Preparation of a Schrödinger's cat

C. Trefzger, M2 – ICFP

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Consider two Bose-Einstein condensates (BEC) in a double-well potential as shown in figure 1.

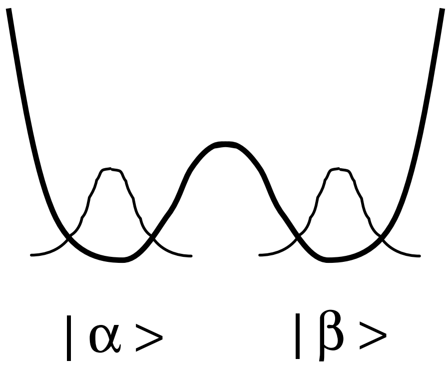

FIG. 1 – Double-well potential with single-particle modes α and β localized respectively in the left well and in the right well.

The system is at zero temperature such that all the atoms occupy the ground state of the double-well ; we will then neglect all excited states. The base of all accessible states is then as follows :

$$|n\rangle = \frac{1}{\sqrt{n!(N-n)!}} \left(\hat{a}^\dagger\right)^n \left(\hat{b}^\dagger\right)^{N-n} |0\rangle, \tag{1}$$

where n is the number of atoms in the left well and N the total number of atoms present in the system. The interactions between the atoms can be modeled by a zero-range potential $V(\mathbf{r}-\mathbf{r}') = g\delta(\mathbf{r}-\mathbf{r}')$.

1. Write the Hamiltonian of the system in terms of the wave functions of the ground state of the double-well and in terms of the coupling constant g.
2. Describe the spectrum of the system in the case where no tunneling is present between left and right well (infinite barrier limit). Are there degenerate levels ?
3. We lower the barrier between the left and the right well and therefore add a tunneling term between the two condensates. Describe the corresponding term of the Hamiltonian as a function of the tunneling amplitude J. Which transition $|n\rangle \to |n'\rangle$ are induced by this coupling ?

4. Use the method of the resolvent, write the effective Hamiltonian $H_{\text{eff}}(z)$ in the subspace $\mathcal{H}_P = |n=0\rangle \oplus |n=N\rangle$ in the limit where the separation between the levels $|n\rangle$ is large compared to the tunneling amplitude J. We will use, after having demonstrated it, the perturbative development of the displacement operator :

$$R(z) = V + V\frac{Q}{z - QH_0Q}V + V\frac{Q}{z - QH_0Q}V\frac{Q}{z - QH_0Q}V + \cdots \tag{2}$$

where we will keep only the dominant terms in V.

5. Using $H_{\text{eff}}(z)$ describe the splitting of degeneracy κ between the states $|n=0\rangle$ and $|n=N\rangle$.

6. Calculate the time evolution of the system starting from the initial state $|n=0\rangle$. What is the state of the system at $\kappa t/\hbar = \pi/4$? Give a physical interpretation.

7. What is the limit of validity of this effective Hamiltonian description of the system ? We give the following Stirling approximation for large N :

$$N! \sim N^N e^{-N}\sqrt{2\pi N}. \tag{3}$$