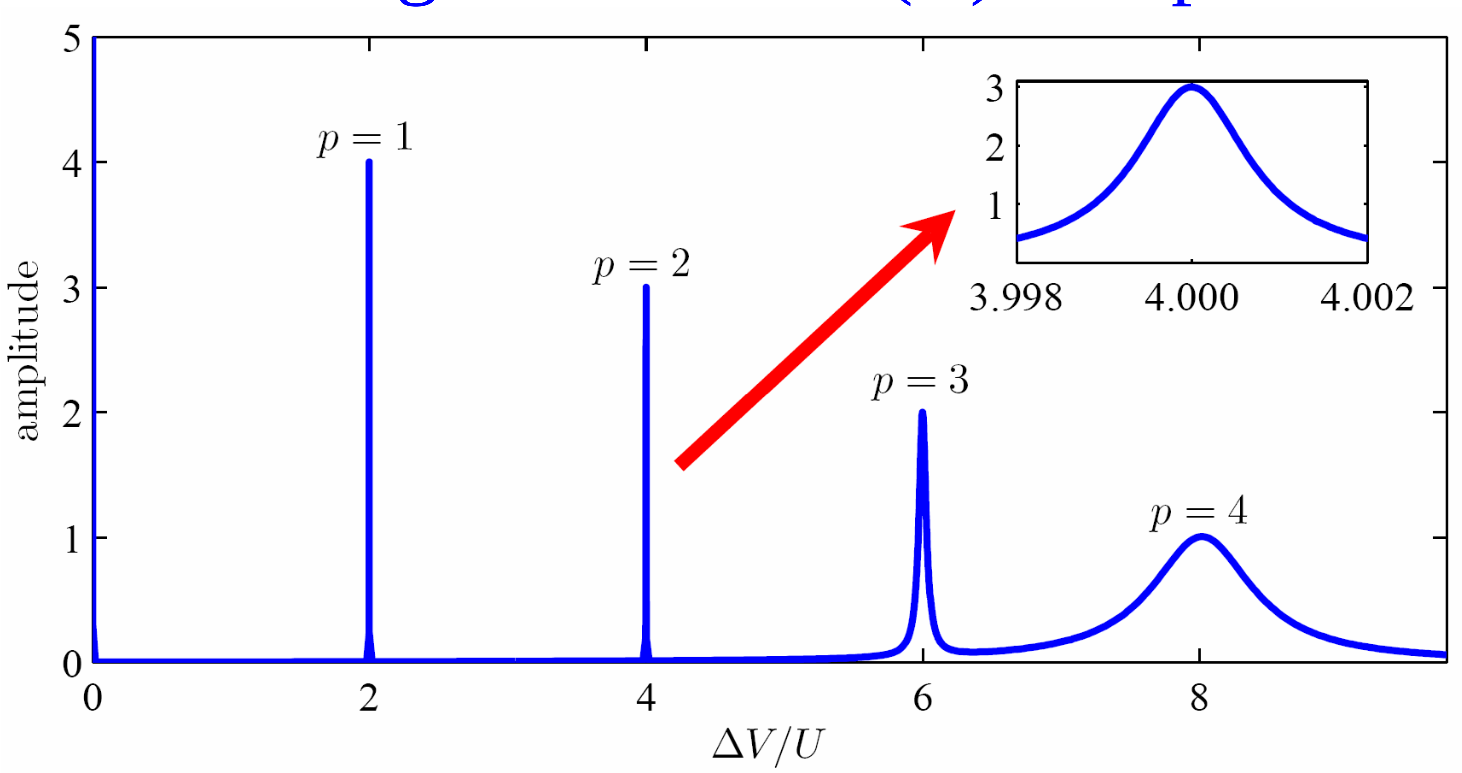
Tunneling Resonances (I): Period



- Tunneling times in a symmetric potential prohibitively long
  - "Self trapping" of Oberthaler experiment

- Tunneling at resonance is hundreds of orders of magnitude FASTER!

Tunneling Resonances (II): Amplitude



- Width of resonance = width of avoided crossing
  - Resonant tunneling suppressed when

$$|\Delta V - \Delta V_p| > 2\Delta\varepsilon_N^p/(N-p)$$

- Tunneling at resonance MUCH MORE ROBUST!

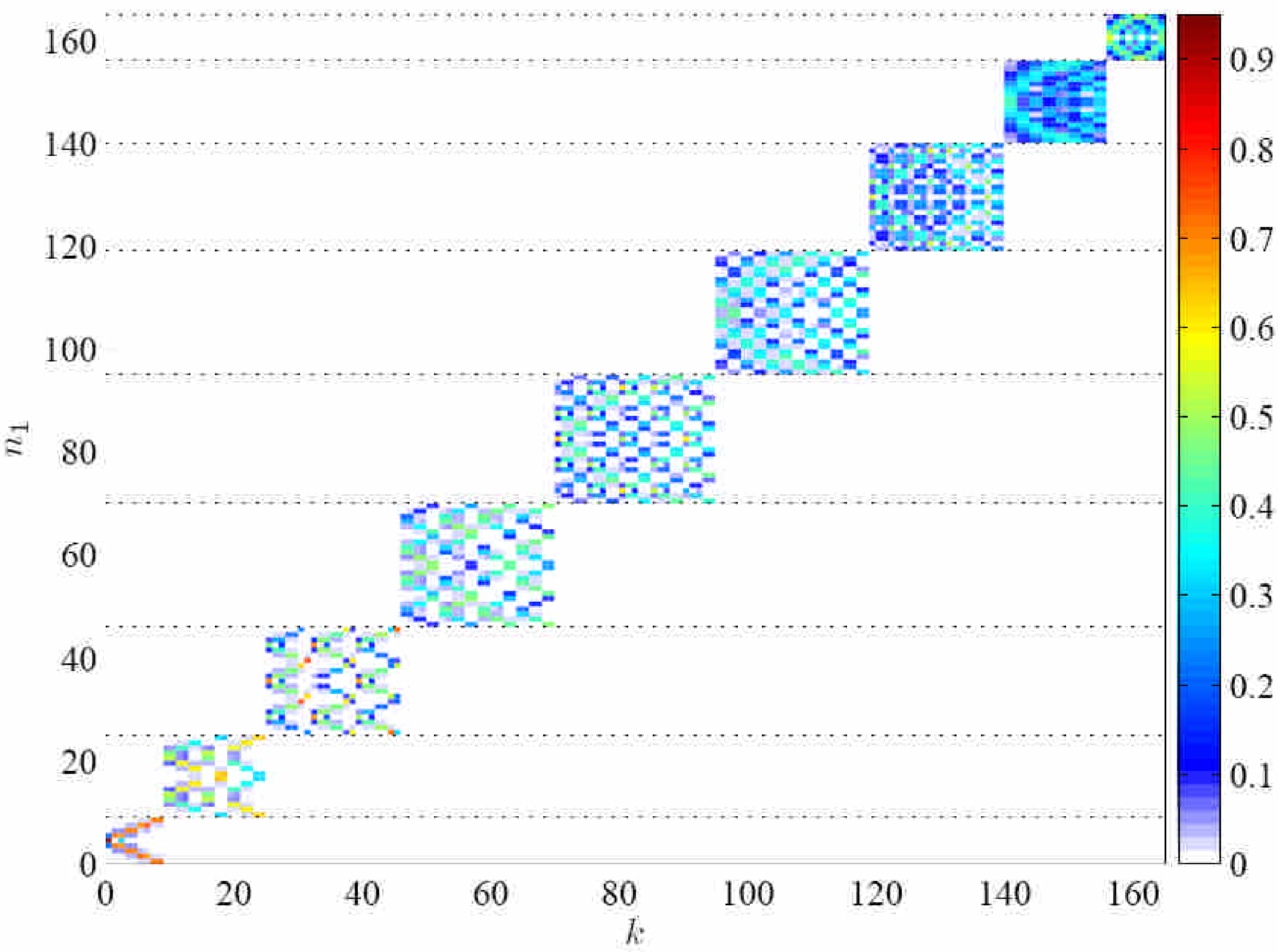
A Few Numbers

- For N=200 atoms
  in ^{87}Rb tilted double well experiment at NIST
  - $T=1.15 \times 10^{635}$ ms
  - Tunneling suppressed for $\Delta V=4.16 \times 10^{-636}$ nK k_B

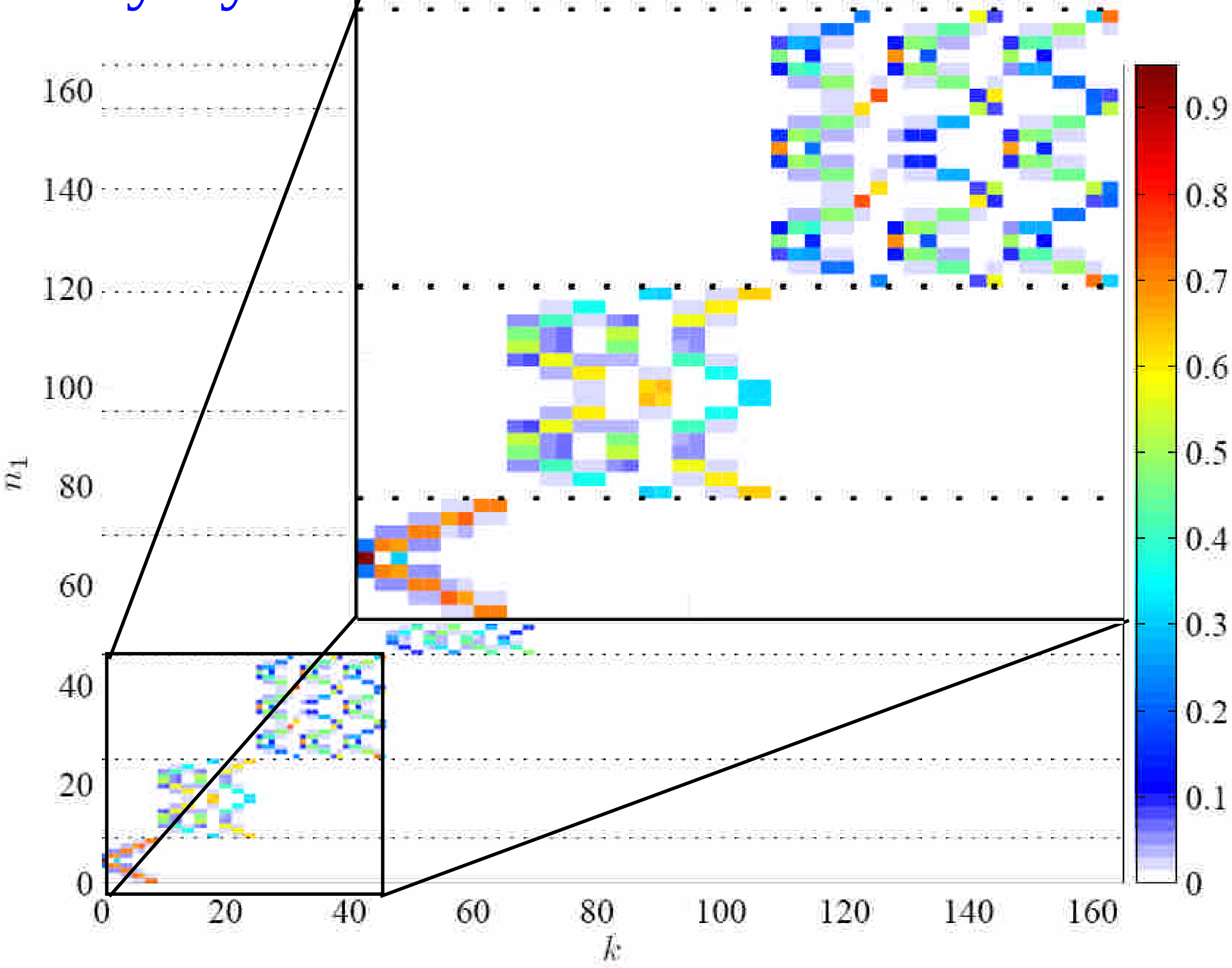
- For N=1,2,3 atoms
  - Tunneling times as long as T=466, 4840, 134000 ms
  - Not experimentally realistic…

- For p=199,198,197 (i.e. N-p=1,2,3)
  - T=33.0,34.3,117 ms
  - Tunneling suppressed for $\Delta V=2.90,1.40,0.273$ nK k_B

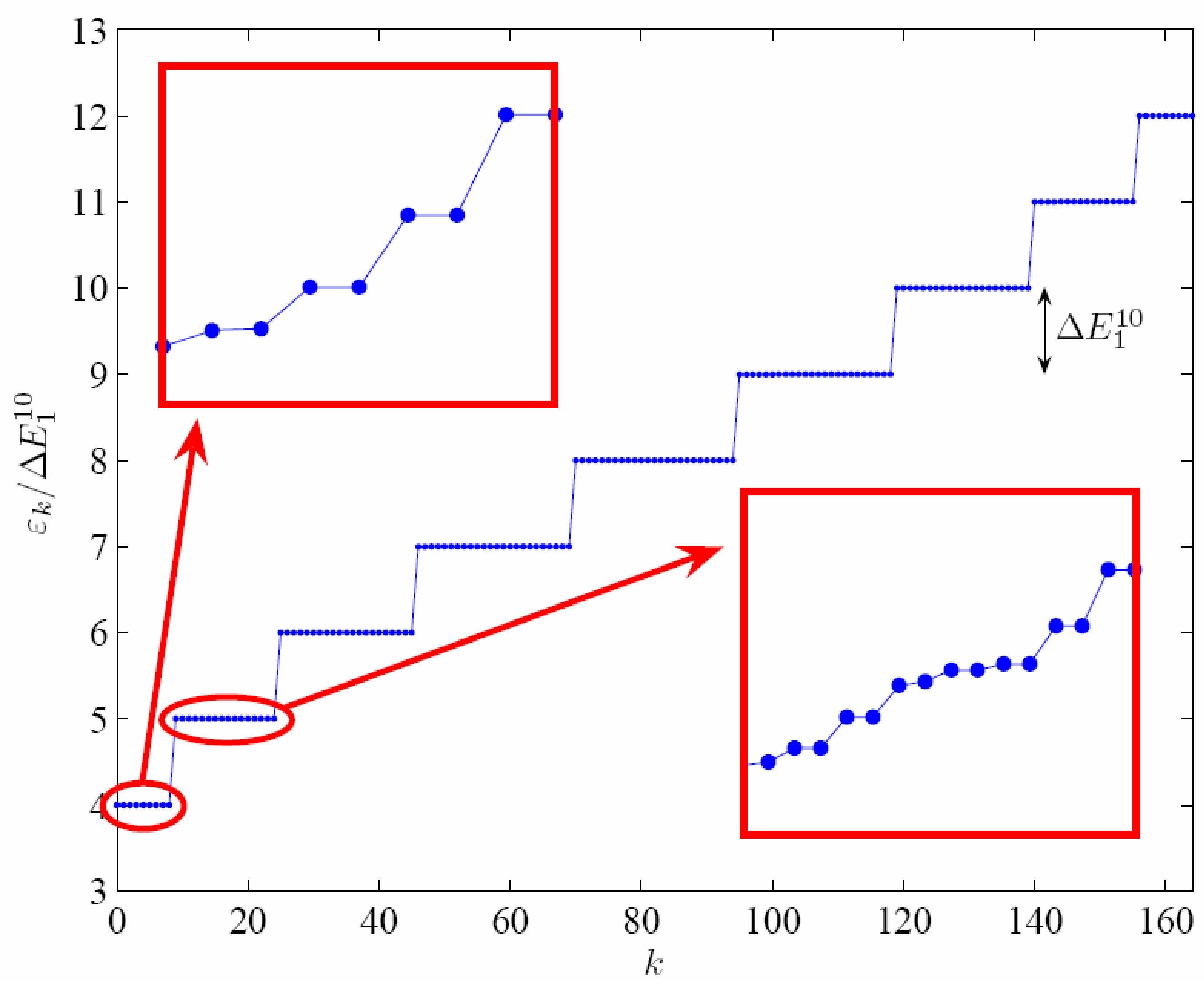
Fly-by View of Four Mode Eigenspace



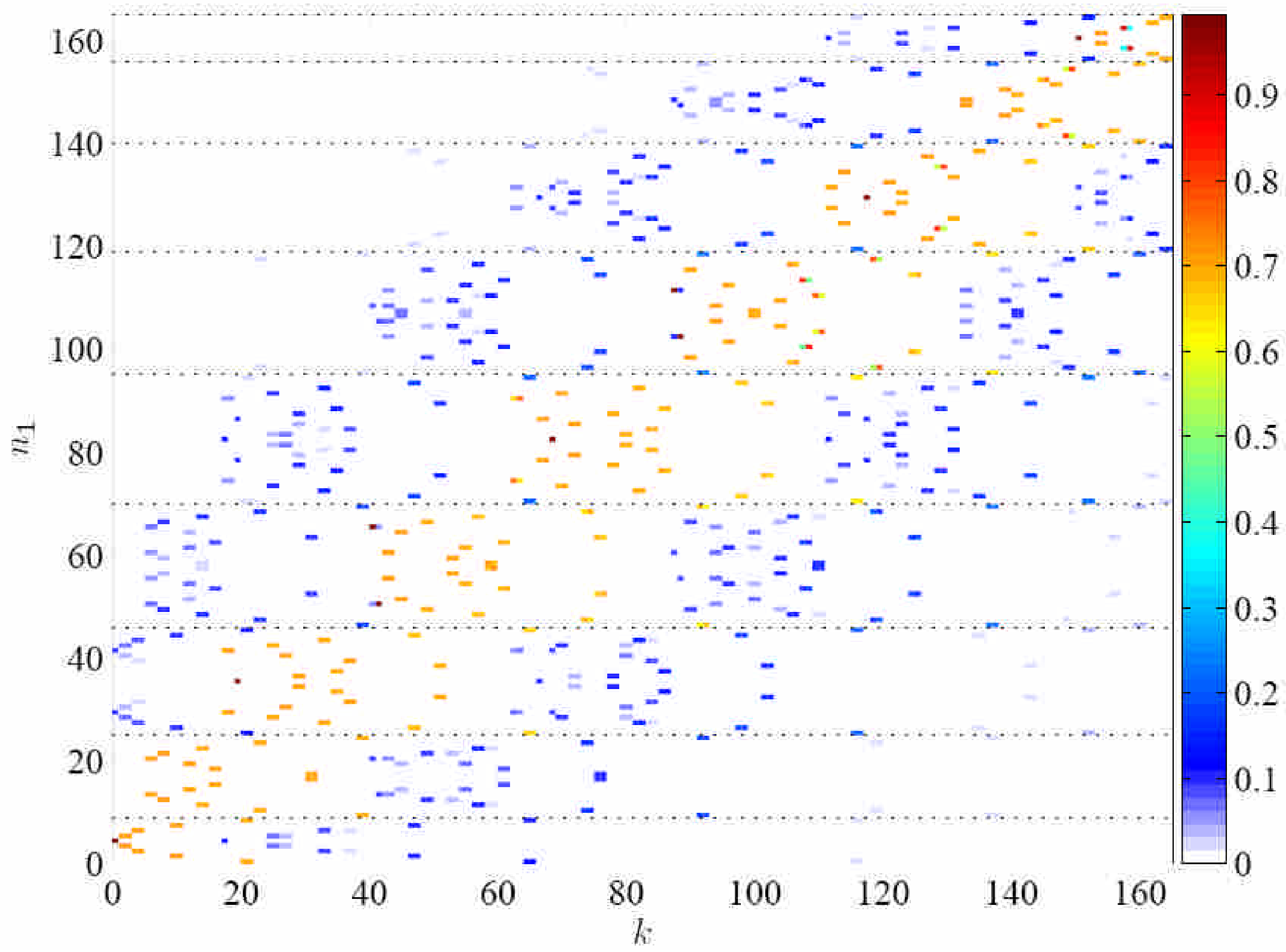
Fly-by View of Four Mode Eigenspace



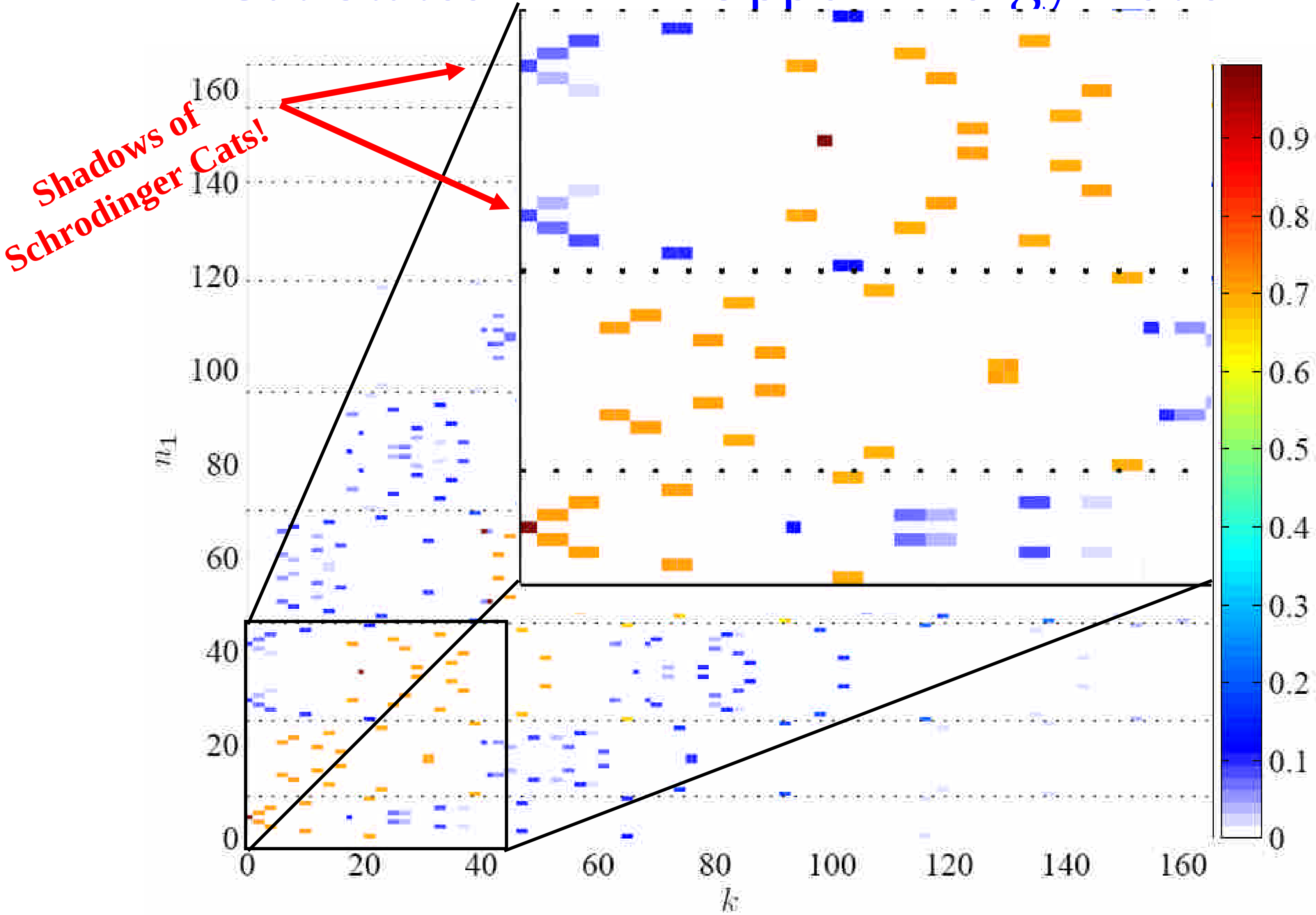
Separation of Eigenstates



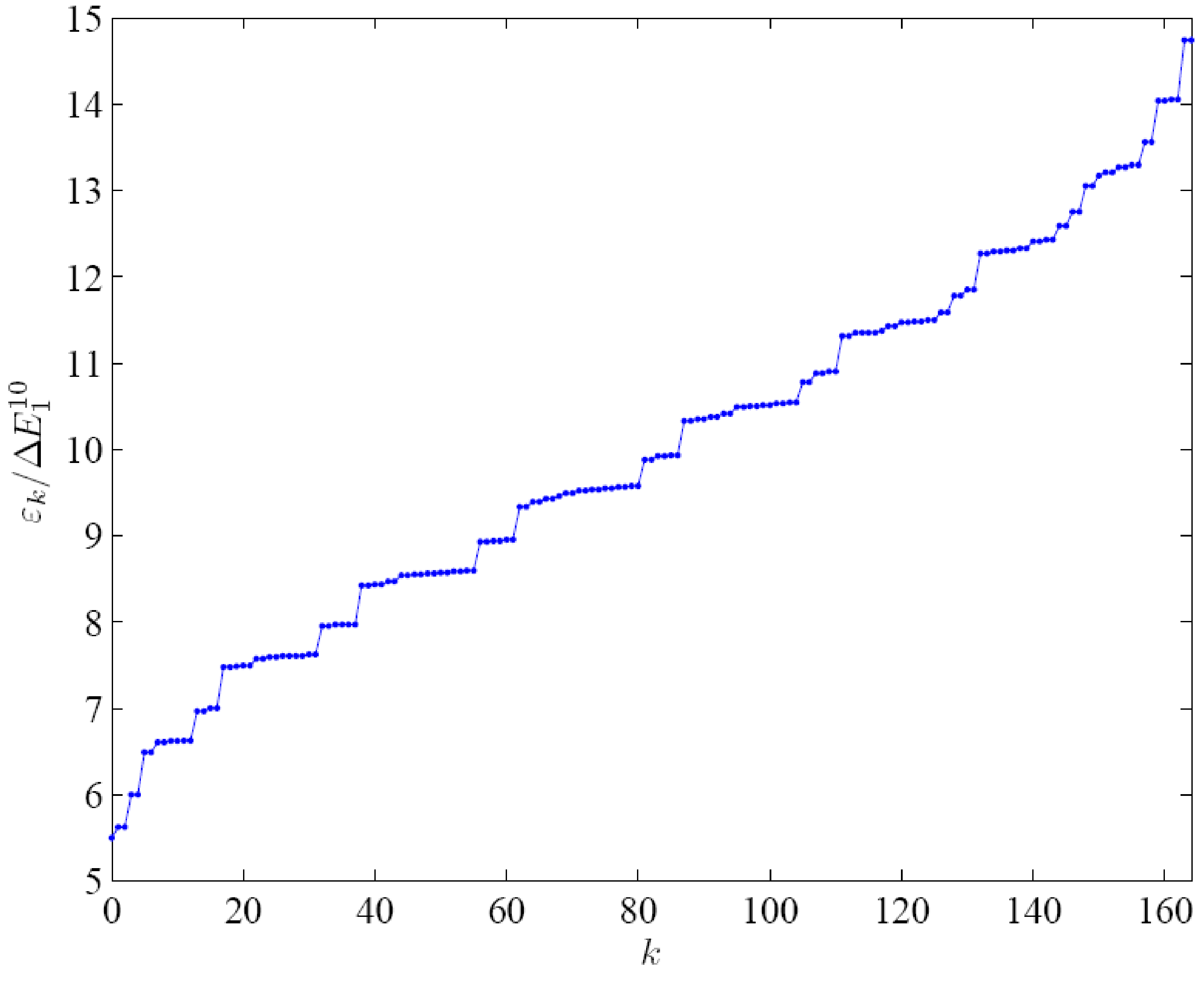
Cat States Mix in Upper Energy Level



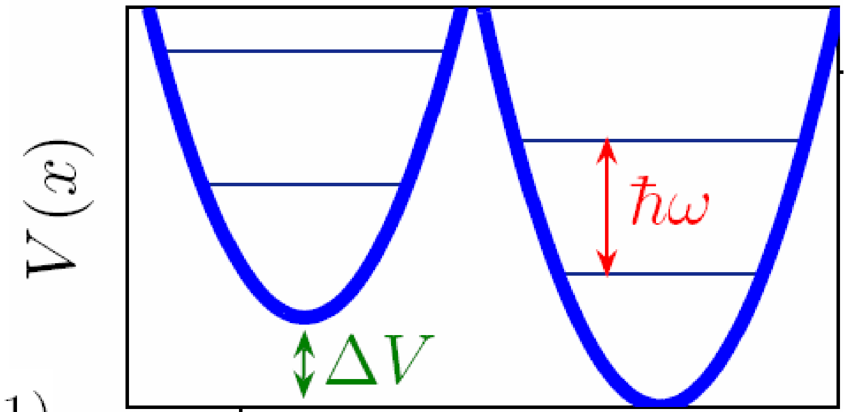
Cat States Mix in Upper Energy Level



Mixing in Spectrum



Formal Bounds on use of Two Mode Model



- From high barrier limit:
  - $U^0 < U^0_{\rm crit} \equiv 2\hbar\omega/(N^2-1)$
  - $|\Delta V| < \Delta V_{\rm crit} \equiv [2\hbar\omega - (N^2-1)U^0]/(2N)$
- From low barrier limit:
  - $N < 1/2 + (\hbar\omega - J^1)/(2J^0)$
- Bounds hold in 1D, 2D, or 3D
- E.g. $N = 100,\ \hbar\omega = 9.7\,\mu\mathrm{K},\ J^0/\mathrm{U}^0 = 0.098$

  $\Rightarrow U_0^{\rm crit} = 1.9\,\mathrm{nK}, \Delta V_{\rm crit} = 76\,\mathrm{nK}$

Conclusions

- Two new energy scales required
  - Tilt, Potential level difference
  - Formal Bounds: $N^2U,\ NJ \leftrightarrow \hbar\omega$
- Many body wavefunction protects Cats
  - Potential Decoherence
  - $|N,0\rangle \pm |0,N\rangle \rightarrow |N-p,p\rangle \pm |p,N-p\rangle$
- Dynamic scheme to obtain Cat-like states
  - Quantum sloshing
  - $|N-p,p\rangle \pm |0,N\rangle$
    - Dounas-Frazer, Hermundstad, and Carr, arXiv:quant-ph/0609119
    - Dounas-Frazer and Carr, arXiv:quant-ph/0610166